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DECOMPOSABLE INVOLUTION CENTRALIZERS INVOLVING EXCEPTIONAL LIE TYPE SIMPLE GROUPS

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1. Introduction

There have been investigations (Janko (1966), Janko and Thompson (1966), Yamaki (1972)) of finite groups G which contain a central involution t whose centralizer (in G) has the form $C(t) = \langle t \rangle \times F$, where F is isomorphic to a non-abelian simple group. Here it is shown such a group cannot be simple when F is isomorphic to an exceptional Lie type simple group of odd characteristic. Specifically the following theorem is proved.

THEOREM 1.1. *Let G be a finite group with a central involution t whose centralizer has form*

$$C(t) = \langle t \rangle \times F,$$

where F is isomorphic to an exceptional Lie type simple group of odd characteristic. Then G has a subgroup of index 2 not containing t.

Theorem 1.1 may be combined with the results of Curran to give:

THEOREM 1.2. *Let G be a finite group with a central involution t whose centralizer has the structure*

$$C(t) = \langle t \rangle \times F$$

where F is isomorphic to any alternating simple group or any classical or exceptional Lie type simple group of odd characteristic. Then G has a subgroup of index 2 not containing t (and so G is not simple), except when $F \approx A_5$ or $F \approx$ PSL$(2, 3^{2n+1})$ $(n \geqq 1)$.

Of course these are true exceptions, for the centralizer of an involution t in the Janko simple group of order 175,560 has the form $C(t) \approx \langle t \rangle \times A_5$ (Janko (1966)); while in the simple Ree groups $C(t)$ has the structure $C(t) \approx \langle t \rangle \times$ PSL$(2, 3^{2n+1})$ $(n \geqq 1)$ (Janko and Thompson (1966)).

The notation follows that of Carter (1972) in which standard facts on the Chevalley groups may be found.

2. Straightforward cases

In this section Theorem 1.1 is proved when F is isomorphic to one of the following simple Lie groups: $G_2(q)$, $F_4(q)$, $E_6(q)$, $E_8(q)$, ${}^3D_4(q^3)$, ${}^2E_6(q^2)$ or ${}^2G_2(3^{2n+1})$ $(n \geqq 1)$. The proof of the theorem is straightforward in these cases, because when $F \approx {}^2G_2(3^{3n+1})$ $(n \geqq 1)$ a Sylow 2-subgroup of G is abelian and appeal to the theorem of Walter (1969) characterizing such groups yields the result; while in the remaining cases every involution in F is a square in F and the theorem follows easily from (2.1) and (2.2) below.

PROOF OF THEOREM 1.1. First consider the case when $F \approx {}^2G_2(3^{2n+1})$ $(n \geqq 1)$, the twisted Ree group of type G_2 over the field $\mathbf{F}_{3^{2n+1}}$ $(n \geqq 1)$. This group has only one class of involutions and as noted above if x is any involution in F, $C_F(x) \approx \langle x \rangle \times \mathrm{PSL}(2, 3^{2n+1})$. Thus a Sylow 2-subgroup of G is elementary abelian of order 8, and so a Sylow 2-subgroup of G is elementary abelian of order 16.

Without loss of generality we may assume $O(G) = 1$, where $O(G)$ is the maximal normal odd order subgroup in G. Then if $O'(G)$ is the minimal normal subgroup of odd index in G, $C(t) \cap O'(G) \lhd C(t)$ and contains a Sylow 2-subgroup of G. Thus $\langle t \rangle \times F \subseteq O'(G)$. But both G and $\langle t \rangle \times F$ have 2-order 16 so by a theorem of Walter (1969) characterizing groups with abelian Sylow 2-subgroups $O'(G) = \langle t \rangle \times F$. Therefore $F \lhd G$ and G/F has order $2k$, k odd. Thus by a theorem of Burnside G/F has a subgroup of index 2 not containing tF and the conclusion of Theorem 1.1 follows.

Now consider the remaining cases. Since t is central, $C(t)$ contains a Sylow 2-subgroup of G of form $S = \langle t \rangle \times M$ where M is a Sylow 2-subgroup of F. We show t is not fused (that is conjugate in G) to any involution in M and use the following lemma of Thompson (1968).

LEMMA 2.1. *Let M be a subgroup of index 2 in a Sylow 2-subgroup S of a finite group G. Let t be an involution in $S \backslash M$ which is not fused to any element of M. Then G has a (normal) subgroup of index 2 not containing t.*

Now the structure of $C(t)$ shows t cannot be fused with an involution which is a square in F. More generally if $\langle L^2 \rangle$ denotes the group generated by the squares of elements of a subgroup L of G, the following holds:

LEMMA 2.2. *t is not fused to any involution $x \in \langle C_F(x)^2 \rangle$.*

PROOF. Suppose on the contrary $x = \Pi_{i=1}^{m} x_i^2$ where $x_i \in C_F(x)$ (m a positive integer) and $t = x^a$, some $a \in G$. Then $t = \Pi_{i=1}^{m}(x_i^a)^2$. But $x_i \in C_F(x) \subseteq C(x)$ so $x_i^a \in C(x)^a = C(t)$. Thus $t \in \langle C(t)^2 \rangle \subseteq F$, a contradiction. But in the remaining simple Lie groups above, every involution is a square and so the conclusion of Theorem 1.1 follows immediately from (2.1) and (2.2). In the table below we list these groups, together with the corresponding Dynkin diagram, a representative of each class of involutions and the order 4 element of which it is a square.

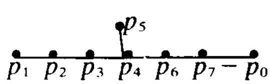
Group	Dynkin Diagram	Representatives	Order 4 elements
$G_2(q)$	1 2	h_1	n_1
$F_4(q)$	1 2 3 4	h_2h_4	n_2n_4
		h_1h_3	n_1n_3
$E_6(q), {}^2E_6(q^2)$	4; 1 2 3 5 6	h_2h_5	n_2n_5
		$h_1h_3h_6$	$n_1n_3n_6$
$E_8(q)$	6; 1 2 3 4 5 7 8	$h_2h_4h_6$	$n_2n_4n_6$
		h_6h_7	n_6n_7
${}^3D_4(q^3)$	1 2 3 4	h_2	n_2

The representatives chosen are found from Iwahori (1970). The element $h_i = h_i(-1)$ and the element $n_i = x_{p_i}(1)x_{-p_i}(-1)x_{p_i}(1)$ is a generator of the subgroup N in the Chevalley group. $n_i^2 = h_i$, and for $i \neq j$ we have $n_in_j = n_jn_i$ if and only if node i and node j are not connected in the Dynkin diagram.

3. The case $E_7(q)$

Finally consider the case when $F \approx E_7(q)$, the adjoint Chevalley group of type E_7 over the finite field $k = \mathbf{F}_q$ of q elements (q odd).

Let Φ be the set of roots of a complex semi-simple Lie algebra of type E_7 relative to a Cartan subalgebra. For a fixed ordering of Φ let Φ^+ be the positive roots containing a fundamental system $\Pi = \{p_1, p_2, \cdots, p_7\}$, with Dynkin diagram



where p_0 is the highest root in Φ^+, $p_0 = p_1 + 2p_2 + 3p_3 + 4p_4 + 2p_5 + 3p_6 + 2p_7$.

Let $W = \langle w_r \mid r \in \Phi \rangle$ be the Weyl group of Φ, and $w_0 \in W$ be the symmetry which interchanges p_3 and p_6, p_2 and p_7, p_1 and $-p_0$, and fixes p_4 and p_5.

Let $E = \langle x_r(t) | r \in \Phi, t \in k \rangle$ be the associated adjoint Chevalley group over the field k, which contains the subgroup $H = \langle h_r(t) | r \in \Phi, t \in \dot{k} \rangle$, where $\dot{k} = \langle \kappa \rangle$ is the multiplicative group of k. The universal Chevalley group of type E_7 has centre $\langle h_1(-1)h_3(-1)h_5(-1) \rangle$ of order 2, so the adjoint group E is generated by the elements $x_r(t)$ ($r \in \Phi$, $t \in k$) satisfying the usual relations for the universal group with the additional relation $h_1(-1)h_3(-1)h_5(-1) = 1$.

Let K be the algebraic closure of k, and for the extension field $\mathbf{F}_{q^2}$ of k in K, let $\dot{\mathbf{F}}_{q^2} = \langle \rho \rangle$. Put $i = \rho^{(q^2-1)/4}$ and $\sqrt{\kappa} = \rho^{(q+1)/2}$. Denote by $\bar{E} = \langle x_r(t) | r \in \Phi, t \in K \rangle$ the connected linear group over K containing E as a subgroup in the natural way, and $\bar{H} = \langle h_r(t) | r \in \Phi, t \in \dot{K} \rangle$.

The mapping $x_r(t) \to x_r(t^q)$ ($r \in \Phi$, $t \in K$) on the generators of $\bar{E}$ extends to the Frobenius automorphism σ of $\bar{E}$. For a subset X of $\bar{E}$ invariant under σ, let X_σ denote the fixed points of σ in X. Then $E^* = \bar{E}_\sigma$ is the group of k-rational points in $\bar{E}$. One knows $|E^* : E| = 2$, $E^* = E.H^*$ and $E \cap H^* = H$, where $H^* = \bar{H}_\sigma$. In fact $E^* = \langle h_1(\sqrt{\kappa})h_3(\sqrt{\kappa})h_5(\sqrt{\kappa}) \rangle.E$.

The proof of Theorem 1.1 requires the classes of involutions and their centralizers in E. First, from the results of Iwahori (1970), we give the classes and their centralizers in $\bar{E}$.

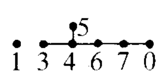
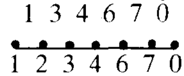
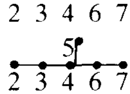
LEMMA 3.1. (i) *There are three classes of involutions in $\bar{E}$ having the following representatives in H^*:*

$$z = h_1(-1)$$

$$u_1 = h_1(i)h_2(-1)h_3(-i)h_5(i)$$

$$u_2 = h_1(i)h_3(-i)h_4(-1)h_5(i)h_7(-1).$$

(ii) *If C^0 is the connected component of the identity for the centralizer C of any of the above involutions then C/C^0 is finite abelian. Coset representatives of C^0 in C are certain $n_w \in N = N_E(H)$, $w \in W$, where the n_w can be chosen to have the same order as w. Further $C_0 = \bar{H}.X$ where X is a connected semi-simple algebraic group and $X \lhd C^0$. The following table lists for z, u_1 and u_2 the simple components of X, the corresponding set of fundamental roots, the order of C/C^0 and the coset representatives in N.*

Representatives	Components	Root Structure	$\lvert C/C^0 \rvert$	n_w
z	A_1, D_6		1	1
u_1	A_7		2	n_{w_0}
u_2	E_6		2	n_{w_0}

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PROOF. If Γ_π is the **Z**-lattice spanned by Π, $\bar{H}$ may be identified with $\mathrm{Hom}(\Gamma_\pi, \dot{K})$. Then (ii) follows from proposition 1 in Iwahori (1970) with $X = \langle X_r(t) | r \in \Phi_\chi, t \in K \rangle$ where the involution $h \in \bar{H}$ corresponds to $\chi \in \mathrm{Hom}(\Gamma_\pi, \dot{K})$ and $\Phi_\chi = \{ r \in \Phi | \chi(r) = 1 \}$. The involutions z, u_1, u_2 correspond to $\chi_{\lambda_2}(2)$, $\chi_{\lambda_5}(2)$, $\chi_{\lambda_1}(1)$ respectively in Iwahori (1970) and the classes for $\bar{E}$ (giving (i)) and the Φ_χ for each of these involutions are given on pages F23, F24 of that paper.

LEMMA 3.2. (i) *There are* 5 *classes of involutions in* E^* *with representatives* z, u_i, u_i^a $(i = 1, 2)$ *where* $n_{w_0} = a^\sigma . a^{-1}$ $(a \in \bar{E})$.

(ii) *There are* 3 *classes of involutions in* E *with representatives*:

$$z,\ u_i\ \ (i = 1, 2) \quad \textit{when } q \equiv 1 \pmod 4.$$

$$z,\ u_i^a\ \ (i = 1, 2) \quad \textit{when } q \equiv -1 \pmod 4.$$

PROOF. By a lemma of Steinberg (10.1 in Steinberg (1968)) if $n_w \in \bar{E}$, $n_w = x^\sigma x^{-1}$ for some $x \in \bar{E}$. Then proposition 6 in Iwahori (1970) shows that for any involution $h \in H^*$ the map

$$C_{\bar{E}}(h)/C_{\bar{E}}^0(h) \to (h^{\bar{E}} \cap E^*)/E^*,$$

(the E^* class of h lying in the $\bar{E}$ class of h), defined by: $n_w \to (h^x)^{E^*}$ (the E^* class of h^x), is a bijection. (i) now follows from (3.1) taking $n_{w_0} = a^\sigma a^{-1}$ $(a \in \bar{E})$ and (ii) is contained in Iwahori (1970) on pages F26, F27.

In order to treat the cases $q \equiv \pm 1 \pmod 4$ uniformly we observe (for $i = 1, 2$),

$$C_{E^*}(u_i^a) = C_{\bar{E}}(u_i^a, \sigma) \underset{a^{-1}}{\approx} C_{\bar{E}}(u_1, \sigma^{a^{-1}}) = C_{\bar{E}_\theta}(u_i)$$

where $\bar{E}_\theta$ is the group of fixed points of $\theta = \sigma^{a^{-1}} = \sigma n_{w_0}$ in $\bar{E}$. So when $q \equiv -1 \pmod 4$ it is convenient to let $E(-1) = E^{a^{-1}}$, of index 2 in $\bar{E}_\theta$, and consider the centralizer of u_i in $E(-1)$, since $C_E(u_i^a) \underset{a^{-1}}{\approx} C_{E(-1)}(u_i)$. Because $\theta = \sigma n_{w_0}$ these centralizers involve twisted Lie groups. We also put $H(-1) = H^{a^{-1}}$, $H^*(-1) = H^{*a^{-1}}$, $E(1) = E$, $H(1) = H$, $H^*(1) = H^*$ and assume $q \equiv \varepsilon \pmod 4$, $(\varepsilon \pm 1)$ in the following.

LEMMA 3.3. (i) $C_E(z) = H.L(z)$ *where* $L(z)$ *is a central product of Lie groups of type* A_1 *and* D_6. *Further* z *is the only central class in* E *and* z *is a square in* E.

(ii). $C_{E(\varepsilon)}(u_1) = \langle h_1 \rangle L(u_1)$, *where* $L(u_1)$ *is of type* $A_7(q)$ $(\varepsilon = 1)$ *or* ${}^2A_7(q_2)$ $(\varepsilon = -1)$, *and* $h_1 \in H(\varepsilon)$.

(iii) $C_{E(\varepsilon)}(u_2) = \langle h_2 \rangle L(u_2)$, *where* $L(u_2)$ *is of type* $E_6(q)$ $(\varepsilon = 1)$ *or* ${}^2E_6(q^2)$ $(\varepsilon = -1)$, *and* $h_2 \in H(\varepsilon)$.

(iv) *For* $i=1,2$ $u_i \in \langle C_{E(\varepsilon)}(u_i)^2\rangle$ *if and only if* $q \equiv \varepsilon \pmod 8$.

PROOF. (i) From (3.1) it follows that $C_E(z) = H.L(z)$, where $L(z) = \langle X_{\pm p_i}(t) | 0 \leqq i \leqq 7, i \neq 2; t \in k\rangle$.

Let $L_1 = \langle X_{\pm p_1}(t) | t \in k\rangle$ and $L_2 = \langle X_{\pm p_i}(t) | i = 0,\ 3 \leqq i \leqq 7;\ t \in k\rangle$, then $L(z) = L_1.L_2$ where L_1 is of type A_1 and L_2 is of type D_6. In fact $Z(L_1) = \langle h_1(-1)\rangle$ so $L_1 \approx \mathrm{SL}(2,q)$, and

$$Z(L_2) = \langle h_0(-1)h_5(-1)h_6(-1)\rangle \times \langle h_3(-1)h_5(-1)\rangle = \langle h_1(-1)\rangle$$

so L_2 is a homomorphic image of $\mathrm{Spin}(12,q)$ which is not $\Omega(12,q)$. Clearly $[L_1, L_2] = 1$ and $L_1 \cap L_2 = \langle h_1(-1)\rangle$ so $L(z)$ is the central product of L_1 and L_2. Further $L(z)$ is of index 2 in $C_E(z)$, because clearly $C_E(z) = \langle h_2(\kappa)\rangle.L(z)$ and $h_2(\kappa)^2 \in L(z)$.

Also z is obviously a square in E ($z = n_1^2$) and a calculation of $|E : C_E(z)|$ shows z is central in E.

(ii) For $i = 1,2$, $C^0_{E(\varepsilon)}(u_i) = E(\varepsilon) \cap C^0_E(u_i)$. So from (3.1) $C^0_{E(\varepsilon)}(u_i)$ is of index 2 in $C_{E(\varepsilon)}(u_i)$ with coset representative n_{w_0}, and $C^0_{E(\varepsilon)}(u_i) = H(\varepsilon).L(u_i)$, with $L(u_i) \lhd C_{E(\varepsilon)}(u_i)$.

Now $L(u_1) = \langle X_{\pm p_i} | 0 \leqq i \leqq 7, i \neq 5\rangle$ is of type $A_7(q)$ ($\varepsilon = 1$) or of type ${}^2A_7(q^2)$ ($\varepsilon = -1$). If $d = (q-\varepsilon, 8)$ and $\gamma = \rho^{(q^2-1)/a}$ then

$$Z(L(u_1)) = \langle h_1(\gamma)h_2(\gamma^2)h_3(\gamma^3)h_4(\gamma^4)h_6(\gamma^5)h_7(\gamma^6)h_0(\gamma^7)\rangle$$
$$= \langle h_1(\gamma^2)h_2(\gamma^4)h_3(\gamma^6)h_5(\gamma^2)\rangle .$$

Thus when $q \equiv \varepsilon \pmod 8$, $Z(L(u_1)) = \langle u_1\rangle$, and when $q \equiv 4+\varepsilon \pmod 8$, $Z(L(u_1)) = 1$. So in the latter case $L(u_1) \approx \mathrm{PSL}(8,q)$ ($\varepsilon = 1$) or $L(u_1) \approx \mathrm{PSU}(8,q)$ ($\varepsilon = -1$).

In fact if $\lambda = \rho^{q-\varepsilon}$ (so $\lambda^q = \lambda = \kappa$ ($\varepsilon = 1$) and $\lambda^q = \lambda^{-1}$ ($\varepsilon = -1$)) it is easily seen that

$$H(\varepsilon).L(u_1) = \langle h_1\rangle.L(u_1)$$

where $h_1 = h_1(\lambda)h_2(\lambda^2)h_3(\lambda^3)h_4(-\lambda^2)h_5(\lambda)$.
Note: if $\sqrt{\lambda} = \rho^{(q-\varepsilon)/2}$, and $g_1 = h_1(\sqrt{\lambda})h_2(\lambda)h_3(\sqrt[3]{\lambda})h_4(-\lambda)h_5(\sqrt{\lambda})$ then $g_1 \in H^*(\varepsilon) - H(\varepsilon)$ and $g_1^2 = h_1$. Since $h_1{}^2 \in L(u_1)$ and $h_1^{n_{w_0}} = h_1^{-1} \equiv h_1 \pmod{L(u_1)}$, $C_{E(\varepsilon)}(u_1)/L(u_1) \approx Z_2 \times Z_2$, the 4-group.
Hence $L(u_1) = L(u_1)' \subseteq C_{E(\varepsilon)}(u_1) \subseteq \langle C_{E(\varepsilon)}(u_1)^2\rangle \subsetneq L(u_1)$.

(iii) $L(u_2) = \langle X_{\pm p_i} | 2 \leqq i \leqq 7\rangle$ is of type $E_6(q)$ ($\varepsilon = 1$) or of type ${}^2E_6(q^2)$ $\varepsilon = -1$), with a centre of order $(3, q-\varepsilon)$. In fact

$$H(\varepsilon).L(u_2) = \langle h_2\rangle.L(u_2)$$

where $h_2 = h_1(\lambda)h_3(\lambda^3)h_4(\lambda^2)h_5(\lambda)h_7(\lambda^2)$. Since $h_2^{(q-\varepsilon)/2} = 1$ and $h_2^{n_{w_0}} = h_2^{-1}$, $C_{E(\varepsilon)}(u_2)/L(u_2)$ is dihedral.

Thus $\langle C_{E(\varepsilon)}(u_2)^2\rangle = C_{E(\varepsilon)}(u_2)'$ is of index 2 in $H(\varepsilon)L(u_2)$.

(iv) Since $h_i^{(q-\varepsilon)/4} = u_i$, $u_i \in \langle C_{E(\varepsilon)}(u_i)^2\rangle$ if and only if $q \equiv \varepsilon \pmod 8$, and in fact when $q \equiv \varepsilon \pmod 8$, $u_i = (h_i^{(q-\varepsilon)/8})^2$ is actually a square in E.

A calculation of $|E(\varepsilon): C_{E(\varepsilon)}(u_i)|$ shows the u_i are not central ($i = 1, 2$) in $E(\varepsilon)$.

We now give the proof of (1.1) when $F \approx E_7(q)$.

PROOF OF (1.1). We put $F = E(\varepsilon)$ where as above $q \equiv \varepsilon \pmod 4$, $\varepsilon = \pm 1$; and relabel z^a, the representative of the central class in $E(-1)$, z. Thus the classes in F have representatives z, u_1 and u_2. We show t is not fused to any of these three involutions and the theorem then follows from (2.2). By (3.3) when $q \equiv \varepsilon \pmod 8$ z, u_1 and u_2 are all squares in F and the conclusion of the theorem follows from (2.1).

Thus assume $q \equiv 4 + \varepsilon \pmod 8$. A maximal set of representatives of the classes of involutions in $C(t) = \langle t\rangle \times F$ is the set $\{t, z, u_i, tz, tu_i \mid i = 1, 2\}$.

(a) Again z is a square in F by (3.3) so $t \not\sim z$ by (2.2).

(b) Suppose $tz \sim t$ in G, say $(tz)^b = t$ some $b \in G$. As $t \in C(tz)$, $t^b \in C(t)$ and is thus conjugate to one of the involutions above. Suppose $t^b \sim u_i$ or tu_i ($i = 1, 2$). In fact we may assume $t^b = u_i$ or tu_i. In either case

$$C(t, z)^b = C(t, tz)^b = C(t, u_i),$$

$$\text{hence } (C(t, z)^{(\infty)})^b = C(y, u_i)^{(\infty)},$$

where for a subgroup L of G, $L^{(\infty)}$ is the last term in the derived series of L.

Now $C(t, z) = \langle t\rangle \times C_F(z)$, so $C(t, z)^{(\infty)} = C_F(z)^{(\infty)} = L(z)$ and similarly $C(t, u_i)^{(\infty)} = L(u_i)$. This gives a contradiction since by (3.3) $L(z)$ and $L(u_i)$ are not isomorphic. Thus $t^b \sim tz$ and again we may assume $t^b = tz$ so $tz \to t \to tz$ under b. Therefore $\langle C(t, z), b\rangle$ is a subgroup of order $2|C(t, z)|$, contradicting the fact that $C(t, z)$ contains a Sylow 2-subgroup of G (since z is central in F). Thus $tz \not\sim t$ in G.

(c) Suppose u_i is conjugate to t in G so $u_i^{c_i} = t$, some $c_i \in G$. Now $u_1^{n_5} = u_1 h_5(-1)$ and $u_2^{n_0 n_1} = u_2 h_0(-1)h_1(-1) = u_2 h_3(-1)h_6(-1)$, where n_5, $n_0 n_1$, $z_1 = h_5(-1)$, $z_2 = h_3(-1)h_6(-1) \in F$. Thus $u_i \sim u_i z_i$ where z_i is central in F (since z_i is clearly a square in F for $i = 1, 2$).Conjugating this relation by c_i, and assuming for the moment $z_i^{c_i} = z_i$, we have $t \sim tz_i$ in G. However $tz_i \sim tz$ in G and this contradicts the result of (b).

To justify the assumption, recall we are assuming $u_i^{c_i} = t$. Then as in (b), since $L(z)$, $L(u_1)$ and $L(u_2)$ are not isomorphic by (3.3), we may suppose $t^{c_i} = u_i$ or tu_i. In either case c_i centralizes $C(t, u_i)$ and so induces an automorphism on $C(t, u_i)^{(\infty)} = L(u_i)$. But $z_i \in L(u_i)$ and by a theorem of Steinberg (1968) every automorphism ψ of a Chevalley group is of form $\psi = fgdi$ where f is a

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field, g, a graph, d a diagonal and i an inner automorphism. As z_i is fixed by field, graph and diagonal automorphisms $z_i^{c_i} = z_i^{f_i}$ some $f_i \in L(u_i) \subseteq F$. Replacing c_i by $c_i' = c_i f_i^{-1}$ we have $u_i^{c_i'} = t$ and $z_i^{c_i'} = z_i$ as assumed.

Therefore t is not fused to any involution in F and the conclusion of the theorem follows from (2.1). This completes the proof of (1.1).

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