PLANEDES IN DOUBLE STARS: THE $\gamma$ CEPHEI SYSTEM

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RESUMEN

Hasta ahora se tiene evidencia de unos 15 planetas en órbita alrededor de estrellas dobles. Todos pertenecen al llamado tipo S, es decir, orbitan en torno a la primaria. Sólo dos de las binarias, Gliese 86 y $\gamma$ Cep, tienen separaciones del orden de las dimensiones de las órbitas en el Sistema Solar. En este estudio, investigamos la estabilidad del planeta en $\gamma$ Cep en relación a los parámetros orbitales de la binaria y del planeta. Además, investigamos la región dentro y fuera de la órbita del planeta ($a = 2.1$ AU). Aún si la masa de un planeta adicional a 1 AU fuera del orden de la masa de Júpiter, el planeta descubierto tendría una órbita estable.

ABSTRACT

Up to now we have evidence for some 15 planets moving in double stars. They are all of the so-called S-type, which means that they are orbiting one of the primaries. Only two of the binaries have separations in the order of the distances where the planets in our Solar system orbit the Sun, namely Gliese 86 and $\gamma$ Cep. In this study we investigate the stability of the recently discovered planet in $\gamma$ Cep with respect to the orbital parameters of the binary and of the planet. Additionally we check the region inside and outside the planet’s orbit ($a = 2.1$ AU). Even when the mass of an additional planet in 1 AU would be in the order of that of Jupiter, the discovered planet would be in a stable orbit.

Key Words: STARS: INDIVIDUAL ($\gamma$ CEP) — STARS: PLANETARY SYSTEMS

1. INTRODUCTION

The existence of stable planets in binaries, where one is a solar-type star, with a separation comparable to the size of the orbit of Uranus is quite important for our search for stable planets in habitable regions. Recently a Jupiter-sized planet was discovered (Cochran et al. 2002) in the binary $\gamma$ Cep orbiting the more massive primary at a distance of about 2 AU. In a search for substellar companions Campbell, Walker & Yang (1988) conjectured that $\gamma$ Cep may host a third body with $M \sin i = 1.7M_{\text{Jup}}$. Later Walker et al. (1992) rejected this assumption and made the rotation of the sun-like star responsible for that period of 2.1 years in the radial velocity curve. Using observations dating back to 1896, Griffin, Carquillat & Ginestet (2002) did a thorough reduction of the data and found a period of 66 years for this spectroscopic binary. We already know another binary – Gliese 86 which hosts a planet at a distance of $a = 0.11$ AU – where the separation of the two stars is in the order of 20 AU. Out of some 15 examples of binaries hosting planets these are the only ones with orbits smaller than 100 AU (see Udry et al. 2004). Here we report of an extension of a recent publication (Dvorak et al. 2003) dealing with the dynamics of planets in $\gamma$ Cep.

2. METHOD OF DYNAMICAL STABILITY STUDIES

The dynamics of planets in double stars is in a certain sense more interesting than stability studies of planetary orbits around single stars. The presence of a massive second star causes important constraints on the regions of motion where a planet may move in binaries. In principle two types of orbits can be realized, namely planets orbiting both primaries (P-type orbits) and planets orbiting one component of the binary staying always in the vicinity of its host star (S-type orbits). In a simplified model one can study these orbits in the restricted three-body problem, where a massless body moves in the gravitational field of two primary bodies in circular orbits around their common barycenter. Taking into account that most binaries have elliptic orbits, the elliptic restricted three body problem (=ER3BP) is the appropriate model. A possible extension is that the third body does not move in the orbital plane of the primaries (for details see e.g. Szebehely 1967). Already some 25 years ago, when no planets around other stars were known to astronomers, dynamical studies of possible planets in double stars were accomplished (e.g. Harrington (1975), Szebehely (1980), Dvorak (1984)). As a simple rule it turned out that P-types may move in stable orbits.

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with semimajor axes larger than 2.5 times the separation of the binary; this value will increase when the binaries move in eccentric orbits. For S-types the same simple stability limit for a stable planetary orbit is approximately 1/4 of the separation of the binary, which again depends also on the eccentricity of the binaries. In the elliptic restricted problem there exist detailed studies of numerical experiments (e.g. Dvorak, 1986; Dvorak, Froeschlé & Froeschlé 1989; Pilat-Lohinger, 2000a and 2000b; Pilat-Lohinger & Dvorak, 2002) which are of special interest for the S-types, because up to now we only know S-type planets in binaries. An empirically stability limit in extension of a work by Rabl & Dvorak (1988) was established by Holman & Wiegert (1999)

\[ ac = \alpha b(0.464 - 0.380\mu - 0.631e + 0.586\mu e + 0.150e^2 - 0.198\mu e^2); \quad (1) \]

where \( ac \) is the critical semimajor axis, defined as the maximum value for still stable, initially circular, orbits, \( \alpha b \) is the binary semimajor axis, \( e \) is the binary eccentricity, and \( \mu \) is the mass ratio.

What are the methods to get results concerning this question? Because no analytical solutions are available one has to use numerical experiments. The advantage is that the straightforward integration of the equations of motion – we used the Lie-integration method with an automatic step-size control (Hanslmeier & Dvorak, 1984; Lichtenegger, 1984) – allows us also to treat more sophisticated models. Besides the ER3BP we used the dynamical model of three massive bodies (binary + massive planet + massless additional planet) and also a 4 body model, where we also investigated the gravitational force of a fictitious 4th body on the existing planet besides the perturbation of the second star. We have undertaken this kind of studies using different models and also different indicators for stability. It turned out that a measure of instability is the possible crossing of the planet with the fictitious planet. Such an encounter would lead to instabilities and therefore such orbits were classified to be unstable. To check the results we used the Fast Lyapunov Indicators (Froeschlé, Lega & Gonczi 1997), which is a quite well-known tool for stability investigations. Although we are aware that chaos does not automatically mean instability all our different comparison studies (e.g. Pilat-Lohinger, Funk & Dvorak 2003) lead to the conclusion that a chaotic orbit always coincides with an unstable one classified by the “crossing criteria”; this means that sooner or later the chaotic orbit will in fact become unstable. As an additional criterion we used the variation of the Delaunay element \( H = \sqrt{a(1 - e^2)^4} \), which turned out to be very sensitive with respect to the stability of an orbit.

### 3. STABILITY STUDY OF A POSSIBLE PLANETARY SYSTEM IN \( \gamma \) CEPHEI

In the former study the main results were that the discovered planet is far inside a stable region in the parameter space and that there exists a small region of stable motion – a stable window – close to 1 AU for an additional planet, which could even have a mass of the order of Jupiter. But this is very unlikely because then in the radial velocity curves the variation would have been discovered. As a consequence we can say only that “the dynamics of the systems allows an Earth-sized planet to move at a distance comparable to the Earth from the Sun in \( \gamma \) Cephei”.

In this new paper we show the results of an extension of the former work: we studied the dynamics of the system for different eccentricities of the binary and the planet. The grid for eccentricity \( e_p \) (\( p \) labels the discovered planet, \( f \) labels the fictitious planet and \( b \) labels the binary) was \( 0.1 < e_p < 0.3 \) with \( \delta e = 0.01 \); for the binaries we fixed the eccentricities \( e_b = 0.3, 0.4, 0.5 \). The initial semimajor axes for the fictitious massless planets were set to \( 0.45 \, \text{AU} \leq a \leq 1.55 \, \text{AU} \) with \( \delta a = 0.05 \, \text{AU} \).

Fig. 1 shows how the orbits of fictitious planets develop close to the stability window for the parameters given in the table. As a check of stability we made use of the Delaunay element defined above.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Host Star A</th>
<th>Star B</th>
<th>Planet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature [K]</td>
<td>4900</td>
<td>3500</td>
<td></td>
</tr>
<tr>
<td>Radius [Solar Rad]</td>
<td>4.7</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Distance to Primary [AU]</td>
<td>0</td>
<td>12 - 32</td>
<td>1.7 - 2.6</td>
</tr>
<tr>
<td>Period [years]</td>
<td>70</td>
<td>70</td>
<td>2.47</td>
</tr>
<tr>
<td>Mass [Solar masses]</td>
<td>1.6</td>
<td>0.4</td>
<td>0.00168</td>
</tr>
<tr>
<td>Semi-major Axis [AU]</td>
<td>0</td>
<td>21.36</td>
<td>2.15</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.44</td>
<td>0.44</td>
<td>0.209</td>
</tr>
</tbody>
</table>

\( \text{TABLE 1} \)

\( \gamma \) CEPHEI 2003 lead to the conclusion that a chaotic orbit always coincides with an unstable one classified by the “crossing criteria”; this means that sooner or later the chaotic orbit will in fact become unstable. As an example, the dynamical evolution of planetary orbits

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*Published online by Cambridge University Press*
Fig. 1. Development of the Delaunay element $H$ (y-axes) for the orbital parameters given in Table 1 for orbits close to the stable window for $a \approx 1$ for $10^5$ years (x-axes).

located in the larger range between $0.5 \text{ AU} < a_{\text{ini}} < 1.35 \text{ AU}$. One recognizes only small variations in $H$ between $0.55 \text{ AU} < a_{\text{ini}} < 0.7 \text{ AU}$ but then we see large variations in $H$ even after a very short time of integration. This is an example of the disappearance of the stable window due to the large eccentricity of the binary(!), which means that even far away from the second star there is a dramatic influence on orbits located there.

Fig. 2. Development of the Delaunay element $H$ (y-axes) for the orbital parameters $e_b = 0.5$ and $e_p = 0.19$ for orbits between with $0.55 \text{AU} < a < 1.35 \text{AU}$ for $10^5$ years (x-axes) as examples of stable and unstable motion.

In Fig. 3 we depict how the stability of the orbits located there changes with the eccentricity of the discovered planet (note that the orbits of the fictitious planets were circular and that the motions were confined to the plane where the binary and the discovered planet move). For values of $e_p < 0.08$ we can see that the region is stable up to $a = 1.3$ AU (black means that for orbits of fictitious planets started there the eccentricity never exceeded $0.1 e_f < 0.1$), then small strips of instability appear. For $0.08 < e_p < 0.12$ we still see very stable orbits close to 1 AU; for larger $e_p$ the region decreases in extent with respect to the initial semimajor axis of the fictitious planets. For the “real” eccentricity of $e_b$ there are two small windows left there (dark grey stands for $e_f < 0.2$) but for larger $e_b$ no stable zone is left for $0.9 < a_f < 2.0$; only orbits close to the primary survive there.

Fig. 4 shows the region outside the planet. It is evident that for the actual values there are no regions where planetary orbits may survive. There is a small strip of stable motion for $e_p < 0.1$ which disappears later. In this region the perturbations of the second star and the planet do not allow planetary motions stable for significant times at all.

Fig. 3. Stability diagram of orbits in $\gamma$ Cephei. The stability of orbits (initial semi-major axes of the fictitious planets (x-axis) versus initial eccentricity of the real planet (y-axis)) is labeled as follows: Black regions are orbits with $e_f < 0.1$, dark grey regions $e_f < 0.2$, light grey and white stand for orbits with eccentricities suffering sooner or later from close approaches to the massive planet and which are unstable.

4. CONCLUSIONS

We continued the exploration of the stability of orbits in $\gamma$ Cep and extended the dynamical study to values of the eccentricity parameters covering partly
the uncertainty in the observed values of $e_b$ and $e_p$. It turned out that the stable window close to $a = 1$ AU disappears for values of $e_b$ different from the one given in Table 1. An interesting point is that the role of the binary’s eccentricity seems to be more important for the stability of additional planets than the eccentricity of the discovered planet moving in the binary. The possible constraint for the formation of planets in $\gamma$ Cep is the following: planets could be formed only at distances as close as 3 AU from the more massive star. According to our studies there is a chance of additional planets with semimajor axes smaller than the orbit of discovered planet in the habitable zone of 1 AU.

R. Dvorak, B. Funk and F. Freistetter wish to acknowledge the support by the Austrian FWF (Project P14375-TPH).

E. Pilat-Lohinger was supported by the Austrian FWF (Hertha Firnberg Project T122).

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DISCUSSION

Scarfe – How well do we know the properties of the secondary star? How do those properties affect your conclusions?

Dvorak – We checked the parameter space for the binary, and even for different mass ratios and eccentricities the real planet would be stable (as shown in one of my figures). This is not so for the fictitious planet. According to e and m1/m2 this region would be large or completely disappears (with the exception of very close orbits around the star A).

Zinnecker – I noticed that the primary star of γ Cephei is a 1.6 $M_\odot$ star. Is it an evolved star? This possibility leads me to ask whether you have also investigated the stability of planetary systems around evolving stars (with mass loss, etc.)

Dvorak – No, we haven’t done it.

Mardling – You are studying dynamical stability rather than secular stability, since you only integrate for $\sim 10^6$ orbits. Long term stability is another matter.

Dvorak – Our “dynamical stability” is equivalent to your definition of secular because we checked all our computations by direct numerical integrations independently with the aid of the Liapunov exponents.

Clarke – If the planet would not be stable outside 3.8 A.U., presumably the same limit would apply to particles in a proto-planetary disc. This places a rather firm upper limit on the radius at which a Jupiter-mass object could have formed. This is interesting because people often argue that giant gas planets must form at large radii ($\gtrsim 5$ A.U.) An alternative explanation would be that the planet formed before the binary, but that would be unconventional.

Dvorak – I agree that the fact of unstable orbits with $a > 3.8$ A.U. (for the actual parameters of the system) would NOT allow planetary formation in this region. I share your opinion that the binary formed before the planet.

Griffin – I too am concerned about the orbit that you have adopted for the stellar companion. How certain are you that the period is about 70 years? I was rash enough to publish a very tentative orbit with about that period a year or two ago, but Gontcharov wrote to me to tell me that if the period were as long as that it would imply transverse motion that ought to be visible, but it is not in historical astrometric data. He favoured an alternative interpretation of the radial velocities (which I cannot refute) with a period of about 30 years. If that is true, it will vitiate your conclusions. It will vitiate mine too!

Dvorak – In fact, with a period of 30 years of the binary the planet would be on the edge of the region of stability. If so, then the presence of the planet can be regarded as confirmation of the 70-year period.