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BIDUALS OF WEIGHTED BANACH SPACES OF ANALYTIC FUNCTIONS

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Abstract

For a positive continuous weight function v on an open subset G of $\mathbb{C}^N$, let $Hv(G)$ and $Hv_0(G)$ denote the Banach spaces (under the weighted supremum norm) of all holomorphic functions f on G such that vf is bounded and vf vanishes at infinity, respectively. We address the biduality problem as to when $Hv(G)$ is naturally isometrically isomorphic to $Hv_0(G)^{**}$, and show in particular that this is the case whenever the closed unit ball in $Hv_0(G)$ is compact-open dense in the closed unit ball of $Hv(G)$.

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If v is a (strictly) positive continuous function on an open subset G of $\mathbb{C}^N$, $N \geq 1$, let $Hv(G)$ and $Hv_0(G)$ denote the Banach spaces of holomorphic functions f on G such that vf is bounded and vf vanishes at infinity on G, respectively, where the norm in both instances is given by $\|f\| = \sup\{v(z)|f(z)| : z \in G\}$. We address the question (*cf.* [1]) as to when $Hv(G)$ is isometrically isomorphic to the bidual $Hv_0(G)^{**}$ of $Hv_0(G)$.

Some affirmative answers to this biduality problem appear in the literature, but only in special settings. Rubel and Shields [11] have shown that $Hv(G)$ is isometrically isomorphic to $Hv_0(G)^{**}$ when $G = D$ is the open unit disk in $\mathbb{C}$ and v is a radial weight on D which vanishes on the boundary ∂D, and they mention that a similar result had been obtained for $G = \mathbb{C}$ and a radial weight v which is rapidly decreasing at infinity by D. L.

Williams [17]. Taking a functional analytic approach, we show (Section 1) in the general context specified above that (1) $Hv(G)$ has a predual X for which there is a natural surjective homomorphism $R: X \to Hv_0(G)^*$, and (2) R is an isometry if and only if the closed unit ball B_0 of $Hv_0(G)$ is dense in the closed unit ball B of $Hv(G)$ with respect to the topology of compact (or, what is the same, pointwise) convergence. From the fact that this density condition can be readily verified in the settings considered by Rubel and Shields [11] and Williams [17], we then recover both of these special cases and indicate extensions to higher dimensions (Section 2). We conclude by giving further descriptions of the predual X of $Hv(G)$ and the corresponding duality $\langle X, Hv(G)\rangle$ in terms of a natural "weighted strict" topology β_v on $Hv(G)$ (Section 3).

A different approach to the biduality problem has been taken by Anderson and Duncan [1] in the context of entire functions on $\mathbb{C}$, and we take the opportunity to thank John Duncan for calling the general problem to our attention. On the other hand, as we learned after obtaining our results, an approach which is similar in spirit to our own can be found in an unpublished manuscript by Shapiro, Shields, and Taylor [14], and we thank Joel Shapiro for making a copy available to us. The main result of [14], which was announced in [*Notices Amer. Math. Soc.* **18** (1971), 181], would also set the ground for our applications in Section 2, but it as well applies to biduality questions in the context of Lipschitz spaces and spaces of bounded linear operators. (For further applications based on the present results, we refer to [3].) Moreover, we express our thanks to the referee for pointing out the present approach to Proposition 3.1, and for a number of other constructive remarks.

1. Density and biduality

In what follows, G will continue to denote an open subset of $\mathbb{C}^N$, $N \geq 1$. Since our methods permit, however, we can somewhat relax the requirements on the weight v, and shall in this section only assume that $v: G \to \mathbb{R}_+$ is an upper semicontinuous function such that $\inf\{v(z): z \in K\} > 0$ for each compact set $K \subset G$. Otherwise our notation remains unchanged. (As pointed out in [1, Section 2], the weight v can sometimes be tied to the space $Hv(G)$ in a better way by replacing v with $\tilde{v} \geq v$ defined by

$$\tilde{w}(z) := \sup\{|f(z)|: f \in B\} \leq 1/v(z) \quad \text{and} \quad \tilde{v}(z) := 1/\tilde{w}(z), \ z \in G.$$

In case $\tilde{w}(z) \neq 0$ for all $z \in G$, then $\tilde{v}: G \to \mathbb{R}_+$ is upper semicontinuous, and we would clearly have that $Hv(G) = H\tilde{v}(G)$.)

We begin by introducing an auxiliary space X consisting of all linear functionals F on $Hv(G)$ such that the restriction $F_{|B}$ of F to the closed unit ball B of $Hv(G)$ is continuous when B is equipped with the compact-open topology κ. Since (B, κ) is compact in view of Montel's theorem, setting $\|F\| = \sup\{|F(f)|: f \in B\}$ defines a norm on X, and we shall assume that X is endowed with this norm. For $F \in X$, we denote the restriction of F to $Hv_0(G)$ by $R(F)$. Finally, we let $\Phi: Hv(G) \to X^*$ be the evaluation map defined by $[\Phi(f)](F) = F(f)$ for $f \in Hv(G)$ and $F \in X$.

THEOREM 1.1. (a) *X is a Banach space (in fact, a closed subspace of $Hv(G)^*$), and the evaluation map Φ is an isometric isomorphism of $Hv(G)$ onto X^*.*

(b) *The restriction mapping R is an isometric isomorphism from X onto $Hv_0(G)^*$ if and only if the closed unit ball B_0 of $Hv_0(G)$ is κ-dense in B.*

PROOF. Since (B, κ) is compact, (a) is a direct consequence of the following result due to Ng [8] (also see Waelbroeck [16, Proposition 1]):

THEOREM A ([8, Theorem 1]). *Let E be a normed space with closed unit ball B. Suppose there exists a (Hausdorff) locally convex topology τ for E such that B is τ-compact. Then E is a dual Banach space. In fact, E is isometrically isomorphic under the evaluation map to the dual of the closed subspace of E^* consisting of all linear functionals F on E such that $F_{|B}$ is τ-continuous.*

Turning to (b), since each $F \in X$ clearly belongs to $Hv(G)^*$, it is immediate that R is a norm-decreasing linear map from X into $Hv_0(G)^*$. We would next show that R is surjective. To this end, fix $F \in Hv_0(G)^*$ and recall (*cf.* [15]) that there exists a bounded Radon measure μ on G such that

$$F(f) = \int_G fv\, d\mu, \qquad f \in Hv_0(G).$$

(In case v is continuous, this follows directly from the Hahn-Banach and Riesz representation theorems.) If we now put $\widehat{F}(f) := \int_G fv\, d\mu$ for each $f \in Hv(G)$, then $\widehat{F} \in Hv(G)^*$ with $\widehat{F}_{|Hv_0(G)} = F$, and so it will suffice to show that $\widehat{F}_{|B}$ is κ-continuous. However, this follows from the inner regularity of μ since v is bounded on each compact subset of G, whereby R is indeed surjective.

If B_0 is κ-dense in B, it is obvious that R is an isometry. Suppose, on the other hand, that B_0 is not κ-dense in B. Then there exist $g \in B$

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and a continuous linear functional F on $H(G)$, the space of all holomorphic functions on G endowed with the compact-open topology κ, such that $|F(f)| \leq 1$ for all $f \in B_0$ and $F(g) > 1$. Since $F_{|Hv(G)} \in X$, we see that R fails to be an isometry, and the proof is complete.

COROLLARY 1.2. *If B_0 is κ-dense in B, then $Hv(G)$ is isometrically isomorphic to the bidual $Hv_0(G)^{**}$.*

REMARKS. 1. Even if B_0 is not κ-dense in B, the proof of 1.1.(b) still serves to show that $Hv_0(G)^*$ is topologically isomorphic to the quotient space $X/\ker R$. Consequently, it follows that R is a topological isomorphism from X onto $Hv_0(G)^*$ if and only if there exists $\varepsilon > 0$ such that εB is contained in the κ-closure of B_0 (in B).

2. On the κ-compact set B, the topology of uniform convergence on the compact subsets of G coincides with the topology of pointwise convergence on G (or any dense subset of G), and hence the density condition of Theorem 1.1 is equivalent to the following *pointwise weighted approximation* condition: For each $f \in Hv(G)$, $\|f\| \leq 1$, there exists a sequence $(f_n)_n \subset Hv_0(G)$ with $\|f_n\| \leq 1$ for each n such that $f_n(z) \to f(z)$ for every $z \in G$. Pointwise *bounded* approximation (that is, the case $v \equiv 1$) by polynomials has been extensively studied in the setting of one complex variable (for example, see Gamelin [7]).

3. Clearly, our arguments work as well for spaces of holomorphic functions on complex manifolds, spaces of harmonic functions, spaces of zero-solutions of linear hypoelliptic partial differential operators, etc. (*cf.* [5, 0.1]).

4. Assertion 1.1.(a) can also be deduced from the Krein-Šmulian theorem (*cf.* [12]). However, the above quoted result by Ng [8, Theorem 1] requires nothing deeper than the bipolar theorem, and thus provides an approach which is both direct and elementary.

In the following section, we shall apply our density criterion to recover the classical results cited in the introduction. Before concluding here, however, we note, for example, that this condition is (trivially) violated when $G = D$ and $v \equiv 1$. In this case, $Hv(G) = H^\infty(D)$ has a unique predual, but is not *isometrically* isomorphic to the second dual of any Banach space (*cf.* Ando [2]). On the other hand, Wojtaszczyk (*cf.* [18]) has shown that $H^\infty(D)$ is *topologically* isomorphic to the bidual of a Banach space.

2. The classical example

EXAMPLE 2.1. Let v be a weight (in the sense of Section 1) on the open unit disk D in $\mathbb{C}$ and assume that v is radial; that is, $v(z) = v(|z|)$ for each $z \in D$. If $\lim_{|z| \to 1_-} v(z) = 0$, then $Hv(D)$ is isometrically isomorphic to $Hv_0(D)^{**}$.

PROOF. Fixing $f \in B$, let $0 < r < 1$, and put $f_r(z) = f(rz)$ for $z \in D$. Then $f_r \in Hv_0(D)$ for each $r \in (0, 1)$. Moreover, given $z \in D$, the maximum modulus theorem yields $\lambda \in \mathbb{C}$ with $|\lambda| = 1$ such that $|f(rz)| \leq |f(\lambda z)|$, and hence

$$v(z)|f_r(z)| \leq v(z)|f(\lambda z)| = v(\lambda z)|f(\lambda z)| \leq \|f\| \leq 1,$$

whereby $f_r \in B_0$ for every $r \in (0, 1)$. Since $(f_r)_r$ converges to f uniformly on any compact subset of D as $r \to 1_-$, B_0 is κ-dense in B, and the desired conclusion now follows from Corollary 1.2.

As the preceding example demonstrates, the biduality result of Rubel and Shields [11] can indeed be readily recovered from our density criterion. Furthermore, the technique of proof clearly allows the conclusion to be carried over to the case where

(i) G is either the open unit polydisk, the open unit ball, or, more generally, any *balanced* (that is, $\lambda z \in G$ whenever $z \in G$ and $\lambda \in \mathbb{C}$ with $|\lambda| \leq 1$) open subset of $\mathbb{C}^N$ for which $\overline{G}$ is a compact subset of $\{z \in \mathbb{C}^N : rz \in G\}$ for each $r \in (0, 1)$, and

(ii) v is a weight on G which vanishes at ∂G and is radial in the sense that

$$(*) \qquad v(\lambda z) = v(z) \text{ for all } z \in G \text{ and every } \lambda \in \mathbb{C} \text{ with } |\lambda| = 1.$$

(For the estimate to show $f_r \in B_0$, fix $z \in G$ and consider the analytic function g of one variable defined by $g(w) := f(wz)$.)

We next show that the corresponding result by Williams [17] in the context of entire functions (of one variable) also follows from our density criterion.

EXAMPLE 2.2. Taking $G = \mathbb{C}$, let v be a radial weight on $\mathbb{C}$ which is rapidly decreasing at infinity (that is, $Hv(\mathbb{C})$ or, equivalently, $Hv_0(\mathbb{C})$ contains the polynomials). Then $Hv(\mathbb{C})$ is isometrically isomorphic to $Hv_0(\mathbb{C})^{**}$.

PROOF. The method of proof for Example 2.1 works as well in the present setting, except that it may not be quite so transparent that the corresponding entire functions f_r belong to $Hv_0(\mathbb{C})$ for each $r \in (0, 1)$. To see this, fix $f \in B$ and $0 < r < 1$. Then, given $\varepsilon > 0$, choose $n \in \mathbb{N}$ with $r^{n+1} < \varepsilon/4$. Next, consider the Taylor series representation $f(z) = \sum_{k=0}^{\infty} a_k z^k$ of f (about 0), and note that the Taylor polynomial $P_n(z) = \sum_{k=0}^{n} a_k z^k$ belongs to $Hv_0(\mathbb{C})$ since v is rapidly decreasing. Consequently, there exists $R > 0$ such that $v(z)|P_n(z)| < \min(\varepsilon/2, 1)$ whenever $|z| > R$. For $|z| > R$, the maximum modulus theorem now yields $\alpha \in \mathbb{C}$ with $|\alpha| = 1$ such that

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$|P_n(rz)| \leq |P_n(\alpha z)|$ whence

$$v(z)|P_n(rz)| \leq v(\alpha z)|P_n(\alpha z)| < \varepsilon/2.$$

On the other hand, if we put $g := f - P_n$, $h(z) = \sum_{k=n+1}^{\infty} a_k z^{k-(n+1)}$ for $z \in \mathbb{C}$, and again consider $|z| > R$, the maximum modulus theorem can be applied once more to get $\lambda \in \mathbb{C}$ with $|\lambda| = 1$ such that

$$\begin{aligned} v(z)|g(rz)| &= v(z)r^{n+1}|z|^{n+1}|h(rz)| \\ &\leq v(z)r^{n+1}|z|^{n+1}|h(\lambda z)| = r^{n+1}v(\lambda z)|g(\lambda z)| \\ &\leq r^{n+1}(\|f\| + v(\lambda z)|P_n(\lambda z)|) < 2r^{n+1} < \varepsilon/2. \end{aligned}$$

Combining these two estimates, we have that $f_r \in Hv_0(\mathbb{C})$, and the proof is complete.

The foregoing proof has the virtue that it also can be extended to higher dimensions and the case of a weight v on $\mathbb{C}^N$, $N > 1$, which is rapidly decreasing at infinity and radial in the sense defined above by $(*)$. In this setting, to verify that $f_r \in Hv_0(\mathbb{C}^N)$ for $f \in B$, one can write the Taylor series of f at 0 as a sum of k-homogeneous polynomials p_k, $k = 0, 1, \ldots$, and take $P_n = \sum_{k=0}^{n} p_k$. If the Euclidean norm of z is larger than some $R > 0$, the first estimate above follows readily. In order to obtain the second one for a fixed $z \in \mathbb{C}^N$ with norm greater than R, consider the entire function g of one variable,

$$g(w) := (f - P_n)(wz) = \sum_{k=n+1}^{\infty} p_k(wz), \ w \in \mathbb{C},$$

and proceed as before.

For entire functions of one variable, however, there is another approach, which we shall sketch since it is in some sense simpler. To begin, given an entire function g (on $\mathbb{C}$), put

$$M(g, r) := \max\{|g(z)| : |z| = r\}, \ r \geq 0,$$

and let $\sigma_n(g)$ denote the Cesàro means of the partial sums of the Taylor series for g about 0. Then it is well-known that

$$M(\sigma_n(g), r) \leq M(g, r)$$

for each $r \geq 0$ and $n \in \mathbf{N}$. Now, fixing $f \in B$ and $n \in \mathbb{N}$, $\sigma_n(f) \in Hv_0(\mathbb{C})$, while the fact that

$$v(z)|[\sigma_n(f)](z)| \leq v(z)M(\sigma_n(f), |z|) \leq v(z)M(f, |z|) \leq \|f\| \leq 1$$

for every $z \in \mathbb{C}$ shows that $\sigma_n(f) \in B_0$. Since $\sigma_n(f)$ converges to f uniformly on each compact subset of $\mathbb{C}$, we conclude that B_0 is κ-dense in B as desired.

As Examples 2.1 and 2.2 illustrate, our density criterion serves to unify (and, to some extent, simplify) the arguments given by Rubel and Shields [11] and Williams [17]. These results would also follow from the main result of [14].

Actually, by using the Cesàro means σ_n of the partial sums of the (k-homogeneous) Taylor series about 0 for a holomorphic function as in the above alternative proof of Example 2.2 plus the fact (due to Fejér) that the corresponding linear operators on the disk algebra are contractive, it is not difficult to deduce the following joint generalization of 2.1 and 2.2.

THEOREM 2.3 (*cf.* [4, Theorem 5]). *Let G be a balanced open subset of $\mathbb{C}^N$ and v a positive continuous function on G which is radial (in the sense of (*)). If $Hv_0(G)$ contains the polynomials, then $Hv(G)$ is isometrically isomorphic to $Hv_0(G)^{**}$.*

We bring this section to a close with some further remarks about radial weights on $\mathbb{C}$.

For $n \in \mathbb{N}$ and $v(z) = (1 + |z|^n)^{-1}$, $z \in \mathbb{C}$, Liouville's theorem yields that $Hv(\mathbb{C})$ is just the space $\mathscr{P}_n$ of polynomials with degree not exceeding n, while $Hv_0(\mathbb{C}) = \mathscr{P}_{n-1}$, and so $Hv(\mathbb{C})$ is certainly not isometrically isomorphic to $Hv_0(\mathbb{C})^{**}$ in this case. On the other hand, it is easy to see that this is the only problem which can arise for a radial weight v on $\mathbb{C}$.

Indeed, for $k \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$, let us put $p_k(z) = z^k$, $z \in \mathbb{C}$. If v is rapidly decreasing on $\mathbb{C}$, then $\{p_k : k \in \mathbb{N}_0\} \subset Hv(\mathbb{C})$. Otherwise, if $n := \min\{k \in \mathbb{N}_0 : p_k \notin Hv(\mathbb{C})\}$, then $\mathscr{P}_{n-1} \subset Hv(\mathbb{C})$ (where we take $\mathscr{P}_{-1} = \{0\}$); we claim that, in fact, $Hv(\mathbb{C}) = \mathscr{P}_{n-1}$. If not, then we can find $f \in Hv(\mathbb{C})$ with $f(z) = \sum_{k=0}^{\infty} a_k z^k$ for $z \in \mathbb{C}$ so that the set $S := \{k \geq n : a_k \neq 0\}$ is nonvoid. Taking $m := \min S$, $h(z) = \sum_{k=m}^{\infty} a_k z^{k-m}$, and $g(z) = z^m h(z)$, $z \in \mathbb{C}$, we have that $g \in Hv(\mathbb{C})$. Moreover, for $z \in \mathbb{C}$, the maximum modulus theorem gives us $\lambda \in \mathbb{C}$ with $|\lambda| = 1$ such that $|a_m| = |h(0)| \leq |h(\lambda z)|$ whence

$$\begin{aligned} v(z)|p_m(z)| &= \frac{1}{|a_m|} v(z)|z|^m |a_m| \leq \frac{1}{|a_m|} v(z)|z|^m |h(\lambda z)| \\ &= \frac{1}{|a_m|} v(z)|g(\lambda z)| \leq \frac{1}{|a_m|} \|g\| < \infty. \end{aligned}$$

Since $p_m \notin Hv(\mathbb{C})$, however, we have reached the desired contradiction. Thus, for a radial weight v on $\mathbb{C}$, $Hv(\mathbb{C})$ *is isometrically isomorphic to* *the second dual* $Hv_0(\mathbb{C})^{**}$ *if and only if* $p_k \in Hv(\mathbb{C})$ *implies* $p_k \in Hv_0(\mathbb{C})$, $k \in \mathbb{N}_0$.

The case of nonradial weights v, on the other hand, remains open; we refer to the last section of [1] for some partial results and an indication of the difficulties that can arise.

3. The predual of $Hv(G)$

Throughout this final section, we assume that the weight v is a strictly positive *continuous* function on G.

In view of Theorem 1.1, it is quite natural to endow $Hv(G)$ with the finest locally convex topology β_v which coincides with κ on B. In the terminology of Cooper [6], β_v is the mixed topology $\gamma(\|\cdot\|, \kappa)$, and $(Hv(G), \beta_v)$ is a *Saks space*. Viewing $Hv(G)$ as the dual of X, however, the weak star topology $\sigma(Hv(G), X)$ is stronger than that of pointwise convergence on G, while its restriction to B is weaker than κ (by the definition of X). Since (B, κ) is compact, these three topologies all coincide on B, and thus β_v is the strongest locally convex topology on $Hv(G)$ which agrees with $\sigma(Hv(G), X)$ on bounded sets; that is, β_v is the (classical) *bounded weak star* topology $bw(Hv(G), X)$ (for example, see Rubel and Ryff [9]). This observation, in turn, leads to a description of β_v as a weighted strict topology.

PROPOSITION 3.1. *Let* $C_0(G)$ *denote the space of continuous functions on* G *which vanish at infinity, and put* $W = vC_0^+(G) = \{v\varphi : \varphi \in C_0(G),\ \varphi \geq 0\}$. *Then* $(Hv(G), \beta_v)$ *is the weighted space* $HW_0(G) = HW(G)$ *of holomorphic functions; that is,* β_v *is the topology given by the system* $(p_w)_{w \in W}$ *of seminorms defined by*

$$p_w(f) = \sup_{z \in G} w(z)|f(z)|, \quad f \in Hv(G).$$

PROOF. Let $C_b(G)$ denote the space of bounded continuous (complex valued) functions on G and β the classical strict topology on $C_b(G)$. The mapping $f \to vf$ is then an isometric isomorphism from $Hv(G)$ onto the closed subspace $vHv(G) = \{vf : f \in Hv(G)\}$ of $C_b(G)$. Since (B, κ) is compact, the unit ball vB of $vHv(G)$ is clearly β-compact. (This fact also follows from [12, Proposition 2].) Thus, $vHv(G)$ is a *normal* subspace of $C_b(G)$ (in the terminology of [12]), and so β coincides on $vHv(G)$ with its bounded weak star topology by [12, Theorem 2]. Simply carrying this back to $Hv(G)$ serves to complete the proof.

The strict topology β on $H1(G) = H^\infty(G)$ was extensively studied by Rubel and Shields [10], while Rubel and Ryff [9] showed that β coincides with the bounded weak star topology in this setting. Shapiro [12] considered the question for a closed subspace E of $C_b(S)$, where S is any locally compact Hausdorff space. In particular, taking $M(S) = C_0(S)^*$ to denote the space of (complex) bounded Radon measures on S, Shapiro [12, Theorem 2] proved that the (sup norm) unit ball of E is β-compact if and only if it is $\sigma(C_b(S), M(S))$-compact and β is the bounded weak star topology $bw(E, M(S)/E^\perp)$. This was carried even further in [13], where these results were extended to the generalized strict topology induced on a left Banach module by a Banach algebra with bounded approximate identity.

Returning to our context, the next proposition follows immediately from standard facts about bounded weak star and mixed topologies.

PROPOSITION 3.2. (a) *β_v is the finest topology on $Hv(G)$ which coincides with the compact-open topology κ on B, and agrees with the topology of uniform convergence on the (norm) null sequences of the predual X of $Hv(G)$. A subset of $Hv(G)$ is β_v-closed if and only if it is β_v-sequentially closed.*

(b) *$(Hv(G), \beta_v)$ is a complete semi-Montel (gDF)-space whose bounded sets are exactly the norm bounded sets (on which β_v and κ agree and which are relatively compact in this topology). Except for trivial cases, this space is neither bornological nor barrelled.*

(c) *$X = (Hv(G), \beta_v)'_b = (Hv(G), \beta_v)'_c$ and $X'_c = (Hv(G), \beta_v)$, where $'_b$ and $'_c$ denote the continuous dual with the strong topology and the topology of uniform convergence on (pre-)compact subsets, respectively.*

(d) *The duality pairing of X and $(Hv(G), \beta_v)$ can be described as follows: Each $F \in X$ has an integral representation of the form*

$$F(f) = \int_G fv\, d\mu, \qquad f \in Hv(G),$$

for some bounded Radon measure μ on G, and isometrically

$$X = (Cv(G), \gamma_v)'_b / Hv(G)^\perp = CW_0(G)'_b / Hv(G)^\perp,$$

where $Cv(G)$ and $CW_0(G)$ are the continuous analogues of $Hv(G)$ and $HW_0(G)$, respectively, and γ_v is the corresponding mixed topology on $Cv(G)$.

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