ROTATION AND STELLAR INTERIORS

(Review Paper)

IAN W. ROXBURGH

Queen Mary College, University of London, England

1. Introduction

Research into the effect of rotation on the internal structure of stars has a long and detailed history. The names of McLaurin, Jacobi, Kelvin and Jeans are associated with detailed work on the structure and stability of rotating liquid masses assuming uniform rotation. Much of this work is summarized in Jeans' (1929) *Astronomy and Cosmogony* and more recent summaries are given in Lyttleton (1953) and Lebovitz (1967).

As more was discovered about the physical conditions inside stars it became clear that stars were gaseous and compressible and Jeans (1929) also considered the effect of uniform rotation on polytropic gases, that is a gas whose pressure and density are related by a power law $P = K\varrho^{1+(1/n)}$. In particular Jeans showed, in an analysis remarkable for its inaccuracies, and even more remarkable for the accuracies of its results, that a uniformly rotating polytrope has a point of bifurcation, that is, admits a non-axially symmetric configuration if $n < 0.8$. For n larger than this the equatorial regions reach Keplerian velocities and the equilibrium series terminates.

During the same period, it became apparent due to the pioneering work of Eddington that in large portions of a star energy was transported by radiation not by convection. This led Milne (1923) and von Zeipel (1924) to consider uniformly rotating radiative stars and led through von Zeipel's theorem to the discovery of internal circulation in stars by Vogt (1925) and Eddington (1925, 1929). The origin of such circulation is easy to see. The hydrodynamic energy and mass conservation equations for a rotating gaseous star are

$$\varrho \frac{\partial v^i}{\partial t} + \varrho v^k \frac{\partial v^i}{\partial x^k} = -\frac{\partial p}{\partial x^i} + \varrho \mathbf{g} + \frac{\partial}{\partial x^j}\left[\eta\left\{\left(\frac{\partial v^i}{\partial x^i} + \frac{\partial v^j}{\partial x^i}\right) - \frac{2}{3}\frac{\partial v^n}{\partial x^k}\right\}\right]$$

$$\varrho T \frac{\partial S}{\partial t} + \mathbf{v}\cdot\nabla S = \nabla\cdot\mathbf{F} - \varepsilon\varrho$$

$$\frac{\partial \varrho}{\partial t} + \nabla\varrho\mathbf{v} = 0$$

where $\mathbf{v}$ is the velocity, S the entropy, η the coefficient of viscosity and the other symbols have their usual meaning. The radiative flux F is given by

$$F = -C\nabla T, \quad C = \frac{4ac}{3}\frac{T^3}{\kappa\varrho}$$

A. Slettebak (ed.), Stellar Rotation, 9–19.

where κ is a known function of pressure and temperature. Von Zeipel assumed that the only motion was uniform rotation and hence deduced that the energy generation must be

$$\varepsilon = \text{const.}(1 - \Omega^2/2\pi G\varrho),$$

a clearly unphysical result. Vogt and Eddington pointed out that this conclusion followed from assuming the only motion was rotation and that it disappeared if meridional circulation was invoked. The reason for such circulation is simple. If we assume no circulation, then due to the departure from spherical form – and the latitude dependence of pressure, temperature, etc., radiative equilibrium cannot be achieved everywhere. This breakdown forces circulation currents to carry excess energy. The speed of such circulation is also simple to estimate. The breakdown in thermal equilibrium in a star is re-adjusted in the Kelvin-Helmholtz time

$$t_{\text{KH}} = GM^2/RL$$

which is essentially the ratio of the gravitational energy to the rate at which energy is radiated. If the breakdown in thermal equilibrium is due to the departure from spherical form, this time-scale is increased by a factor

$$GM/\Omega^2 R^3$$

giving

$$t_{\text{circ}} = G^2 M^3/\Omega^2 L R^4$$

and a velocity R/t of

$$v_{\text{circ}} = \Omega^2 L R^5/G^2 M^3 .$$

For the sun the circulation time-scale is of the order of 10^{12} years – a lot longer than its evolutionary time-scale. More generally stars on the main sequence have $L \propto M^4$ and as the evolutionary time-scale

$$t_{\text{evol}} \propto M/L \propto 1/M^3$$

we have

$$\frac{t_{\text{circ}}}{t_{\text{evol}}} = \left(\frac{\Omega^2 R^3}{GM}\right)\frac{M}{R} = \lambda 10^3$$

where λ is the ratio of centrifugal force to gravity and M/R is a slowly varying function of mass. For rapidly rotating stars $\lambda \simeq 1$ and the meridional circulation completes several circulations and hence radically alters the angular velocity field.

The original work of Vogt and Eddington has been revised and improved by Sweet (1950) and Baker and Kippenhahn (1959) and a review is given by Mestel (1965).

This discovery of the circulation immediately gave rise to the assumption that rapidly rotating stars were kept chemically homogeneous by the circulation. This is not necessarily true and Mestel (1953) showed that the chemical composition changes prevented such a mixing by showing that the circulation produced a non-spherically symmetric distribution of chemical composition which 'choked' back the circulation.

2. Steady State Solutions

For those stars where the rotation is sufficiently strong to drive circulation currents many times round the star while the star is still on the main sequence, some redistribution of angular momentum will take place and we may expect some steady state distribution to be achieved. Such solutions have been investigated by Schwarzschild (1947) and Roxburgh (1964). The difficulty is that the viscosity in stellar interiors is so small that unless there are enormous gradients of angular velocity the viscous forces are negligibly small. The balance of forces in the azimuthal direction then reduces to

$$\mathbf{v}\cdot\nabla\Omega\omega^2 = R_\phi(\Omega, \eta) = 0$$

where R_ϕ is the azimuthal component of the viscous force. The only solutions of this equation that have been found are $\mathbf{v}=0$, no meridian circulation, although the present author did find solutions for $v \neq 0$ in the case of polytropes. With no circulation the equations determining the steady state are

$$\nabla P/\varrho = -\mathbf{g} + \Omega^2\boldsymbol{\omega}$$
$$\nabla\cdot c\nabla T = 0$$

where the nuclear energy generation is negligible except in the very central regions.

The form of the solutions of this equation are readily seen by taken $C=$constant, writing $P=P_0+P_1$, etc., and then solving the perturbation equations

$$\nabla P_1 = -\varrho_1\mathbf{g} + \varrho_0\Omega^2\boldsymbol{\omega}$$
$$\nabla^2 T_1 = 0$$
$$P_1/P_0 = \varrho_1/\varrho_0 + T_1/T_0 .$$

Elimination of ϱ_1 gives

$$\nabla\left(\frac{P_1}{P_0}\right) = +\frac{\mathbf{T}_1}{T_0}\frac{\varrho_0}{P_0}\mathbf{g} + \frac{\varrho_0}{P_0}\Omega^2\boldsymbol{\omega}$$

on taking the curl we have

$$\nabla T_1 \times \frac{\varrho_0\mathbf{g}}{T_0 P_0} + \operatorname{curl}\left(\frac{\varrho_0}{P_0}\Omega^2\boldsymbol{\omega}\right) = 0 .$$

Now ϱ_0, T_0, P_0, etc., are functions only of radial distance r, and if we expand T_1 in spherical harmonics

$$T_1 = \sum T_n P_n , \quad T_n = A_n r^n + B_n r^{-(n+1)}$$

we have

$$\sum T_n \frac{\partial P_n}{\partial\theta} = \frac{T_0^2}{g}\operatorname{curl}\left(\frac{\Omega^2\omega}{T_0}\right).$$

A solution of this equation with $\Omega=\Omega(r)$ is readily obtained by taking $n=2$. In this case

$$\operatorname{curl}\left(\frac{\Omega^2}{T}\omega\right) = r\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{\Omega^2}{T}\right)\sin\theta\cos\theta$$

https://doi.org/10.1017/S0252921100026993 Published online by Cambridge University Press

and so Ω is given by

$$\frac{T^2}{g}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{\Omega^2}{T}\right) = Ar + \frac{B}{r^3}$$

where the integration constants are determined by the boundary conditions. This is in essence the solution obtained by Roxburgh (1964) where he included the dependence of c on ϱ and T. This solution is given in Table I where $\beta = \Omega^2/\Omega_c^2$ is tabulated against fractional radius $x = r/R$.

It should be emphasized that this is only the first step in a procedure to determine

TABLE I

Variation of $\beta = \Omega^2/\Omega_c{}^2$ for a steady state solution with no circulation currents

x	β	x	β	x	β
0.00	1.000	0.34	0.612	0.68	0.469
0.02	1.000	0.36	0.592	0.70	0.467
0.04	1.000	0.38	0.574	0.72	0.465
0.06	1.000	0.40	0.558	0.74	0.463
0.08	1.000	0.42	0.545	0.76	0.462
0.10	1.000	0.44	0.534	0.78	0.460
0.1217	1.000	0.46	0.524	0.80	0.459
0.14	0.984	0.48	0.515	0.82	0.458
0.16	0.950	0.50	0.507	0.84	0.457
0.18	0.905	0.52	0.501	0.86	0.456
0.20	0.858	0.54	0.495	0.88	0.455
0.22	0.812	0.56	0.490	0.90	0.454
0.24	0.769	0.58	0.485	0.92	0.453
0.26	0.730	0.60	0.481	0.94	0.452
0.28	0.695	0.62	0.478	0.96	0.452
0.30	0.663	0.64	0.475	0.98	0.451
0.32	0.636	0.66	0.472	1.00	0.451

the steady state solution. With the Ω so determined the viscous force is non-zero so that there must be a non-zero circulation velocity driven essentially by the viscous force. In the complete steady state solution there will be differential rotation and meridian circulation. Such refinements, while of marginal interest for stellar interiors, could be of importance for the structure of planetary atmospheres.

Other steady state solutions have been determined when magnetic fields are introduced (Roxburgh, 1967), the most realistic case being one where a magnetic field maintains uniform rotation in spite of the meridian circulation. The transport of angular momentum by the circulation is just balanced by a magnetic torque. More recently Maheswaran (1968) has looked at the effect of slight departures from non-uniform rotation but still constant on field lines.

3. Stability of Steady State Solutions

In the last few years there has been considerable progress in examining the stability of differential rotation, notably by Fricke (1968) and by Goldreich and Schubert (1967). For a rotating liquid the condition of equilibrium requires

$$\partial\Omega/\partial z = 0 \qquad \text{Taylor-Proudman Theorem.}$$

While the condition of dynamical stability requires

$$\partial(\Omega^2\omega^4)/\partial\omega > 0 \qquad \text{Rayleigh condition.}$$

If either of these two conditions is violated then the system changes in time.

For a compressible fluid the situation is different. The Taylor-Proudman Theorem is no longer valid and equilibrium solutions with $\Omega = \Omega(\omega, z)$ are possible. Again, Rayleigh's criterion is no longer true for adiabatic displacements, the stability effect of the stratification being dominant. The analogue of the Rayleigh condition is (Randers, 1942)

$$\frac{1}{\omega^3}\frac{\partial}{\partial\omega}(\Omega^2\omega^4) + \mathbf{g}\cdot\left(\frac{\nabla\varrho}{\varrho} - \frac{1}{\gamma}\frac{\nabla P}{P}\right) < 0\,.$$

However in a gas there are other possibly unstable modes where the radiative diffusion is important and such modes have been analysed by Fricke (1968) and Goldreich and Schubert (1967) assuming axisymmetric disturbances. For a perturbation that is small compared to a scale height they find instability if

$$\frac{k_z}{\Omega^3\gamma}\left(g_\omega + \Omega^2\omega - \frac{k_\omega}{k_z}g_z\right)\left(k_\omega\frac{\partial}{\partial z} - k_z\frac{\partial}{\partial\omega}\right)\ln\left(\frac{T}{\varrho^{\gamma-1}}\right)$$
$$+ \frac{2}{\gamma\omega\Omega}k_z\frac{\sigma}{\Omega}\left(k_\omega\frac{\partial}{\partial z} - k_z\frac{\partial}{\partial\omega}\right)(\Omega\omega^2) + \frac{\sigma}{\gamma}\frac{v^2k^4}{\Omega^3} < 0 \quad \text{for any}\,(k_\omega, k_z)$$

where (k_ω, k_z) is the wave number of the disturbance, $k^2 = k_z^2 + k_\omega^2$, and σ is the Prandtl number, the ratio of the radiative conductivity to kinematic viscosity. In the conditions that prevail inside stars σ is large $\simeq 10^5$–10^6. For infinite σ the stability criterion reduces to

$$\partial\Omega/\partial z = 0 \quad \partial/\partial\omega(\Omega^2\omega^4) > 0\,,$$

which is just the Taylor-Proudman Theorem and the Rayleigh condition. For sufficiently large thermal diffusivity a disturbance can radiate away its buoyancy without diffusing its momentum so that the fluid behaves like a classical liquid, and we recover the stability conditions applicable to a liquid.

For the real case the differential rotation that can be stabilized is

$$\Delta(\Omega\omega^2)/\Omega\omega \leqslant g/\sigma\Omega^2\omega$$

where Δ is either $\partial/\partial\omega$ or $\partial/\partial z$. For rapidly rotating stars $g/\Omega^2\omega \simeq 1$ and we have just the Rayleigh and Taylor-Proudman conditions, since $\sigma \gg 1$. For slowly rotating stars the possibility arises that $g/\sigma\Omega^2\omega > 1$ and a substantial degree of differential rotation can be tolerated. The value of the Prandtl number is therefore important. The viscosity has two contributions, molecular and radiative; these give

$$\nu = \frac{2m^{1/2}(kT)^{5/2}}{\varrho e^4 \ln \Lambda} + \frac{16\sigma T^4}{15c^2\kappa\varrho^2}$$

where

$$\Lambda = \frac{3kT}{e^2}\left(\frac{m}{\varrho}\right)^{1/3},$$

whereas the radiative diffusivity is

$$k = 16\sigma T^3/3\kappa\varrho^2 c_v .$$

For stellar conditions this gives

$$\sigma = (k/\nu) \sim 10^6 .$$

Only in very slowly rotating stars can Ω vary with z or $\Omega\omega^2$ decrease outwards. In such stars the meridian circulation would be so slow that a steady state could not be achieved in the lifetime of the star. In any case where rotation is important stability conditions require

$$\Omega = \Omega(\omega); \quad \frac{\delta}{\delta\omega}(\Omega\omega^2) > 0 .$$

On the other hand there are no steady state solutions with $\partial\Omega/\partial z = 0$ (Roxburgh, 1966), and we therefore reach the important conclusion that there are *no stable steady state* solutions for rotating stars. Inside a rapidly rotating star we expect a battle between the meridian circulation and the turbulence driven by the differential rotation the circulation produces. It seems probable that such turbulence produces uniform rotation.

4. Rotation and Convective Zones

In recent years the problem of the differential rotation of the sun has been attacked with great vigour, from many different viewpoints – some of which will be discussed subsequently during this meeting.

The two types of approach can be described as turbulent and liquid. The difficulty about all these attempts is that we know virtually nothing about turbulence on the one hand, and the sun is not an incompressible Boussinesq liquid on the other hand. The most striking result and probably the reason why almost all attempts to explain the equatorial acceleration meet with success is that meridian circulation in a viscous spherical shell can produce equatorial acceleration.

To show this consider a steady state situation in a rotating spherical shell. Let the rotation be Ω and the meridian circulation be given by a stream function $S(r, \theta)$.

$$\varrho \mathbf{v} = (\mathbf{k} \times \nabla S)/(r \sin\theta) .$$

For simplicity we assume constant velocity and density (although this is not necessary).

The azimuthal component of the equation of motion is then

$$k \times \nabla S \cdot \nabla(\Omega r^2 \sin^2 \theta) = r\left(\sin\theta \frac{\partial^2(\Omega r^2)}{\partial r^2} + \frac{\partial}{\partial\theta}\frac{\partial\Omega}{\sin\theta\,\partial\theta}\sin^2\theta\right).$$

For large viscosity this equation can be solved by successive approximation. The first approximation is $\Omega = \Omega_0 =$constant and the departure from Ω_0, say Ω_1, is then given by

$$\frac{\partial S}{\partial z}\Omega_0 = v\sin\theta\frac{\partial^2\Omega_1}{\partial r^2} + \frac{\partial}{\partial\theta}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\Omega_1\sin^2\theta.$$

Obviously by a suitable choice of $S(\omega, z)$, Ω_1 can be made to give equatorial acceleration. In particular a stream function

$$S = \Lambda(r)\sin^2\theta\cos\theta$$

gives an angular velocity distribution

$$\Omega = \Omega_0 + \omega_1(r) + \omega_2(1)\sin^2\theta$$

in agreement with the observed equatorial acceleration of the sun provided $\mathrm{d}\Lambda/\mathrm{d}r > 0$ at the surface.

The turbulent theories have their origin in work of Wasiutinski (1946) who developed the viscous stress tensor for turbulence. This derivation has been improved by Elsasser (1966) and Roxburgh (1969). The essential point is that if the turbulence is anisotropic differential rotation can result (cf. Wasiutinski, p. 86). A similar point was made, Biermann (1951), and subsequently developed by Kippenhahn (1963), Cocke (1966), Roxburgh (1963). Assuming various forms for the viscous stress tensor these authors arrived at various results – all explaining the equatorial acceleration. The point should be emphasized that almost any departures from spherical symmetry will give equatorial acceleration.

A slightly different approach was adopted by Durney and Roxburgh (1969). Arising out of the observations by Dicke (1967) of the oblateness of the sun Roxburgh (1967) was led to deduce a latitude dependence in the convective efficiency of heat transport, an inference supported by the work of Chandrasekhar (1961) and Durney (1968a, b; 1970). Thus although the turbulence may be locally isotropic the effective 'eddy conductivity' varies with latitude due to the latitude effect of rotation. Global thermal equilibrium then demands a circulation which maintains the equatorial acceleration.

The other method of approach has been to test the stability and steady state convection of a Boussinesq (incompressible) liquid in a spherical shell. This approach has been used by Durney (1968a, b, 1970) and Busse (1969). They find that when rotation is included the most unstable mode is a 'banana-shaped mode' proportional to a surface harmonic T_l^l where l is determined by the thickness of the shell ($l \simeq 10$). This convective mode produces a transport of angular momentum towards the equator which maintains the differential rotation against viscous braking. Durney (1970) has followed the evolution of the system until an asymptotic state is reached using the

Herring approximation for developed convection. He finds equatorial acceleration and a preference for energy to go through the equatorial regions. This may be linked to Dicke's oblateness measurements (see Durney and Roxburgh, 1969).

Other work on the equatorial acceleration of the sun has invoked Rossby waves travelling around the sun as the momentum transport mechanism. The asymptotic periodic time dependence found by Durney is very similar to such waves and not surprisingly both give similar results.

5. Effect of Rotation on the Main Sequence

Considerable effort has been expended on this problem in recent years. Following earlier work, Sweet and Roy (1953) determined the effect of uniform rotation on a simple Kramer's opacity Cowling Model star for slow rotation.

Other models have since been constructed by Roxburgh *et al.* (1965) and a set of

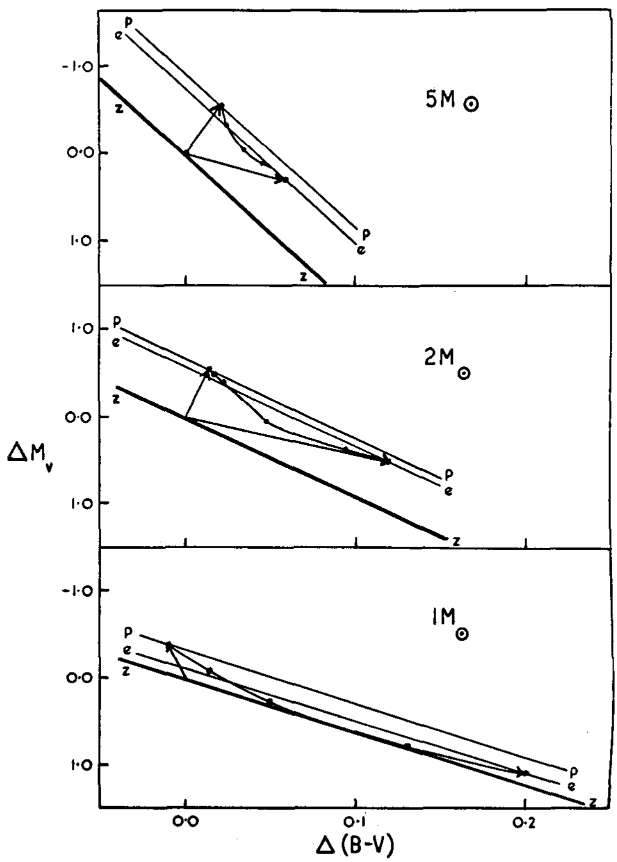

Fig. 1. Variation of ΔM_V and $\Delta\,(B-V)$ with aspect angle $i=0$, to $\pi/2$, and maximum rotational velocity (reprinted courtesy of the *Astrophysical Journal*).

https://doi.org/10.1017/S0252921100026993 Published online by Cambridge University Press

accurate models were calculated by Faulkner *et al.* (1968). The effects of gravity darkening and distortion on the observed properties of stars was first determined by Collins (1963) which was followed by a determination including the effect of rotation on the interior carried out by Roxburgh and Strittmatter (1965). Recent work by Faulkner *et al.* (1968) has provided a set of detailed main-sequence models for a range of mass and angular velocity, and a simple prescription for determining any other models. The results of this latter work are given in Figures 1 and 2.

In the last few years interest has been growing in the effect of non-uniform rotation for many reasons. Firstly the effect of rotation is enhanced if the central regions spin faster than the surface and the work of Mark and Ostriker illustrates this effect. Secondly, it has been suggested by Dicke that the interior of the sun may be more rapidly rotating than the outside. Thirdly, substantial differential rotation could lead to fissional instability and the formation of binary stars as I suggested in 1966. I am sure we will have many other reasons for looking at non-uniform rotation before this colloquium is over.

I have not attempted in this talk to be comprehensive and I have left out several important topics. I make no apology for this since I am sure they will all be covered during the course of the meeting, and a recent review article by Strittmatter (1969) also covers much of this material.

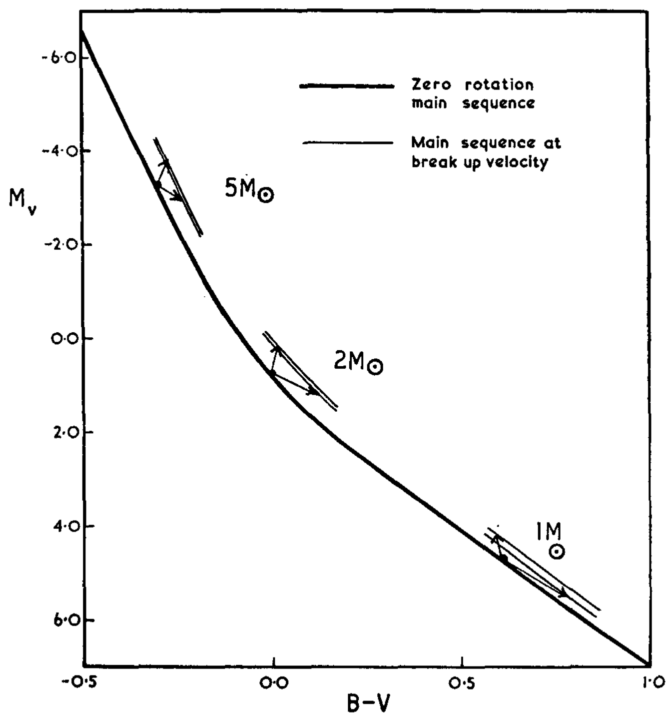

Fig. 2. Effect of rapid rotation on the main sequence (reprinted courtesy of the *Astrophysical Journal*).

https://doi.org/10.1017/S0252921100026993 Published online by Cambridge University Press

References

Baker, N. and Kippenhahn, R.: 1959, *Z. Astrophys.* **48**, 140.
Biermann, L.: 1951, *Z. Astrophys.* **28**, 304.
Busse, S.: 1969, to be published.
Chandrasekhar, S.: 1961, *Hydrodynamic and Hydromagnetic Stability*, Oxford.
Cocke, W. J.: 1966, *Astrophys. J.* **150**, 1044.
Collins, G. W.: 1963, *Astrophys. J.* **138**, 1134.
Dicke, R. and Goldenberg, M.: 1967, *Phys. Rev. Letters* **18**, 313.
Durney, B.: 1968a, *J. Atmos. Sci.* **25**, 372.
Durney, B.: 1968b, *J. Atmos. Sci.* **25**, 771.
Durney, B.: 1970, this volume, p. 30.
Durney, B. and Roxburgh, I. W.: 1969, to be published.
Eddington, A. S.: 1925, *Observatory* **48**, 73.
Eddington, A. S.: 1929, *Monthly Notices Roy. Astron. Soc.* **90**, 54.
Elsasser, K.: 1966, *Z. Astrophys.* **63**, 65.
Faulkner, J., Roxburgh, I. W., and Strittmatter, P. A.: 1968, *Astrophys. J.* **151**, 203.
Fricke, K.: 1968, *Z. Astrophys.* **68**, 317.
Goldreich, P. and Schubert, G.: 1967, *Astrophys. J.* **150**, 571.
Jeans, J.: 1929, *Astronomy and Cosmogony*, Cambridge.
Kippenhahn, R.: 1963, *Astrophys. J.* **137**, 664.
Lebovitz, N.: 1967, *Adv. Astron.* Vol. 5, p. 465.
Lyttleton, R. A.: 1953, *Stability of Rotating Liquid Masses.*
Maheswaran, M.: 1968, *Monthly Notices Roy. Astron. Soc.* **140**, 93.
Mestel, L.: 1953, *Monthly Notices Roy. Astron. Soc.* **113**, 716.
Mestel, L.: 1965, 'Meridian Circulation in Stars', in *Stellar Structure* (ed. by Allen and McLaughlin), Chicago.
Milne, E. A.: 1923, *Monthly Notices Roy. Astron. Soc.* **83**, 118.
Ostriker, J. P. and Mark, J. W.: 1968, *Astrophys. J.* **151**, 1075.
Randers, S.: 1942, *Astrophys. J.* **95**, 454.
Roxburgh, I. W.: 1963a, Fellowship Thesis, Churchill College, Cambridge.
Roxburgh, I. W.: 1963b, *Monthly Notices Roy. Astron. Soc.* **126**, 67.
Roxburgh, I. W.: 1964, *Monthly Notices Roy. Astron. Soc.* **128**, 157.
Roxburgh, I. W.: 1966, *Astrophys. J.* **143**, 111.
Roxburgh, I. W.: 1967, *Nature* **213**, 1077.
Roxburgh, I. W.: 1969, *Solar Hydrodynamics and Hydromagnetics*, National Center for Atmospheric Research.
Roxburgh, I. W., Griffith, J. S., and Sweet, P. A.: 1965, *Z. Astrophys.* **61**, 203.
Roxburgh, I. W. and Strittmatter, P. A.: 1965, *Z. Astrophys.* **63**, 15.
Schwarzschild, M.: 1947, *Astrophys. J.* **106**, 407.
Sweet, P. A.: 1950, *Monthly Notices Roy. Astron. Soc.* **110**, 548.
Sweet, P. A. and Roy, A. E.: 1953, *Monthly Notices Roy. Astron. Soc.* **113**, 701.
Strittmatter, P. A.: 1969, *Adv. Astron.*, Vol. 6, p. 665.
Wasiutinski, J.: 1946, *Astron. Norv.* **4**.
Vogt, H.: 1925, *Astron. Nachr.* **223**, 229.
Von Zeipel, H.: 1924, *Monthly Notices Roy. Astron. Soc.* **84**, 665.

Discussion

Clement: Is an equilibrium distribution of angular velocity possible for which departures from cylindrical symmetry are small?

Roxburgh: If there is no magnetic field, then the answer is no. Cylindrical rotation with circulation would require velocities parallel to the rotation axis, i.e., matter leaving the star. Equilibrium solutions with no circulation are approximately spherically symmetric.

Clement: It is not correct to say that equilibrium solutions with no circulation are spherically symmetric. Spherical symmetry is only a special case (cf. Clement: 1969, *Astrophys. J.* **156**, 1051).

Mark: Concerning the statement that 'large differential rotation will cause large changes in stellar interior parameters': in fact, only $\Omega_p/\Omega_e \simeq 2$–3 can give very large changes. What seems to matter is the kind of differential rotation that does not give an equilibrium model with an equatorial cusp which terminates the sequence.

Roxburgh: I call an inward increase of 2–3 in Ω large; this increases the effect of rotation in the central regions by a factor of 4–9 and therefore the change in, say, luminosity is increased by a similar factor. This could give a luminosity change of 30 to 70% on an extrapolation of the uniform rotation results.

Fricke: You mentioned in connection with Randers' work, that each Ω-distribution should be dynamically stable in a gaseous star due to the stable stratification of the pressure-temperature-density field. This is not the case. You can well construct unstable angular momentum distributions for any given stratification against adiabatic motions.

Concerning the Rayleigh criterion, I may point out that it is generally not a sufficient criterion for the stability of a rotating liquid. Shear instabilities may occur even if Rayleigh's criterion is satisfied.

Roxburgh: What you say is true. If the angular momentum gradient is sufficiently large then an adiabatic instability can exist. For this to be the case, the ratio of the scale of variation of the square of the angular momentum to the radius of the star must exceed the ratio of gravity to centrifugal force. If the scale of variation of the angular momentum is of the same order as the radius of the star then the distribution will be, in general, stable. Of course, shearing instabilities could occur but this is usually a more severe criterion than the Rayleigh criterion. Near the axis of a star it may be possible to have shearing instabilities and be stable in the Rayleigh case.

Ostriker: I would like to make a small objection to an argument which was forwarded by Roxburgh as only schematic. It appears that a contracting rotating star in which viscous and magnetic forces may be neglected will never shed mass at the equator. Rotational 'mass loss' does not occur – at least not under the stated conditions.

Roxburgh: I agree that there are difficulties in understanding how the mass loss occurs but this may just be our ignorance. My own conjecture is that when the gravity and temperature in the equatorial regions is the same as that in pulsating stars a pulsation instability sets in which ejects material – this may be the cause of Be stars.

Dicke: Perhaps it should be remarked that the Goldreich-Schubert instability is model-dependent and that it would be dangerous to conclude that a rapidly rotating stellar core is *impossible.* The stabilizing effects of compositional gradient and meridional flow or oscillation are omitted. Also the complexity and possibly stabilizing effects of a magnetic field (if it should occur) has been omitted.

Roxburgh: Of course this is true and composition gradients could indeed stabilize differential rotation although they are only to be expected in the center of a star. It is more difficult to see how meridional velocities would produce a stabilization since they are so very slow. If they were fast enough they would be able to do so but I doubt if this is the case for a real star.

A magnetic field is more difficult, but I think Klaus Fricke will talk about this later in the meeting.