THE EFFECT OF NON-STATIONARY ACTION ON THE MOTION OF PARTICLES IN PROTOPLANETARY NEBULAE

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Dynamical evolution of a plane system of non-interacting particles, which are in a certain smoothed-out regular field, is of a certain interest in a number astronomical applications. Among them are the plane subsystems of the spiral galaxies, the rings of the giant planets and protoplanetary nebulae. The problem of determining the stationary state of these systems has been studied enough [1]. But one seldom comes across studies concerning with the reaction of the non-stationary field.

We consider various types of the non-stationary actions, which may be in certain Cases strictly periodic, but it may be quasiperiodic in the sense that the amplitude and the phase change gradually from one period to the other. In the general form this phenomenon is well known in the theory of oscillations, for instance, when superpositioning the oscillations with close frequencies (beats). Various mechanisms of generating these oscillations are admissible. In Case of nebulae, the non-linear effects or resonances can be a cause of such quasiperiodic disturbances.

First, we are trying to elaborate a suitable procedure for describing small disturbances of statistical nature to which belong the enumerated plane subsystems due to their extension along the radius and, particularly, because of dispersion of velocities. Secondly, special interest for us is one aspect of the problem, namely difference of the reversible changes from the irreversible ones.

At first we give a model disturbing potential in the form

$$U(r,\theta) = U^{(0)}(r) + \varepsilon U^{(1)}(r)\cos\nu(\theta - A(r) - \int \Omega(t)dt) \quad (1)$$

where $U^{(0)}(r)$ is the main part of the potential, ε is the small parameter, $A(r)$ is a certain function depending on the concrete form of the spiral, Ω is the angular velocity of revolution of the particles in the nebula, ν is the azimuthal wave number (we consider ν as the arbitrary positive integer), t is time, r is the radius.

In fact, further we shall restrict ourselves to rotation of a rigid body only and to the near circular orbits.

Let us go over to the action-angle variables. The disturbing part of the potential must be also expressed in terms of these variables (it appears to be a periodic function of the angular variables). In the Case when this periodic dependence is a fast oscillating, it may be of convenience to carry out an averaging with respect to angles, so that the averaged Hamiltonian appears to be a function of variables

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action only. Then the averaged trajectories are easily found. Averaging with respect to the epicycle motion is a physical meaning of this operation. The circular epicycle occurs rather often, but we use the epicycle as an ellipse with the arbitrary ratio of the semiaxes.

The different situation exists in a Case of slow oscillations. Then, the principle of adiabatic invariance does not work, and the adiabatic invariants are defined in a more complicated way. This Case appears to be especially typical for the corotation. Except for corotation, the singularities in application of the adiabatic invariants arise from Lindblad's resonances as well.

The way at once suggests itself to take the angular momentum $\mathcal{I}$ (the integral of the motion), characterizing the regular rotation of the population, as one of the variables of action. The other action variables, say, N, we associate with a peculiar motion of particles. One may show that the expression for N starts with a quadratic form, the coefficients of which can be found in terms of the epicycle frequency κ and the angular velocity Ω. Then we find, after some, simple in concept, computations, the angular variables θ'' and φ. Finally, the equations of the undisturbed motion are defined by the formulae:

$$\frac{dN}{dt} = \frac{d\mathcal{I}}{dt} = 0, \qquad \frac{d\theta''}{dt} = \frac{\partial E}{\partial \mathcal{I}}, \qquad \frac{d\varphi}{dt} = \frac{\partial E}{\partial N} \tag{2}$$

(E is the energy).

Let us pass to the coordinate system rotating together with a wave with a constant angular velocity ω. The quantity θ is now counted off relative to the new coordinate system, that is θ is equal to the former value $\theta - \omega t$. The corresponding canonical equations keep the previous form (2) with the Hamiltonian in the form $H_1 = H - \omega\mathcal{I} = E - \delta U - \omega\mathcal{I}$, where, according to (1), δU is a known function.

Then we find $\widehat{\delta U}$, (the averaged part of δU) and introduce the derived $\widehat{\delta U}$ into the equations of the motion. The averaged problem has one degree of freedom and in accordance with composition of our equations the motion of a particle is as if reduced to moving along the level curves in the plane $(\mathcal{I}, \theta'')$.

In general, resonant zones cover rather narrow zones along r. In majority of Cases in their limits the function $U^1(r)$ may be regarded as constant and equal, say, to b. Respectively, the function $A(r)$ is supposed to be the linear one: $A(r) = \alpha(r - r^*)$, where r^* is the radius of the middle line of the resonance zone.

After linearization with respect to $r - r^*$, the Hamiltonian $\widehat{H_1}$ takes the form

$$\widehat{H_1} = E_1 + b\cos\nu\theta'' - \alpha(r - r^*)\sin\nu\theta'',$$

where the E_1 is a totality of terms, which do not depend on N and θ''. If one gives the specific value h for $\widehat{H_1}$, it is seen after some reasoning that under $h > b$ the corresponding curve falls into the two isolated branches and under $h < b$ the real value $\tilde{y} = r - r^*$ corresponds to the only possible interval of changing the variable θ''. This gives an oval-like curve, which does not envelope the center of the system. In the intermediate Case, separatrixes are obtained, which are topologically similar to those of the well-known problem of the motion of the mathematical pendulum with its rotatory and oscillatory types of behavior [2, 3].

If one goes to the wave, depending smoothly on time, it is necessary to take into account the adiabatic invariance of the quantity

$$T = \oint \mathcal{I} d\theta''.$$

Here, the integral is taken along the level curve $\widehat{H_1} = const$. The closure of the way is understood in the sense of keeping the variable θ'' of the previous value or the value, changed by $\pm 2\pi$.

Three Cases take place.

Case 1. – Responds to the trapped orbits, when in the rotating coordinate system the particle performs the pendulum-like motions about the libration center and, consequently, moves with the mean angular velocity Ω of the disturbed field.

Case 2. – Corresponds to the flying by orbits, which are not synchronous to the disturbing field, that is, they lag behind or pass ahead the wave.

Case 3. – Has an intermediate character. The motion here takes place along the separatrix.

Let us take into account consequences of the dependence on time of the parameter Ω, when the disturbance has the quasiperiodic character. Again, neglecting the comparatively unimportant dependence on N of the Hamiltonian and restricting ourselves to the Case $\alpha = 0$, we obtain the modified Hamiltonian, for which the classification of the orbits could be made in the same way as before (although in a more complicated form). In particular, a question can be raised on the transformation of the family of trapped particles into the flying by ones.

Here we have the following picture of the non-stationary disturbance effects. There is a category of the trapped particles which move together with the resonant zone and fill the phase space is some domains, not closing with each other. The other (flying by) particles pass in the intervals between these domains and are not subject to such essential action of disturbance. It is also clear that under the sufficiently large amplitude of changing the quantity Ω even if it returns finally to the previous value, the irreversible changes of the type corresponding to the phase intermixing are accumulated. The latter take place due to the redistribution of the phase domains of various behavior and at expense of "chaotization" of the motion in the resonance zones, in the vicinity of the separatrixes. According to Liouville's theorem, any irreversibility can lead to the diffusion of the phase diagram only, that is usually equivalent to increasing the peculiar velocities. It is not expected that such "heating" tends to decrease the density of the particles in the resonance zones.

All this entitles us to draw the following conclusions: Effects of non-stationary actions lead to the local temporary decrease of the density during the time interval as long as the specific zone is a resonance one.

References

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