

A time constant for hemispheric glacier mass balance

Arthur M. GREENE*

*International Research Institute for Climate and Society, The Earth Institute at Columbia University, Palisades,
New York 10964-8000, USA
E-mail: amg@iri.columbia.edu*

ABSTRACT. The notion is developed of a mass-balance time constant applicable to the Northern Hemispheric glacier inventory taken as a whole. Ice dynamics are incorporated only implicitly in its estimation, which follows directly from a consideration of observed mass-balance and hemispheric temperature time series. While such a parameter must certainly be related to the rate at which glacier hypsometry adjusts to variations in climate, as are time constants derived via dynamic considerations, the parameter discussed herein differs with respect to its statistical character. For an ensemble of Northern Hemisphere glaciers a time-constant value on the order of a century is estimated. It is shown that such a value is consistent with the hemispheric near-equilibration of glaciers that prevailed around 1970. A 'reference climate' is defined, such that the mass balance in a given year is a function only of the difference between that year's climate and the reference. This difference was small during the hemispheric near-equilibrium that prevailed around 1970, implying that the glacier wastage of the late 20th century is essentially a response to post-1970 warming. It is shown that precipitation fluctuations play a compensating role in the hemispheric net mass budget, in that they are strongly anticorrelated with fluctuations in temperature-induced melting. However, the contribution of precipitation does not override that of temperature, which remains the dominant influence on hemisphere-wide glacier fluctuations.

1. INTRODUCTION

A variety of time constants for glacier adjustment to a change in climate state have been studied, including those based on kinematic wave theory (Nye, 1963), those based on a filling time (Jóhannesson and others, 1989) and time constants that take account of positive and negative feedbacks associated with transient changes in glacier configuration (Harrison and others, 2001). In general, the objective has been to understand how idealized individual glaciers might respond to changes in climate.

In this investigation a somewhat different approach is adopted, in that winter, summer and net mass-balance data for the entire Northern Hemisphere are considered in the form of hemispherically averaged time series. A time constant is then computed with respect to these series; such a parameter is then applicable to the hemispheric glacier inventory taken as a whole. The time constant itself is computed without making any explicit assumptions about the details of glacier geometry, but rather, is based on observed time series of mass balance and temperature.

The averaging together of the behavior of many glaciers of differing sizes and latitudes may obscure differences in their behavior. In order to investigate the degree to which these different categories of glaciers might bias the computed value, the population of glaciers is then coarsely stratified with respect to both glacier size and latitude, and the estimation repeated. This procedure is combined with other parameter variations in order to provide some idea of the associated uncertainty range.

As is shown by Elsberg and others (2001), the 'conventional' glacier mass balance includes information about both the climate and the degree of disequilibrium between

climate and glacier state, or more simply put, about both the climate and the glacier itself. In this regard, the 'reference-surface balance' defined by these authors represents an attempt to separate the climatic and non-climatic components of the mass balance, in part to facilitate study of the glacier's climate-driven response. In the present study, however, it is not the reference-surface balance, but rather the conventional mass-balance record that is utilized. Rather than attempting to extract its climatically driven component, attention is instead focused on the rate at which glaciers equilibrate to shifts in the forcing climate. This rate can also be thought of as the rate at which a glacier 'forgets' about the climate it has experienced in the past, in the sense that its physical configuration becomes less and less dependent on the past climatic influences to which it has been subjected, and more and more a function only of the present, or recent, climate.

To the extent that the mass balance differs from zero, it can be said that a disequilibrium exists between climate and glacier state. At the same time, however, it is not unreasonable to suppose that an ambient climate might be identified with which the subject glacier, in its present configuration, *would* be in equilibrium. For example, if a glacier is observed to be losing mass each year, one might suppose it was likely to be in equilibrium with a cooler climate. The climate so identified is herein referred to as the *reference climate*.

In effect, then, one can say that the mass balance depends not on one, but on *two* climates: that of the year in which the measurement is made, and that in which the mass balance, given the glacier's physical configuration in the measurement year, would have been null, i.e. the reference climate. More specifically, the mass balance depends on the difference (or, more generally, the *distance*, if one thinks of climate state as a multi-component vector) between these two climates.

* Formerly at: Lamont-Doherty Earth Observatory, Columbia University, Palisades, New York 10964, USA.

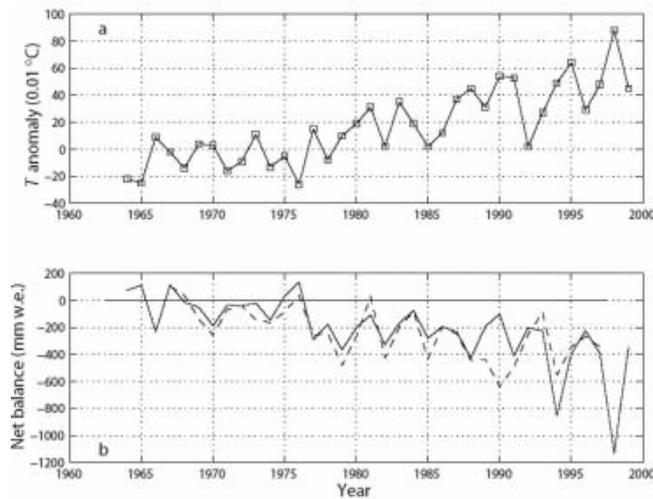


Fig. 1. (a) The GISTEMP JJA Northern Hemisphere temperature anomaly record corresponding to the years of the truncated dataset, 1964–99. (b) The primary net balance series (solid line) and the seasonally derived net balance, the latter representing the sum of summer and winter balance values.

The existence of a mass-balance relaxation time constant implies a finite memory for glaciers, insofar as concerns the climate information encoded by the mass balance. (Just what is meant by glacier ‘memory’ in this context is discussed more fully in section 7.) While it has been suggested that the widespread glacier recession of the 20th century represents a response to the recovery from the low temperatures of the Little Ice Age, it is shown here that a time constant of a century or even longer is not inconsistent with the hemispheric near-equilibration of glaciers that prevailed by around 1970. This, taken together with the temperature record itself, implies that the hemispheric *reference* climate after this time must be at least as warm as it was around 1970 and, thus, that the glacier wastage of the late 20th century is essentially a response to post-1970 warming.

The work discussed here takes a statistical perspective in examining the hemispheric-wide behavior of the mass balance of mountain glaciers and small ice caps. In the painting of such a picture, many details are necessarily blurred by the broad brush strokes employed. It is hoped, however (and aside from the examination of estimation error conducted in section 9), that such a perspective may serve as a useful complement to more detailed modeling studies that, of necessity, are limited to a small number of representative ice bodies.

2. TERMINOLOGY

As used herein, the term *mass balance* shall be taken to mean the net gain or loss of mass by a glacier over a single measurement year. The term *net* here means the combined effects of both accumulation (principally snowfall) and ablation (principally melting), integrated over the entire glacier surface. The traditional unit for mass balance is that of *length*, expressing a depth of water or ice equivalent; since the data compilation employed provides values in millimeters of water equivalent (mm w.e.), it is this unit that shall generally be used, although occasionally values are given in meters of water equivalent (m w.e.).

In addition to the net balance, seasonal (summer and winter) balance values will be discussed. These correspond to the accumulation (winter) and ablation (summer) seasons. In theory, the net balance is equal to the sum of the winter and summer values, these generally being greater and less than zero, respectively; while there are additional complexities, such as summer snowfall and winter ablation, these are not broken out in the dataset employed, and are not taken into account here. The terms ‘net balance’ and ‘mass balance’ are used interchangeably.

3. DATA

The recent compilation of Dyurgerov (2002), comprising most, if not all, of the extant mass-balance data, was employed. Since almost all of these data (265 out of 282 glaciers) derive from the Northern Hemisphere, and since the mass-balance records are to be compared with those of summer temperature (section 5), it was decided for purposes of simplicity to limit the analysis to glaciers that lie north of the Equator. Since Northern and Southern Hemispheric temperature histories are somewhat different (Hansen and others, 1999, 2001), this choice should have the beneficial effect of bringing into better relief the temperature–mass-balance relationship we seek to quantify.

In order to obtain statistically homogeneous results, the net balance data used were further restricted to the years 1964–99, for which a minimum of 55 observations exist for each year, and summer and winter data to 1967–97, during which at least 36 observations are provided for each year, for each of these two glacier seasons. In a few cases, only one of the two seasonal-balance values was present. If the net value was also given for these years, the other seasonal value was computed via simple subtraction. If the net value was not present, the datum was discarded. Thus, in the final seasonal dataset the number of winter and summer balance values for each year is identical, and there are precisely two degrees of freedom in each year’s set of winter, summer and net balance. The fraction of years requiring that a value be computed in this way was small.

The final, time-restricted datasets comprise a total of 2928 and 1454 observations, for the net and (each of the) seasonal-balance series, respectively. The spatial distributions of glaciers contributing to the two series, comprising the familiar glaciated regions of the world, are similar, although the data density is necessarily reduced in the case of the latter group. For both net and seasonal-balance datasets, only a few glaciers lie south of 30° N. Thus, the data examined essentially represent the behavior of mid-latitude to subpolar glaciers in the Northern Hemisphere.

The data are not evenly distributed over the world’s glaciated regions, but tend to be concentrated in areas such as the Alps, which are close to major population centers and where there is a long tradition of glacier study. Many fewer data derive from relatively inaccessible regions remote from populated areas. In an attempt to reduce possible bias from this uneven spatial distribution, the data were first binned into 10° × 10° latitude–longitude boxes, and an average value computed for each box and year. Annual hemispheric mean time series were then produced by spatially averaging each year’s box values, area-weighted by the cosine of latitude. This weighting compensates for the differing areas of boxes at different latitudes. Surface areas of the contributing glaciers themselves are not considered in the

computation of the hemispherically averaged series. Thus, the series values do not reflect an aggregate response that can be interpreted in terms of sea-level change, for example. For such an interpretation see Dyurgerov (2002).

It is of interest to compare the decadal means of the net balance time series thus computed with values derived from a globally averaged series produced by Meier and others (2003, p. 129). The most directly comparable series shown by these authors would be the one labeled 'b₂₀' in their table IV; for the periods 1961–76, 1977–87 and 1988–98 they find globally averaged mass-balance values of –93, –208 and –400 mm w.e., respectively, while for the area-weighted series utilized herein we find, for the periods 1964–75, 1976–87 and 1988–99, values of –33, –191 and –409 mm w.e., respectively. The greatest discrepancy, for the first of the three periods, can largely be ascribed to the difference in averaging period, since 1961–63 was a period of fairly large negative balances that is omitted from our first 'decadal' mean. Means for our net balance series computed for the periods utilized in Meier and others (2003) are –67, –217 and –415 mm w.e. a⁻¹, respectively, the differences now presumably due to the binning and area-averaging performed in this study and the screening performed by those authors. This comparison suggests that the time series resulting from the binning and area-averaging procedures employed herein do not differ drastically from those that would be obtained by simple averaging over glaciers.

As a check on the consistency of summer, winter and net balance series, the last of these is plotted in Figure 1b, along with the sum of the first two. It can be seen that these two representations of the net balance are quite similar, particularly as concerns their overall behavior with respect to the y axis, or zero mass-balance line. For comparison, the NASA Goddard Institute of Space Studies (GISS) surface temperature analysis (GISTEMP) Northern Hemisphere June–August (JJA) temperature anomaly series (Hansen and others, 1999, 2001) is shown in Figure 1a. It can be seen from this figure that the mass-balance and temperature series are anticorrelated, with respect to both overall trend and interannual fluctuations. The relationship between temperature and net balance is examined in more detail in section 5.

It should be mentioned that although the number of annual observations in the time-restricted datasets remains fairly constant from year to year, the specific subset of glaciers from which each year's mean is computed varies considerably. Of the 258 glaciers that contribute to the time-restricted Northern Hemisphere net balance dataset, 82, or 32%, are represented by three or fewer measurements, and 134, or more than half, by six or fewer measurements. The full 36 years of net balance data are provided for only 19 glaciers, one-third of these being in the Alps and another third in Scandinavia. Thus, even if all the measurements contributing to this dataset had been made with equal precision, standard errors of the estimates from different parts of the globe would still differ considerably. Box values were not weighted by number of measurements (in fact, this would tend to undo the effect of the binning described above) or otherwise manipulated to compensate for this possible source of bias.

4. ROLE OF PRECIPITATION

Figure 2 shows the winter and summer balance series (top and bottom, respectively), and their sum, which represents

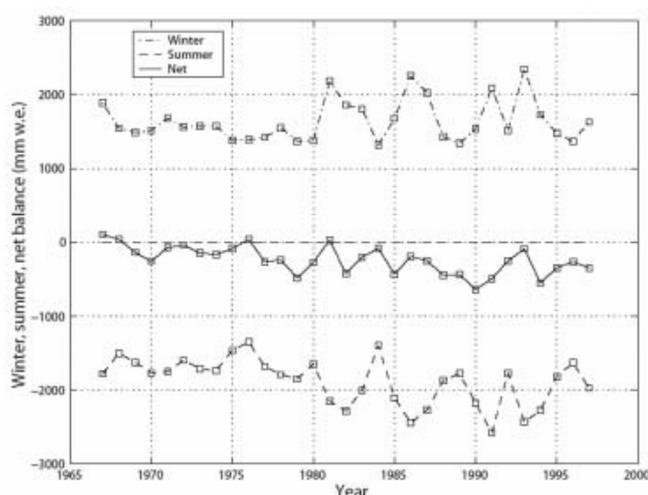


Fig. 2. Time series of the winter (top), summer (bottom) and net balance for the subset of glaciers for which seasonal data exist. The net balance is the sum of the winter and summer values. Correlation coefficient for the winter and summer series, $r = -0.81$.

the net balance for the subset of glaciers in the seasonal-balance data subset. The winter and summer balance series are rather strongly anticorrelated ($r = -0.81$), meaning that as temperatures increase, driving the summer balance toward more negative values, snowfall at glacier sites also tends to increase. Because of this strong anticorrelation, only the relationship between mass balance and temperature will be considered in the sections that follow.

As was shown in Figure 1, the *net* balance is also anticorrelated with temperature, both from year to year and from decade to decade. Precipitation is thus seen to play a *compensating* role in the hemispheric mean, offsetting, but only in part, the increased melting due to higher temperatures. This in turn implies that the increasingly negative net balance values – the wastage, in other words – of mountain glaciers during the last decades of the 20th century has effectively been driven by a *rise in temperature*, and while solid precipitation at the sites of glaciers has increased as temperatures have risen, this increase has only partially offset the anomalous melting due to increasing temperatures.

Comparing mean values for the first and last decades of the seasonally derived net balance record shown in Figure 2, in quantitative terms, between the first and last decades of the seasonally derived net balance record shown in Figure 2, net balance went from –71 to –386 mm w.e. a⁻¹, a decrease of 315 mm w.e. Of this shift, the change in summer balance (melting) contributed –402 mm w.e., which was offset by a winter balance (precipitation) increase of 88 mm w.e. Thus, for this three-decade period, precipitation has compensated the melting due to rising temperatures by 22%.

We note in passing that variability of both the summer and winter balance series appears to have increased rather abruptly around 1980. The variance of their sum shows a smaller increase, the difference due in large part to the strong anticorrelation discussed above. We do not attempt to explain the abrupt change in seasonal-balance variability itself; such a shift may be related to changes in one of the large-scale modes of climate variability, such as the Pacific Decadal Oscillation, which is thought to be responsible for a

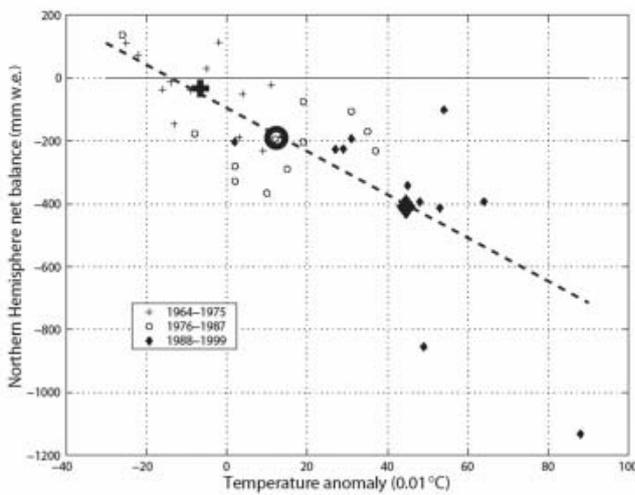


Fig. 3. Scatter plot, Northern Hemispheric net balance vs JJA temperature anomaly from the GISTEMP dataset, 1964–99. The three decadal-length periods (see plot legend) are shown with distinct symbols, with decadal-mean values represented by matching, larger symbols. A regression line for the entire period is shown.

climatic ‘regime change’ around this time (Graham, 1994; Mantua and others, 1997). Meier and others (2003) have made a similar observation regarding a shift in mass-balance variability around this time, and discuss specific details of the atmospheric circulation in connection with it.

5. NET BALANCE AND TEMPERATURE

5.1. General observations

A scatter plot of the Northern Hemisphere net balance against the JJA Northern Hemisphere temperature anomaly, the latter taken from the GISTEMP dataset (Hansen and others, 1999, 2001), is shown in Figure 3. A summer temperature index was used because ablation is most sensitive to temperature anomalies during this season (Paterson, 1994; Greene and others, 1999, 2002). The 36 year period of record is broken into three equal subperiods, the data points for each subperiod being plotted with distinct symbols. Decadal (actually, 12 year) means for these periods are also plotted, with corresponding larger symbols; a regression line for all points is shown. This regression is significant, with $R^2=0.58$ and a p value less than 1×10^{-7} .

The drift of points (or, perhaps easier to see, the progression of decadal means) to the right and downward with the passage of time indicates that hemispheric temperatures rose, while net balance became more negative, over the course of the 36 year data period. This can be taken to mean that the disequilibrium between climate and glacier state was *widening* during these decades, i.e. that temperatures were rising more rapidly than glaciers were able to ‘keep up’, by adjusting their area–altitude distributions in such a way as to maintain a balance between accumulation and ablation. The rise in hemispheric temperature follows an approximately linear trend (Fig. 1a). As will be shown, when taken together with the increasingly negative net balances, this suggests that the relaxation time constant is likely to be long, compared with the length of the data period plotted.

The collinearity of the three decadal means suggests that a nearly linear proportionality between hemispheric temperature and mass balance existed during the period of record. As a test of this apparent linearity, similar plots (not shown) were generated using a 6 month (April–September) warm season, for both the GISTEMP and HADCRU2v (Hadley Center Climate Research Unit, version 2) datasets. In general, linearity is somewhat more precise for the GISTEMP data and for the 6 month warm season than for the HADCRU2v data and the 3 month warm season. While the rather precise alignment of decadal means shown in Figure 3 may thus be an artifact of sampling variability, it can be said that there exists an approximate proportionality between hemispheric temperature and mass balance.

In postulating a simple model for the temperature–mass-balance relationship (section 6), a linear sensitivity function will be assumed, the value determined by the slope of a straight line fitted to the mass-balance and temperature data. Such a fitted line is illustrated in Figure 3. This value would then represent an effective ensemble sensitivity for the period of record. This approximation would seem to be reasonable, given the data reduction employed in the analysis. The latter is described in detail in section 6, where further remarks on this approximation are offered.

Numerically, the mass balance would be proportional, not to the value of the anomaly as given in the temperature dataset, but to the temperature anomaly computed with respect to the x -intercept value of the regression line. This intercept has a special physical significance: it represents the climate, in this case embodied by the hemispheric temperature anomaly, for which the average mass balance over the period of the regression would be null. By definition, this is the climate with which glaciers, given their physical configurations during this period, would have been in equilibrium. Since the mass balance depends only on the anomaly as measured from the x -intercept temperature, the latter may be thought of as a *reference temperature*, because glaciers in effect refer to it when deciding whether to gain or lose mass in a given year. More broadly, the x intercept may be thought of as representing a ‘reference climate’, in this case for the dataset interval over which the regression has been computed.

The reference temperature for the 1964–99 period (x intercept in Fig. 3) is quite close to the ambient temperature that prevailed at the start of the period. This, taken together with the small absolute value of the decadal-mean mass balance for 1964–75, signals that glaciers were not far from equilibrium during the first of the three data decades considered here. Indeed, scattered glacier advances were reported around 1980 (Osborn and Luckman, 1988; Herren and others, 1999; Kääb and others, 2002), suggesting the proximity, on the hemispheric scale, of ambient and reference climates. During the two subsequent decades, hemispheric temperature increased along an approximately linear trend, while net balance became increasingly negative, suggesting that the reference climate was warming much more slowly than the ambient climate during this time. However, the data at hand prove too noisy to quantify the rate of increase of the reference temperature with any confidence by computing x intercepts for the three individual decades.

5.2. A note on the mass-balance sensitivity

The slope of the regression line in Figure 3, an indication of the sensitivity of net balance to temperature on the

hemispheric scale, is -0.69 ± 0.21 m w.e. $(^{\circ}\text{C})^{-1}$ ($\pm 2\sigma$). This is near the center of the range of -0.12 to -1.15 m w.e. $(^{\circ}\text{C})^{-1}$ estimated by Oerlemans and Fortuin (1992) for a group of 12 glaciers representative of various mid-latitude and subpolar climate regimes, using a detailed mass-balance model tuned to represent local conditions for each glacier considered. The data point in the far lower-right corner of Figure 3, which represents the warm El Niño year of 1998, does influence the regression-line slope, but only to a limited degree; without it the regression yields a sensitivity of -0.54 ± 0.21 m w.e. $(^{\circ}\text{C})^{-1}$. This comparison offers some assurance that the approach taken here is proceeding on reasonable numerical grounds, although it should be noted that the mass-balance values used for calibrating the model of Oerlemans and Fortuin are probably included in the dataset employed herein.

6. ESTIMATING THE RELAXATION TIME

6.1. A simple model for the evolution of mass balance

In order to estimate the hemispheric mass-balance relaxation time, a simple model, based on the proportionality between mass balance and temperature anomaly discussed above, is postulated. Since the temperature anomaly is indicative of the disequilibrium between ambient climate and glacier state, and since the mass balance in all cases acts to incrementally *reduce* the climate–glacier disequilibrium, we can say that the rate at which this disequilibrium changes is proportional, with a change of sign, to the value of the mass balance itself. A forcing term, representing the temperature anomaly, must be added, yielding

$$\frac{dX}{dt} + kX = -\frac{dU}{dt}, \quad (1)$$

where X is the mass balance, dX/dt the rate of glacier equilibration, k a decay constant and U the temperature anomaly forcing. The minus sign corresponds to the fact that higher temperatures drive more negative mass balances. Note that when $dU/dt=0$ this equation describes pure exponential decay, which is what we would expect for the conventional mass balance under a constant climate.

The idea of a linear dependence of the mass balance on the degree of climate–glacier disequilibrium is not in itself novel (see e.g. Elsberg and others, 2001; Harrison and others, 2001), although in the present instance the postulated relationship is based, not on a consideration of the theoretical mechanical response of an individual glacier, but on the empirical relationship between the hemispherically averaged mass-balance and temperature records, as illustrated in Figure 3 and discussed earlier. The underlying mechanism, namely, the readjustment of glacier hypsometry, via the flow of ice, so as to conform with the perturbed mass-balance *profile* that characterizes a changed climate, is likely to be the same in both cases, but the terms in Equation (1) refer specifically to hemispheric mean values, and thus have a statistical, rather than deterministic, character. In fact, no *explicit* assumption is made here regarding the details of the equilibration process; rather, we concern ourselves only with what is ‘observable’, which in this case is the state of the hemispheric-mean mass balance, and its relationship with the temperature.

Also in this vein, we note that Equation (1) is identical in *form* to equation (5) in Harrison and others (2001), the difference being that the latter describes the dependence of

a glacier’s *cumulative* volume change on the area-integrated reference-surface balance rate, while Equation (1), besides referring to an ensemble of glaciers, describes the dependence of the conventional balance rate on the time rate of change of climate forcing. Given a specified (constant) reference-surface balance rate, the cumulative volume change described by equation (5) in Harrison and others asymptotically approaches some finite value, whereas the conventional balance described by Equation (1) always decays to zero. Alternately, one might think of a given reference-surface balance rate as characterizing a particular climate. Then dU/dt , on the righthand side of Equation (1), would be analogous to the time rate of change of climate.

Equation (1) does depend on the *implicit* assumption that the hemispheric glacier response for the entire period under investigation can be characterized by a single decay constant (there is but a single constant in the equation). In fact, since glaciers are constantly changing their sizes and configurations, even a decay constant averaged over glaciers would be likely to be subject to some variation with time. However, in order to estimate such a constant empirically, the mass balance and temperature must be observed over a period that is at least comparable to the decay time itself, in order that the decay mechanism act for a long enough time to produce an appreciable effect. This presents a conundrum, in that over long periods the ‘constant’ is likely to vary – yet long periods are required in order to recover an estimate! Ultimately, it must be accepted that the model given by Equation (1), like all other models, represents an idealization; for computational purposes, the parameter in question may be thought of as a long-term effective constant, about which the year-to-year, or even decade-to-decade, value may fluctuate. The present discussion may then be considered a prelude to more sophisticated analyses that account for a time-varying decay parameter. Additional remarks regarding possible variations in the decay constant over the analysis period appear in section 6.2.

There are two forcing-solution pairs for Equation (1) that will be useful in the estimation of a relaxation time constant. One of these involves the case of a static climate; the other, a climate that warms or cools with a linear trend. Both of these have simple solutions: In the first case we have

$$X = Ae^{-kt} \quad (2)$$

and in the second,

$$X = Ae^{-kt} - C/k, \quad (3)$$

where A is a constant of integration, and the relaxation time constant, $\tau = 1/k$. In Equation (3), $C = dU/dt$ is the (linear) rate of change of the temperature forcing. Note that the decay constant is the same in both cases, but in Equation (3) the solution asymptotically approaches $-C/k$, rather than zero. Thus, when temperature increases (decreases) linearly with time, the mass balance decays toward a negative (positive) constant value. The absolute value of this constant becomes larger for larger C (more rapid change in temperature) and for larger τ (slower glacier response), both intuitively plausible responses.

As suggested above, an inherent difficulty arises when attempting to estimate τ directly from the data series, owing to the structure of the model and its solutions: Since the magnitude of τ is expected to lie in the range of decades to centuries, precise estimation involves discriminating between very small differences in k . However, numerical

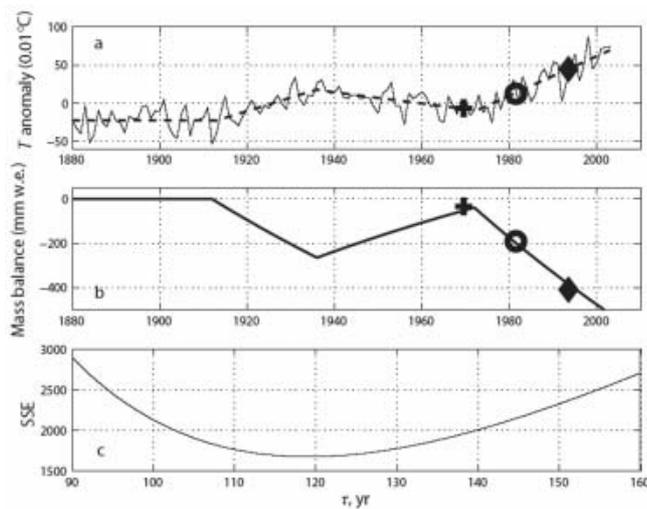


Fig. 4. (a) GISTEMP Northern Hemisphere JJA temperature anomaly from 1880, fitted with four linear trends, for the periods 1880–1912, 1913–36, 1937–72 and 1973–2003. The cross, circle and diamond correspond to the decadal means plotted in Figure 3. (b) Modeled mass-balance sequence utilizing Equations (2) and (3) and the temperature trends, as well as an assumed null mass balance at the beginning of the record, and fit to the three decadal means from the mass-balance dataset. These means are also shown by the three symbols corresponding to those of Figure 3. The best-fit decay constant corresponds to a relaxation time of 119 years. (c) Sum of the squared error as a function of τ , for the modeled mass balance. The sum applies to the three decadal-mean data points, as compared with the corresponding decadal averages for the fitted curve. See text for discussion.

experiments indicate that the data are noisy enough to render such a course of action impractical, if not impossible. Thus, a somewhat ad hoc procedure was developed for the estimation of τ . In essence, the temperature record, extended back to 1880, is represented as piecewise linear, while the mass balance is modeled using Equations (2) and (3), and additional observations near the beginning of the period are used to infer a necessary initial condition. The essential long-term structure of the temperature series is not greatly distorted by what is essentially a smoothing procedure, while noise in the mass-balance series during the period of record is considerably reduced by considering only the decadal-mean values for this time interval. The time period thus considered is less likely to be short, as is the 36 year data interval, compared with the time constant to be estimated.

6.2. Detailed procedure

The linear trends were fitted to the temperature record in two stages: First, the record was parsed by inspection, to identify the approximate time intervals over which coherent linear trends might reasonably apply. The first section of the temperature record, beginning in 1880, exhibits only a very slight trend, which was evaluated using the Mann–Kendall test (Kendall, 1938) and found not to be statistically significant. The early part of the record was thus taken to represent a period of constant temperature. An upward trend was identified beginning some time around 1915 and extending to a local maximum around 1936. This approximate interval was fitted with a straight line, and the period extended until this line met the zero-trend section that

begins in 1880, thus forming a natural ‘joint’ between segments (the actual junction in our reconstruction occurs at 1913). The remainder of the record was fitted accordingly, in the forward direction. Trends of all segments other than the first were found to differ significantly from zero, again using the Mann–Kendall test. The slope of each fitted trend provides the value of C in Equation (3), which is then used to model the time evolution of the mass balance. An initial value for the entire period is still required, however, and for this it is necessary to venture beyond our core dataset.

While glaciers have generally been in retreat since the culmination of the Little Ice Age (LIA), this retreat has slowed or even reversed momentarily from time to time (Houghton and others, 2001). Three such episodes have occurred since the end of the LIA: around 1890, 1915 and 1980. The last of these episodes can be viewed as a response to the long cooling trend of the mid-20th century; as discussed above, by the time this trend ended, hemispheric mass balance was close to zero, signaling that, on this large scale, glaciers were not far from equilibrium. We suggest that the period *prior* to the rising trend commencing around 1915 (i.e. around the turn of the century) may have been comparable, with glaciers near equilibrium and average mass balance around zero. Thus, an initial value of zero is assumed for modeling the extended mass-balance sequence. The sensitivity of the estimated relaxation time to this initial value is considered below.

For each temperature segment, then, beginning in 1913, Equation (3) is used to compute the exponential progress of the mass balance, given the slope of the corresponding temperature trend and the final mass-balance value of the preceding segment. For the years 1964–99 the mass-balance dataset provides the three decadal means already discussed and shown in Figure 3; it is these three points that are fitted, by least squares, to the computed mass-balance sequence, in order to identify the best-fit value of k . The long GISTEMP record and its linearization, the fitted mass-balance sequence and a plot of the sum of squared errors for this fit for different values of k , are shown as Figure 4a, b and c, respectively. Figure 4c exhibits a well-defined minimum at $\tau = 119$ years. (Note that here numerical values returned by the estimation procedure are given to the nearest year, in order to facilitate correspondence with the values shown in Table 1, which is discussed more fully in section 9. Such high precision in the estimates is not implied.)

The glacier advances reported around 1915 were not large, the period of advance not long, and the deep cold of the LIA not far in the past at this time. If glaciers had not yet come into equilibrium, as suggested above, it is thus most likely that net balance would still have been negative, reflecting continuing adjustment to post-LIA warming. For example, mass balance may have been similar to that of 1970, when scattered advances were also recorded after a period of cooling, but hemispheric net balance was still slightly negative. If the assumed initial net balance value is taken as $-50 \text{ mm w.e. a}^{-1}$, rather than zero, the corresponding estimate for τ is 87 years, while an assumed initial value of $-100 \text{ mm w.e. a}^{-1}$ yields an estimate of 71 years. Thus, more negative initial values correspond to shorter relaxation time estimates.

The estimated response times become shorter with increasingly negative initial values because the exponential segment *shapes* depend, apart from k , only on the temperature trend. Thus, a more negative mass-balance

starting value (but the same temperature trend) for the 1915–37 period would result in the corresponding mass-balance segment simply being displaced downward on the plot (Fig. 4b). This negative offset would then propagate through all the segments of the simulated mass-balance series, ultimately forcing the curve away from the decadal-mean value of 1970 and increasing the sum of squared errors (Fig. 4c). A better agreement with the decadal means would thus only be obtainable if τ is reduced (and the exponential segments become less ‘straight’) as the initial mass-balance value becomes more negative, and vice versa. However, a smaller value for τ would also imply a more rapid recovery from the LIA, and thus a greater likelihood that the pre-1915 balance *would* be close to zero. The initial balance value would thus appear to be constrained by these two opposing tendencies. Presumably, a ‘most likely’ initial value could be computed, by iteration if necessary, but high precision would not have great significance here, given the other uncertainties inherent in the estimation of τ . Sensitivity of the final estimate to uncertainties in the initial mass-balance value is indicated in Table 1, which is discussed in more detail in section 9. Allowing for the possibility that full equilibration had not occurred by 1915, and seeking a good round number, we might thus settle on a hemispheric mass-balance relaxation time on the order of a century, which would correspond to an initial value for the long series of $-25 \text{ mm w.e. a}^{-1}$.

In effect, the estimation procedure utilized amounts to a weighted and constrained regression, with the decadal points receiving all of the weight, the initial value providing a constraint, and little control, other than the form of Equation (3), in the middle. If many more decadal-mean data points were available in the early and central portions of the record, for example, a more dependable determination of k , as well as its possible variation with time, might be made. In this regard, we note that the mass-balance record does extend back to 1946, albeit with fewer contributing glaciers. Decadal means centered at 1951 and 1962 (not plotted) lie *below* the modeled mass-balance line shown in Figure 4, suggesting that the net balance might indeed have been somewhat negative before about 1915. Given both the reduced number of observations and the loss of spatial coverage for these earlier years, no attempt was made to further refine the estimate of k using these points, although, with care, this might be one way in which the present work could be extended.

The exponential mass-balance segments of Figure 4 appear as nearly straight lines, owing to the relative rapidity with which hemispheric temperature trends have changed sign during the simulation interval, compared with the relaxation time. Thus, even were the time constant considerably longer than 100 years, the plot would not appear substantially different, since these line segments cannot be much ‘straighter’ than they already are. Even a relatively long relaxation time constant, then, is not incompatible with the near-equilibration of glaciers around 1970. Figure 4 also demonstrates graphically why noise renders the precise estimation of τ difficult, since, starting from a baseline value of 100 years, the shape of the exponential to be fitted has only a weak dependence on τ , in particular as τ increases beyond this value. The asymmetry associated with the limiting shape of these exponential line segments can be seen in the slight asymmetry of the curve of Figure 4c.

7. GLACIER MEMORY

In the use of this common metaphor it is necessary to first draw a distinction, between what might be termed the *capacity* of a glacier’s memory and the *contents* of this memory. One may think of the memory capacity of a glacier as related to its response time. If a glacier is subject to a step change in climate, the response time governs the rate at which the old climate is ‘forgotten’, as the glacier comes into equilibrium with the newly imposed climate. For a glacier with a large memory capacity this period will be long. At the same time, one may think of the contents of a glacier’s memory as reflected by the mass balance. If the glacier has equilibrated and the balance (suitably averaged over a number of years) has fallen to zero, one can say that the glacier’s memory is now empty, in the sense that it knows only about its present climate. Going forward, it is this climate that now becomes the criterion (i.e. the reference) with which future climates will be compared, in the determination of the mass balance. Climates in the glacier’s more remote past have ceased to be relevant. The glacier’s capacity to remember has not changed, however; it is just that there is nothing *to* remember at this point. Because the mass balance depends solely on the difference between ambient and reference climates, a glacier’s memory (referring now to the content) cannot extend further back than the time at which the present reference climate last prevailed in actuality. (Again, this notion must be understood in the sense of a time-smoothed climate: because of natural interannual variability, a single anomalously warm year, for example, does not imply the ‘prevalence’ of a warm climate. This is why the decadal means for the hemispheric temperature are shown in Figure 3 and emphasized in the text.)

It has sometimes been claimed that the widespread glacier recession of the late 20th century represents a continuing recovery from the cold climate of the Little Ice Age, but Figure 4 shows that this is not likely to be the case, even given an exponential mass-balance relaxation time of a century or longer. The essential point on which this observation turns is not the value of the relaxation time constant, but rather, the near-zero net balance of the first data decade. This is because, once equilibrium is reached and net balance has decayed to zero, ambient and reference climates converge, and the content of glacier memory has effectively been depleted. Thus, while the small, residual negative balance remaining at the end of the mid-century cooling trend may indeed preserve some vestigial memory of the LIA, the sharp negative trend in mass balance since that time has arisen in the context of a reference climate that nearly coincides with the ambient climate around 1970. The glacier wastage of the last decades of the 20th century can thus be viewed primarily as a result of post-1970 warming.

8. PATH DEPENDENCE OF THE REFERENCE CLIMATE

The solutions to Equations (2) and (3) that were employed to model the mass-balance sequence of Figure 4 both involve the identical decay constant k . However, it was also observed that the *absolute* rate of glacier adjustment is variable – in fact, this was one of the underpinnings of the argument that led to the basic model, as represented by Equation (1). Thus, given a temperature forcing of the appropriate form, it is theoretically possible to induce more

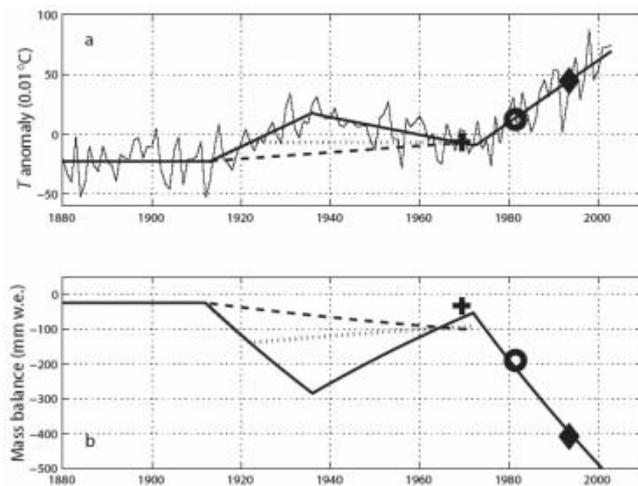


Fig. 5. (a) Hypothetical temperature scenarios for the 1913–70 period, and (b) the corresponding mass-balance responses. The solid lines and the series with interannual fluctuations duplicate what is shown in Figure 4a and b. The dotted lines show a temperature history (a) that rises with the same trend as the actual data until reaching the 1970 level, remaining constant thereafter. Once the temperature stops increasing, the mass balance decays toward zero with time constant τ . The dashed lines show a temperature history that rises along a single long trend between the two end-points. In this case, the mass balance decays, with an unbroken exponential, toward a negative value. In both hypothetical cases, the final balance values are further from zero than is the observed balance, indicating smaller shifts in the respective reference climates.

rapid changes in the reference climate than those that would be governed by a single exponential of the form Ae^{-kt} prevailing over the entire time period considered.

In fact, this phenomenon can be observed indirectly in the record at hand. During the three or so decades of mid-century cooling, temperatures were *warmer* than that of the ambient climate at the end of the period. Because of this, the *reference* temperature rose more rapidly during these decades than it would have, had the ambient climate simply warmed to the 1970 level and remained there, or alternatively, had it warmed in a single linear trend. This is not because the relaxation time constant varied, but because the temperature trend was broken into two segments, rendering it effectively non-linear over the 1915–70 interval. In the first part of this interval, temperatures rose higher than the 1970 level, forcing the mass balance to more negative values than otherwise would have been the case. Although the second part of the temperature trend was downward, evoking an upward-tending exponential in the mass balance, this cooling was not enough to compensate for the melting in the earlier part of the period, with the net result that reference and ambient climates were closer (glaciers were closer to equilibrium) in 1970 than they otherwise would have been. Thus, ‘piecewise linear’ is not the same as ‘linear’, and the reference climate warmed more rapidly during the 1915–70 period than the relaxation time alone would lead one to expect.

The situation is illustrated in Figure 5, where two alternative temperature and mass-balance scenarios are plotted on reproductions of the top two plots of Figure 4. In the first case (dotted lines), temperature rises along the same trend as in the real data until the 1970 decadal-mean

temperature is reached, then remains constant until 1970. The rise to the 1970 level requires just 10 years. As a response to this temperature structure, the mass balance follows the previously computed trend for the first 10 years, but then begins to decay toward zero, as dictated by the solution described by Equation (2). In the second case (dashed lines) the temperature rises along a single long trend between 1913 (the computed junction of trends in our synthesized series) and 1970. The mass-balance response here is an unbroken exponential decaying toward a negative value, as dictated by the solution described by Equation (3). By 1970, when the real and hypothetical temperature sequences converge, it is the mass-balance value corresponding to the former that is closest to zero. The real temperature series, in other words, has produced a warmer reference climate (just slightly warmer in this example), and a more nearly equilibrated glacier population, than either of the two hypothetical scenarios. The evolution of the reference climate is thus not simply a matter of the time constant, but also of the *path* followed by the evolving ambient climate.

9. GLACIER SIZE DISTRIBUTION AND OTHER SOURCES OF ERROR

The glacier size distribution within the sample population is skewed, with median and mean glacier areas of 3.8 and 226 km², respectively. The population also spans a large latitude range (although this factor would not be expected to be completely independent of glacier size). To explore the degree to which the single computed response time might be masking differences among glaciers in differing categories, the regression of Figure 3 and the estimation of τ described in the previous section were repeated for a number of population subgroups, whose ranges were restricted with respect to both size and latitude. The results are summarized in Table 1.

The table shows that, in general, pre-screening for size or latitude produces only modest variations in computed values of τ , with most estimates somewhat higher than that for the population as a whole. In this context it may be worth noting that there is not necessarily a monotonic relationship between response time and glacier size (Bahr and others, 1998).

There are four values given for τ for each subgroup (righthand columns): the first three correspond to initial series mass-balance values of -25 , -50 and 0 mm w.e. a⁻¹ and are labeled as τ^{-25} , τ^{-50} and τ^0 , respectively, while the fourth, labeled $\tau^{b_1-1\sigma}$, refers to the value of τ obtained when the regression-line slope utilized in the conversion of temperature to mass balance is reduced by one standard deviation. These values provide some idea of the sensitivity of τ to both the initial mass-balance value and the slope of the mass-balance–temperature regression line.

For each case, SSE designates the sum of squared errors for the three decadal-average points corresponding to the years 1964–75, 1976–87 and 1988–99, as in Figure 3, and utilizing the initial value of -25 mm w.e. a⁻¹. The SSE values are similar, with the subgroup having $1 < A < 100$ km², where A is the glacier area, providing the lowest value and thus a somewhat better fit than the others, and the subgroup having $5 < A < 50$ km² the highest estimate of τ and the poorest fit. Note that the SSE values may not be completely comparable, since in each case they represent a best fit to a different set of

Table 1. Left columns: regression of the hemispheric mass balance on temperature, corresponding to Figure 3, for a variety of population subsets. Right columns: some statistics for the estimation of τ , the mass-balance relaxation time, for these subsets. An initial mass-balance value of $-25 \text{ mm w.e. a}^{-1}$ was employed, except for experiments described in the third and fourth columns to the right of the vertical line, which have initial mass-balance values of -50 and 0 mm w.e. a^{-1} , respectively. Note that τ is given to the nearest year, in order to facilitate correspondence with references that occur in the main text; other table entries can provide some sense of the actual precision associated with the estimation of τ

Screening	n	R^2	$b_1 \pm 1\sigma$	τ^{-25}	SSE	τ^{-50}	τ^0	$\tau^{b_1-1\sigma}$
Entire population	282	0.58	-6.9 ± 1.0	100	2193	87	119	55
$A < 100 \text{ km}^2$	253	0.57	-7.1 ± 1.1	109	2760	94	133	59
$1 < A < 100 \text{ km}^2$	187	0.67	-6.8 ± 0.8	139	1237	115	181	76
$A < 5 \text{ km}^2$	166	0.45	-7.0 ± 1.3	129	1469	108	162	58
$5 < A < 50 \text{ km}^2$	72	0.60	-6.1 ± 0.9	196	3811	148	311	86
$30^\circ < \text{Lat.} < 60^\circ$	146	0.49	-7.5 ± 1.3	107	2416	93	128	53
$\text{Lat.} > 60^\circ$	113	0.11	-3.1 ± 1.6	169	697	116	386	36

Notes: n is number of glaciers in the subgroup; A is glacier surface area; R^2 is coefficient of determination, regression of net balance on temperature anomaly; b_1 is slope of the regression line; τ^{-25} is mass-balance relaxation time constant, assumed initial value of $-25 \text{ mm w.e. a}^{-1}$; SSE is sum of squared errors for the estimation of τ ; τ^{-50} , τ^0 are best-fit values for τ , given initial values of -50 and 0 mm w.e. a^{-1} , respectively; $\tau^{b_1-1\sigma}$ is best-fit value of τ , utilizing a value of $b_1 - 1\sigma$ for the regression-line slope.

decadal-mean mass-balance points, as discussed next. In any event, it can be seen that the pre-screening implemented here produces neither greatly differing values of τ nor large improvements in the precision of the estimate for the various glacier subgroups.

The lefthand columns of Table 1 refer to the regression of mass balance on temperature anomaly, for which Figure 3 serves as the model. In addition to the value of R^2 , the number of glaciers in the subgroup and the slope of the regression line are provided as n and b_1 , respectively. Both R^2 and the SSE value on the right must be considered when evaluating the estimation of τ , since a low value of SSE, for example as shown by the Arctic subgroup with latitudes $> 60^\circ \text{ N}$, is rendered less meaningful in light of the low R^2 statistic.

The column labeled $\tau^{b_1-1\sigma}$ indicates the effect on τ of reductions of magnitude 1σ in b_1 . For an increase of like amount in b_1 , however, SSE did not exhibit local minima over any range tested, up to $\tau = 1000$ years, for any of the subgroups. This may be viewed as a shortcoming of the method, related to the way in which the fit itself is constructed using only the three decadal-mean values. Varying b_1 while leaving these points in their original positions introduces an inconsistency, but b_1 , even if it were accompanied by a value for the intercept, is not sufficient to determine the locations of these points. Estimates of the sensitivity of τ to changes in b_1 should be viewed with this in mind.

Factors such as snowfall seasonality and cloudiness (the latter in effect a proxy for variations in the surface radiation balance) have also been found to play a role in glacier fluctuations (e.g. Greuell and Genthon, 2003). The significance of these, as well as other possible local influences, is difficult to assess when data are considered only in the hemispherically averaged sense, however, and such influences are not considered in the present study.

10. SUMMARY

A mass-balance time constant applicable to the Northern Hemispheric glacier inventory taken as a whole is investigated, and its value estimated. The estimation was

performed using a simple model, based on a linear proportionality between temperature anomaly forcing and mass-balance response. This relationship was justified by both the approximately linear proportionality shown in mass-balance–temperature plots and the data reduction used in the final computation. It was noted that opportunities may exist for refinement of the method, perhaps using a time-varying constant of proportionality.

When fitted to decadal-mean mass-balance values for the 1964–99 period, using a long temperature series that had been smoothed by piecewise linear representation, the model yielded an estimate for the hemispheric mass-balance relaxation time constant of about a century. This value was shown to be consistent with the near-equilibrium state of Northern Hemisphere glaciers during the first decade of the data period.

A ‘reference climate’ was defined; the near-equilibration described above, and the corresponding proximity of ambient and reference climates during the first data decade, suggest that late 20th-century glacier wastage is primarily a response to post-1970 warming, as the Northern Hemisphere glacier inventory post-1970 retained little collective memory of Little Ice Age climate.

A significant ancillary finding, on which much of the above analysis is based, is that, on the hemispheric scale, winter accumulation and summer melting are strongly anticorrelated. This indicates a primary role for temperature in forcing glacier fluctuations, since the net balance is negatively correlated with hemispheric temperature. This observation may be useful in the detection and/or attribution of climate change, since we can say with additional certitude that it is an increase in temperature, and not a reduction in precipitation, that is signaled by the continuing disappearance of terrestrial ice.

ACKNOWLEDGEMENTS

The author would like to express his appreciation to R. Alley, W. Haeberli, W. Robinson and J. Shepherd for helpful discussions and suggestions, and to T. Jóhannessen and an anonymous reviewer for comments that measurably improved the manuscript. Financial support for this work was

provided by the US National Oceanic and Atmospheric Administration Climate and Global Change Program, grant NA96GP0405. This is Lamont–Doherty Earth Observatory contribution 6761.

REFERENCES

- Bahr, D.B., W.T. Pfeffer, C. Sassolas and M.F. Meier. 1998. Response time of glaciers as a function of size and mass-balance. 1. Theory. *J. Geophys. Res.*, **103**(B5), 9777–9782.
- Dyurgerov, M. 2002. *Glacier mass-balance and regime: data of measurements and analysis*. Boulder, CO, University of Colorado. Institute of Arctic and Alpine Research. (INSTAAR Occasional Paper, 55.)
- Elsberg, D.H., W.D. Harrison, K.A. Echelmeyer and R.M. Krimmel. 2001. Quantifying the effects of climate and surface change on glacier mass balance. *J. Glaciol.*, **47**(159), 649–658.
- Graham, N.E. 1994. Decadal-scale climate variability in the Tropics and North Pacific during the 1970s and 1980s: observations and model results. *Climate Dyn.*, **10**, 135–162.
- Greene, A., W.S. Broecker and D. Rind. 1999. Swiss glacier recession since the Little Ice Age: reconciliation with climate records. *Geophys. Res. Lett.*, **26**(13), 1909–1911.
- Greene, A.M., R. Seager and W.S. Broecker. 2002. Tropical snowline depression at the Last Glacial Maximum: comparison with proxy records using a single-cell tropical climate model. *J. Geophys. Res.*, **107**(D8), 4061 (10.1029/2001JD000670.).
- Greuell, W. and C. Genthon. 2003. Modelling land ice surface mass balance. In Bamber, J.L. and A.J. Payne, eds. *Mass balance of the cryosphere: observations and modelling of contemporary and future changes*. Cambridge, Cambridge University Press.
- Hansen, J., R. Ruedy, J. Glasoe and M. Sato. 1999. GISS analysis of surface temperature change. *J. Geophys. Res.*, **104**, 30,997–31,022.
- Hansen, J.E. and 7 others. 2001. A closer look at United States and global surface temperature change. *J. Geophys. Res.*, **106**, 23,947–23,963.
- Harrison, W.D., D.H. Elsberg, K.A. Echelmeyer and R.M. Krimmel. 2001. On the characterization of glacier response by a single time-scale. *J. Glaciol.*, **47**(159), 659–664.
- Herren, E.R., M. Hoelzle and M. Maisch. 1999. *The Swiss glaciers, 1995/96 and 1996/97*. Zürich, Swiss Academy of Sciences. Glaciological Commission; Federal Institute of Technology. Laboratory of Hydraulics, Hydrology and Glaciology. (Glaciological Report 117/118.)
- Houghton, J.T. and 7 others. 2001. *Climate change 2001: the scientific basis. Contribution of Working Group I to the Third Assessment Report of the Intergovernmental Panel on Climate Change*. Cambridge, Cambridge University Press.
- Jóhannesson, T., C.F. Raymond and E.D. Waddington. 1989. A simple method for determining the response time of glaciers. In Oerlemans, J., ed. *Glacier fluctuations and climatic change*. Dordrecht, etc., Kluwer Academic Publishers, 343–352.
- Kääb, A., F. Paul, M. Maisch, M. Hoelzle and W. Haeberli. 2002. The new remote-sensing-derived Swiss glacier inventory: II. First results. *Ann. Glaciol.*, **34**, 362–366.
- Kendall, M.G. 1938. A new measure of rank correlation. *Biometrika*, **30**, 81–93.
- Mantua, N.J., S.R. Hare, Y. Zhang, J.M. Wallace and R.C. Francis. 1997. A Pacific interdecadal climate oscillation with impacts on salmon production. *Bull. Amer. Meteor. Soc.*, **78**(6), 1069–1079.
- Meier, M.F., M.B. Dyurgerov and G.J. McCabe. 2003. The health of glaciers: recent changes in glacier regime. *Climatic Change*, **59**, 123–135.
- Nye, J.F. 1963. The response of a glacier to changes in the rate of nourishment and wastage. *Proc. R. Soc. London, Ser. A*, **275**(1360), 87–112.
- Oerlemans, J. and J.P.F. Fortuin. 1992. Sensitivity of glaciers and small ice caps to greenhouse warming. *Science*, **258**(5079), 115–117.
- Osborn, G. and B.H. Luckman. 1988. Holocene glacier fluctuations in the Canadian Cordillera (Alberta and British Columbia). *Quat. Sci. Rev.*, **7**(2), 115–128.
- Paterson, W.S.B. 1994. *The physics of glaciers. Third edition*. Oxford, etc., Elsevier.

MS received 30 April 2004 and accepted in revised form 10 April 2005