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Claude Aime & Farrokh Vakili

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Expectations for the Early *TPF-C* Mission

Robert A. Brown

Space Telescope Science Institute, 3700 San Martin Drive, Baltimore, MD 21218, USA;
email: rbrown@stsci.edu

Abstract. We use Monte Carlo techniques to estimate the results and character of the early *TPF-C* mission. Using 10^8 samples to represent the planets of interest, we compute the completeness of the first search observations of prioritized target stars with optimized exposure times that sum to one year total. Assuming simple observing protocols and decision rules for searching, verifying, and characterizing observations, and taking into account ranges of probabilities for confusion sources ($P_{\mathrm{confusion}}$) and the occurrence of planets of interest (η), we compute 10^5 samples of the *TPF-C* schedule and observational outcomes for the first year of exposure time. For example, for Earth-like planets on habitable-zone orbits, assuming no observing overheads or pointing restrictions, and for the values $P_{\mathrm{confusion}} = 0.5$ and $\eta = 0.1$, we find that a median 2.2 planets are found, verified, and characterized in one year of exposure time, with the 68 highest priority stars searched.

Keywords. instrumentation: high angular resolution, space vehicles: instruments, techniques: high angular resolution, (stars:) planetary systems

1. Introduction

The task of finding and characterizing Earth-like planets around other stars is focused and well defined. Because of this sharp definition, the process of mission modeling—estimating results from projected technical performance—is especially useful for planet-finding missions, for evaluating instrument designs, exploring observing strategies, and ensuring robustness against a range of natural uncertainties. This paper discusses expectations for the early *Terrestrial Planet Finder* (*TPF-C*) mission, based on mission modeling.

The concept of completeness is the cornerstone of mission modeling for *TPF-C*. Completeness is the fraction of planets that an instrument can detect in exposures of given depth. Assuming a star has a planet, the number of planets that are found by searching—zero or one—is a Bernoulli random variable with probability equal to the completeness. The expectation value of the number of planets found is also equal to the completeness. For direct-detection techniques, searching a star at different epochs will accumulate completeness. Furthermore, the expectation value of the total result of searching multiple stars is the sum of their accumulated completenesses. Because of this nexus, probability theory and Monte Carlo experiments are emerging as powerful tools for understanding and optimizing the *TPF-C* mission.

The basic mission model for the planet-finding coronagraphic instrument on *TPF-C* comprises five components: (1) the *instrument* (five performance parameters); (2) the *planets* (size, two photometric characteristics, and two orbital characteristics); (3) the *stars* (input catalog and five characteristics related to planetary detectability); (4) the *protocols & priorities* (defining and ranking three types of observations; and (5) the *strategic unknowns* (exozodiacal light, occurrence rate of planets of interest (η), and probability of confusion sources ($P_{\mathrm{confusion}}$)). We combine these components into four structures: (i) *completeness*, based on the star, instrument, and planets; (ii) *exposure time*, based on the star, instrument, planets, and exozodiacal light; (iii) *pure-search results*, based on completeness and exposure time; and (iv) *mission results*, based on completeness, exposure time, protocols & priorities, η, and $P_{\rm confusion}$.

Instrument. As currently envisioned, *TPF-C* is a visible-light camera with fore-optics that strongly suppress starlight in an annular detection zone around a target star. Currently, the telescope is projected to have a 3.5-by-8 meter elliptical entrance aperture. Planned for launch after 2015, the *TPF-C* spacecraft will be stationed near the second Lagrange point and operate for at least five years. It will search a selected subset of nearby stars for terrestrial planets in the habitable zone, where water might occur in liquid form—an assumed precondition of life. *TPF-C* will obtain the spectra of any found planets, looking for atmospheres compatible with—or even reflecting the presence of—life.

The five most important parameters of the *TPF-C* coronagraph for understanding its early accomplishments are: (1) $A_{\rm eff} \equiv A\ QE$, the effective area—the product of the area of the entrance aperture and the quantum efficiency of the optical system; $A_{\rm eff}$ governs the information rate of *TPF-C*. (2) *IWA*, the "inner working angle," the angular radius of the central field obscuration, which introduces a selection effect, hiding planets with smaller apparent angular separations. (3) *OE*, the observational efficiency, which is reduced by operational overheads and scheduling constraint. (We assume $OE = 1$ in this paper.) (4) ζ, the residual level of starlight after suppression, which increases the exposure time to achieve the desired signal-to-noise ratio (*SNR*) on a limiting source. (5) The instability of speckles in the residual starlight, which causes them not to be canceled by image subtraction, which introduces systematic error and restricts the achievable sensitivity to the limit $\Delta mag_{0,\rm max}$, expressed as a limiting delta magnitude with respect to the star. While the values of these performance parameters are not yet fixed, they can nevertheless be reasonably estimated or assumed for purposes of computing *SNR* and completeness.

Planets of interest. The brightness of a planet reflecting starlight depends on the size, derived from the mass (M_p) and average density (ρ), the geometric albedo (p), the phase function (Φ), and the position of the planet. The planets position is specified by the semimajor axis (*SMA*) and eccentricity (*EPS*), the anomaly, and the three random Euler angles orienting the orbit in space.

In this paper, we study two cases of instrumental parameters and the planetary population of interest, which are described in Tables 1 and 2. Case 1 was analyzed in detail by Brown (2005); this paper extends that analysis to include single-visit optimization. Case 2 introduces variable planet sizes, better limiting sensitivity, and a lower *SNRGoal*, the threshold for "detection."

Stars. There are 1408 stars closer than 30 pc with no known close companion that also have main-sequence lifetimes long enough for life to develop under suitable conditions ($B - V < 0.3$). Spread uniformly over the sky, these stars constitute the input catalog for the *TPF-C* mission. Five stellar characteristics are relevant to searching observations with *TPF-C*. (1) V, the visual magnitude, governs the information rate. (2) L, the stellar luminosity in solar units, determines the physical size of the habitable zone. L affects both the relative brightness (Δmag) and apparent physical separation (s, in AU) of planets of interest. (3) d, the stellar distance in pc, determines the resolved fraction of the habitable zone. (4) M_s, the stellar mass, governs the evolutionary time scale of planetary separation and brightness. (This paper treats only initial searches, so M_s is not utilized.) (5) *RA, Dec*, the position on the celestial sphere, determines the days of the year that the star can be observed, given the solar-avoidance angle of the telescope. (We ignore solar avoidance in this paper.) Comprehensive information on potential *TPF-C* target stars is available

Table 1. Adopted design parameters of *TPF-C* mission

A	70000π cm^2	area of entrance aperture
QE	0.17	quantum efficiency
$A_{\rm eff}$	11900π cm^2	effective area (A QE)
λ	550 nm	central wavelength of searching passband
$\Delta\lambda$	110 nm	width of searching passband in nm
IWA	0.05672 arcsec	inner working angle
Ω_x	1.18×10^{-15} str	solid angle of critically sampling pixel
n_x	28.6	noise pixels in photometric aperture
ζ	5×10^{-11}	starlight suppression from theoretical Airy peak
ξ	0.001	dark count rate in sec^{-1} pixel^{-1}
C^2	4	read noise equivalent counts in pixel^{-1}
OE	1	observational efficiency
Case 1 (Brown 2005)		
$\Delta mag_{0,\max}$	25	maximum limiting delta magnitude
$SNRGoal$	10	photometric definition of "detection"
Case 2		
$\Delta mag_{0,\max}$	26	maximum limiting delta magnitude
$SNRGoal$	5	photometric definition of "detection"

Table 2. Adopted planetary populations of interest

ρ	planet density	1 Earth density	
Φ	phase function	Lambertian	
Case 1 (Brown 2005)			
p	geometric albedo	0.33	
SMA	semimajor axis	$0.7\sqrt{L} \leqslant SMA \leqslant 1.5\sqrt{L}$ AU	power law = 0 distribution
EPS	eccentricity	$0 \leqslant EPS \leqslant 0.35$	power law = 0 distribution
M_p	mass	$M = 1$ Earth mass	delta-function distribution
Case 2			
p	geometric albedo	0.2	
SMA	semimajor axis	$0.75\sqrt{L} \leqslant SMA \leqslant 1.8\sqrt{L}$ AU	power law = 0 distribution
EPS	eccentricity	$0 \leqslant EPS \leqslant 0.1$	power law = 0 distribution
M_p	mass	$0.33 \leqslant M \leqslant 10$ Earth mass	power law = −1 distribution

in the *TPF* Target List Database, at http://sco.stsci.edu/tpf_tldb/, which was developed by Margaret Turnbull.

Protocols and priorities. Assume the following observing protocols: There are three types of observations: searching, confirming, and characterizing. The searching observation uses one filter; the exposure time at each roll is *tExpOpt*, assumed the same for all filters and equal to the optimal exposure time discussed in the previous section. Three roll angles are searched, due to the elliptical aperture; the total time cost of searching a star is 3×*tExpOpt*. Confirming observations use three filters—to permit disambiguation by both color and motion, for which the total time cost is (1, 2, or 3)×3×*tExpOpt*, because confusion sources can appear at 1, 2, or 3 rolls; a planet can appear only at 1 roll. The three colors will help—along with the relative motion—differentiate speckles and background objects from planets. Characterizing observations cost 70×*tExpOpt*; this assumes the resolving power of the spectrometer is 70 times that of a filter.

Assume the following observing priorities: We prioritize target stars based on their discovery rate. If no confirming or characterizing observation is pending, then we search the highest priority, as yet unsearched star on the prioritized target list. If a searching observation finds a candidate feature—which could be either a planet or confusion

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source—at any roll angle, then we perform a confirming observation at each such roll. A confirming observation always confirms a planet and eliminates a confusion source. If a confirming observation confirms a planet, then we make a characterizing observation. We continue until one year of exposure time is exhausted.

Strategic unknowns. Three additional—and currently unknown—parameters will influence the course of the *TPF-C* mission. The occurrence probability, η, is assumed the same for all potential target stars. The confusion probability per searching exposure, $P_{\mathrm{confusion}}$, can be either astronomical or instrumental in origin. (Confirming observations are needed to disambiguate confusion sources from planets of interest.) The level of zodiacal light around a target star, (μ, in units of the solar-system zodiacal light) increases exposure times. In this paper, we assume $\mu = 3$, as in Brown (2005).

In the following sections, we compute completeness, estimate the yield of planets in one year of exposure time for searching observations, and explore the stochastic evolution of the *TPF-C* mission for ranges of η and $P_{\mathrm{confusion}}$.

2. Completeness

As developed by Brown (2004, 2005), there are various types of completeness, distinguished by qualifying adjectives. "Specific" or "ensemble" refer to whether we treat a specific orbital size and shape (*SMA* and *EPS*) or an ensemble of sizes and shapes. "Visit," "program," or "design" indicates whether we treat a single observing visit, a program of observations (between which the planetary positions and brightnesses evolve), or exhaustive observations, assuming unlimited observing time. Finally, "photometric" and/or "obscurational" refers to whether the completeness is a function of planetary flux (i.e., limited by Δmag_0) or a function of apparent separation (i.e., limited by *IWA*)—or both. Brown (2004) treats specific and ensemble obscurational completeness for the visit, program, and design cases (*SVOC, SPOC, SDOC, EVOC, EPOC*, and *EDOC*). Those results apply to any centrally obscured instrument with unlimited photometric sensitivity, in either thermal radiation or reflected starlight. Brown (2005) treats ensemble, visit, photometric and obscurational completeness (*EVP&OC*) for the case of reflected starlight, without optimization. This paper gives some optimized results for *EVP&OC*. Hunyadi, Shaklan, and Brown (2006) treat both optimized *EVP&OC* and program completeness (*EPP&OC*). Brown and Lisman (2006) treat design completeness (*EDP&OC*), which is the best an instrument can do.

To compute photometric and obscurational completeness, we represent the planetary population of each star by a large Monte Carlo sample drawn from the planetary population of interest. Initially, the planets are placed in their orbits at random mean anomalies. Knowing the orbit, we can compute the positions of planets at future epochs, and then we can compute s and Δmag from

$$\Delta mag = -2.5 \log \left(p \; \Phi(\theta) \left(\frac{R}{r} \right)^2 \right) \quad , \tag{2.1}$$

where θ is the planetary phase angle (planetocentric angle between the star and the observer), R is the planetary radius (derived from M_p using ρ), and r is the physical distance between the planet and star. To determine whether a planet is detected or not, we compare s with d *IWA* and Δmag with Δmag_0. The fraction of planets of interest for which $s > d$ *IWA* and $\Delta mag < \Delta mag_0$ is *EVP&OC*, the completeness at the epoch of initial searching. Figures 1 and 2 in Brown (2005) show the computed probability density

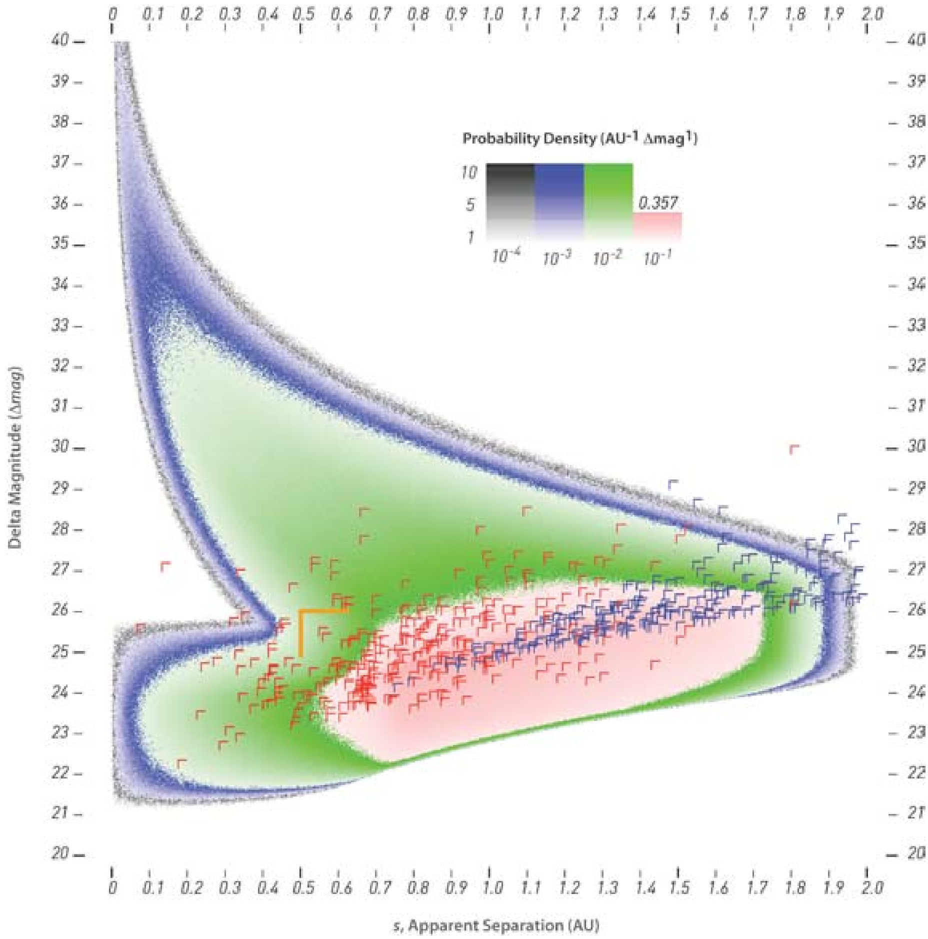

Figure 1. Distribution of probability density vs. s and Δmag for terrestrial planets in the habitable zone in case 2, based on 100 million random planets and shown for $L = 1$. The corners mark the integration zones for computing the completeness on individual valid stars (equivalent positions for $L = 1$). Red: the 198 stars with highest priority in an optimized first search using a total of one year of exposure time. Blue: the 247 lower priority but still valid stars. Orange corner is the fiducial case of Earth at 8.82 pc. The completeness is the probability density integrated below and to the right of a corner. The high priority stars are shown at their optimized values of Δmag_0.

distribution of planets vs. s and Δmag and *EVP&OC* vs. d *IWA* and Δmag_0 for case 1. Figures 1 and 2 in this paper show the corresponding completeness results for case 2.

The number of planets found (0 or 1) is a Bernoulli random variable with probability equal to the accrued completeness times η. This product is also the expectation value of the number of planets found. For multiple observations of multiple stars, the expectation value of the total number of planets found is the sum of the products of accrued completeness times η.

For cases 1(2), only 455(445) stars out of the 1408 in the input catalog are valid targets. "Valid" means that some portion of the habitable zone is unobscured. For each valid star, the search completeness is a unique function of exposure time—unique because the exposure time to achieve *SNRGoal* for any specified Δmag_0 depends on the stellar

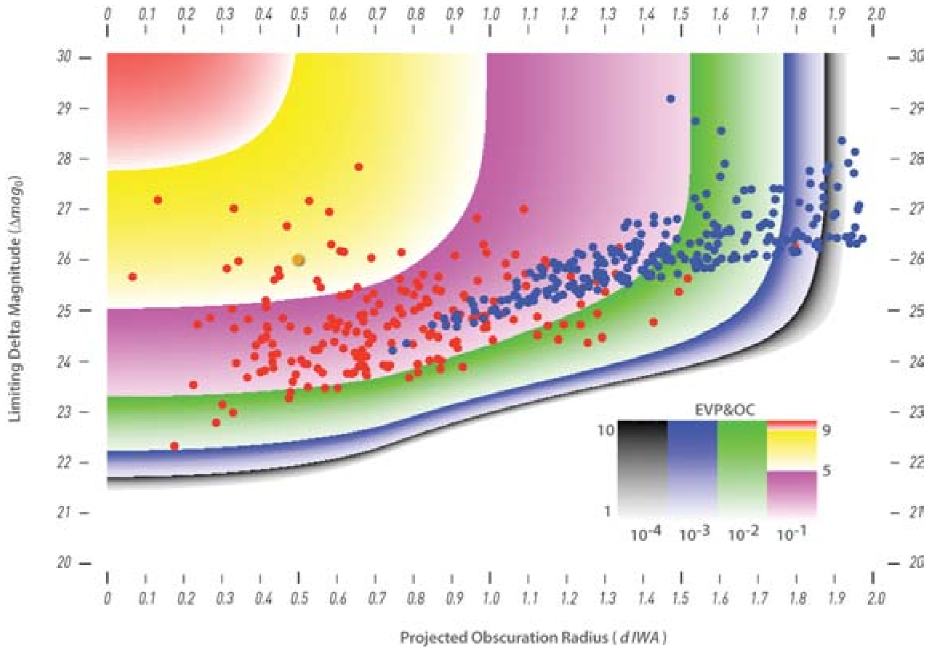

Figure 2. First-visit search completeness (EVP&OC) vs. projected *IWA* and Δmag_0. The points are for valid stars in case 2, based on the corners in Figure 1. The color code is the same as in Figure 1.

quantities V (information rate), d (resolved fraction of the habitable zone, which determines completeness vs. Δmag_0), and μ.

Figure 3 shows completeness vs. exposure time for typical valid stars in case 2. In general, the completeness function has three regimes: an initial time delay before any completeness is accumulated, followed by a monotonic rise, followed by flattening to a terminal value of completeness for exposure times longer than required to achieve *SNRGoal* on $\Delta mag_{0,\max}$, beyond which point the sensitivity is capped by systematics, not limited by photons.

The purely searching observing program described in Brown (2005) is not optimal because it assumed exposing to reach $\Delta mag_{0,\max}$ on all stars. As discussed by Hunyadi, Shaklan, and Brown (2006), this exposure strategy does not make the best use of limited exposure time, because the slope of completeness vs. exposure time on some stars flattens before $\Delta mag_{0,\max}$ is reached. With a limited total budget, it may be advantageous to withdraw some exposure time from such stars and invest it in a new star. An optimized observing program adjusts the exposure times on individual stars and re-prioritizes them dynamically to maximize the sum of completeness for all the stars observed in a given period of time. In the current analyses, the exposure time budget is one year. Figures 1 and 2 show the optimized, single-visit results for individual stars in case 2. The brightest stars are untouched by optimization, because their discovery rates are already high due to short exposure times, and fainter stars show increasingly greater gains in discovery rate from exposure-time optimization.

As shown in Table 3, for one year of exposure time spent purely searching, the non-optimal approach for cases 1(2) produces an expected $31\eta(15\eta)$ discovered planets after searching 117(57) stars. The results for case 2 are lower than case 1, despite increased sensitivity, because the great time costs of the uniformly deeper exposures reduce the

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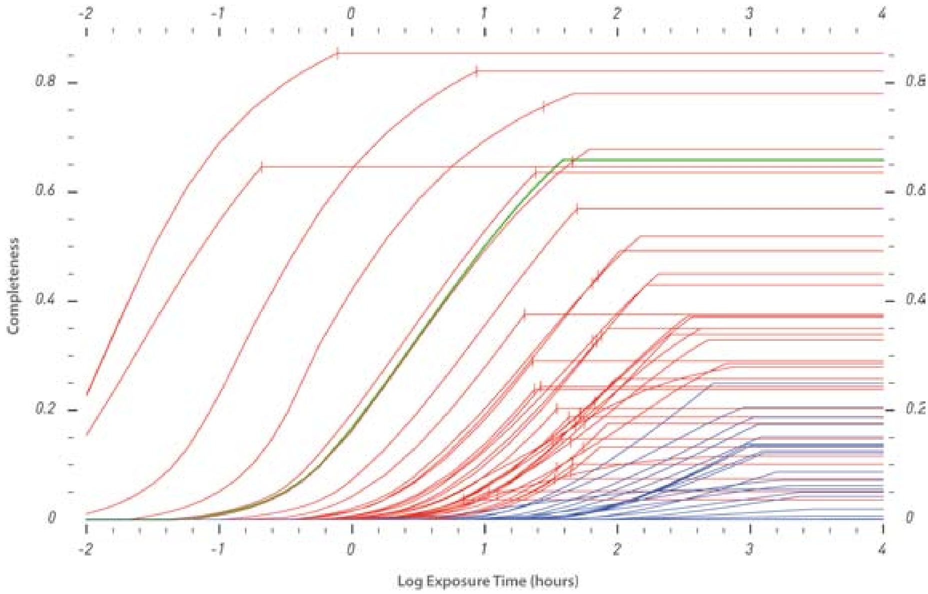

Figure 3. Completeness vs. exposure time for typical stars in case 2. The color code is the same as in Figure 1, except that the fiducial case is now green. Only a random 10% of the 445 valid stars are shown. The marks on high-priority stars show the exposure time after optimization.

Table 3. Pure-search results. Exposure time optimization improves planet yield, moreso for greater Δmag. Planet yield approximately doubles when Δmag is increased from 25 to 26.

Case 1		
Valid stars	455 out of 1408	
Δmag_0	Fixed at 25	Optimized
Planets (stars)	$31\eta(117)$	$33\eta(135)$
Case 2		
Valid stars	445 out of 1408	
Δmag_0	Fixed at 26	Optimized
Planets (stars)	$15\eta(57)$	$56\eta(198)$

number of stars searched. The optimized approach produces an estimated 33(56) planets after searching 135(198) stars. This demonstrates that optimal use of limited exposure time is essential to securing the benefits of greater sensitivity.

3. Mission modeling

We can also use Monte Carlo techniques to model the early *TPF-C* science mission itself, for which we provide the following simple demonstration. In each Monte Carlo trial, we use a Bernoulli random deviate with probability η to assign each target star a number of planets $n_p = 0$ or 1. In each searching observation, the number of planets detected is a Bernoulli random deviate with probability $n_p \times EVP\&OC$. If a planet is detected, we assign it randomly to one of the three rolls. The number of confusion sources detected at each of the three rolls is a Bernoulli random deviate with probability $P_{\text{confusion}}$. Table 4 shows the results of 10^5 Monte Carlo trials for particular values of η and $P_{\text{confusion}}$. Figures 4 and 5 show the histograms of results for the middle entry in the table.

From these results for one year of exposure time, we see that if η approaches unity, the mission becomes spectroscopy dominated, the number of stars searched is ~10% the

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Table 4. Results of mission simulations with 10^5 trials for case 1.

η	$P_{\rm confusion}$					
	0.0		0.5		1.0	
	planets	stars	planets	stars	planets	stars
0	9	135	0	108	0	88
0.05	1.47	115	1.15	73	0.99	58
0.1	2.70	99	2.20	68	1.94	56
0.2	4.69	77	4.02	59	3.65	51
1	11	23	11	23	11	23

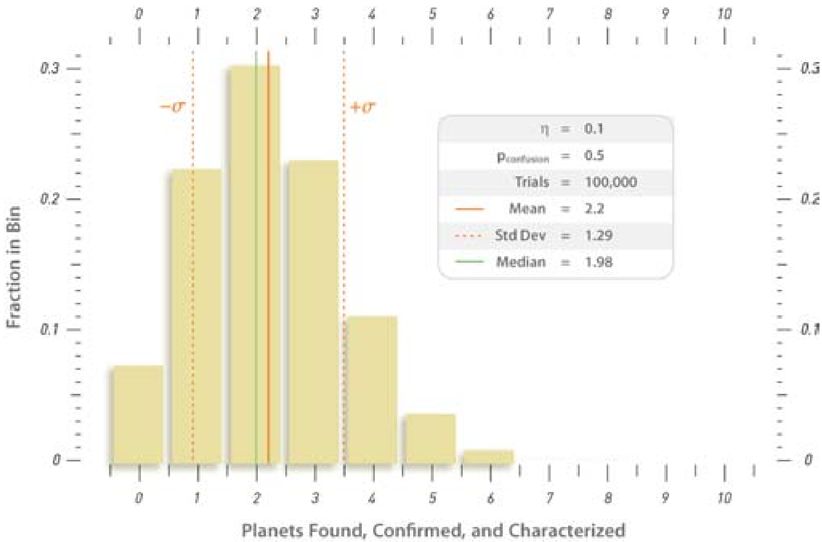

Figure 4. Typical distribution of the number of planets found, confirmed, and characterized by one year of exposure time. Results for the center case in Table 3.

number for small η, and about half the stars searched will have characterized planets. If η approaches zero, the mission becomes search dominated and confusion limited. For intermediate η, the number of planets is proportional to η, but the number of stars searched becomes independent of η as $P_{\rm confusion}$ approaches unity.

4. Commentary

The completeness formalism provides a useful tool for planning and estimating the search results of the early *TPF-C* mission. We can use discovery rate, which is completeness per exposure time, to optimize exposure times, prioritize target stars, and compute the probability of finding a planet of interest. We can compute completeness for any star from instrumental parameters, the specification of the planetary population of interest, the exposure time, and an assumed value of μ, the level of exozodiacal light.

Because completeness is the expectation value of the number of planets found, it has become the primary performance metric of NASA's *TPF-C* project, used for design trades and verifying that the optical design is compatible with the science requirements.

Mission modeling based on completeness is a useful tool for exploring the character of the *TPF-C* mission for values of the strategic unknowns—η, $P_{\rm confusion}$, and μ. Including observing overheads and solar avoidance will increase the fidelity of mission modeling.

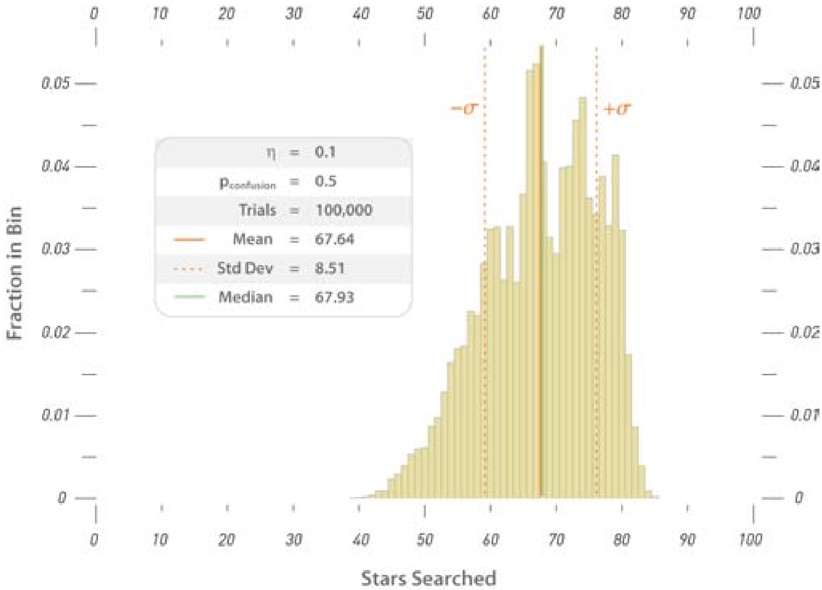

Figure 5. Typical distribution of the number of stars searched for the center case in Table 3.

The *TPF-C* project will use mission modeling to verify that the mission design will meet the science requirements for reasonable ranges of the strategic unknowns.

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