zoomable (implying inter alia that none of the remaining six semiregular tilings are zoomable.)

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92.24 Counting faces on Archimedean solids

On an Archimedean solid all vertices are congruent and all faces regular. (In the special case of a Platonic solid the faces are of only one kind). This congruency allows us to use Descartes' angle defect formula to count vertices and proceed to count faces without invoking the Euler relation.

On an Archimedean solid there are \( V \) identical vertices, where \( k_i \) regular \( n_i \)-gons meet.

By Descartes' formula, working in whole-turns,

\[
V = \frac{2}{D}, \quad \text{where } D \text{ is the angle defect at each vertex.}
\]

Also, \( D = 1 - \text{sum of interior angles} \)

\[
= 1 - \sum k_i \left(\frac{1}{2} - \frac{1}{n_i}\right)
= 1 - \frac{1}{2} \sum k_i + \sum \frac{k_i}{n_i}.
\]

Thus

\[
V = \frac{2}{D} = \frac{2}{1 - \frac{1}{2} \sum k_i + \sum \frac{k_i}{n_i}} = \frac{4}{2 - \sum k_i + 2 \sum \frac{k_i}{n_i}}.
\]

To find the number of \( n_i \)-gons \( F_i \) we multiply \( V \) by the sum of the \( n_i \)-gon fractions \( f_i \) meeting in each vertex, given by

\[
f_i = k_i \frac{1}{n_i} = \frac{k_i}{n_i}.
\]

So

\[
F_i = V f_i = \frac{4 \frac{k_i}{n_i}}{2 - \sum k_i + 2 \sum \frac{k_i}{n_i}}, \quad \text{and } F \text{ is given by the total number of faces, } F = \sum F_i = \frac{4 \sum \frac{k_i}{n_i}}{2 - \sum k_i + 2 \sum \frac{k_i}{n_i}}.
\]

On a Platonic solid, where there is only kind of face, the formula reduces to

\[
F = \frac{4k}{2n - kn + 2k}.
\]

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