Professor George Chrystal, M.A., LL.D. By Dr J. Sutherland Black and Professor C. G. Knott.

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Part I.—Life and Career. By Dr J. S. Black.

George Chrystal was born on the 8th of March 1851, at Mill of Kingoodie, in the parish of Bourtie, some thirteen miles to the north-west of the city of Aberdeen. His father, William Chrystal of Gateside, who achieved some success both in agriculture and in commerce, is described as having been a man who made his way, without any initial advantages, by sheer force of character and the exercise of great natural ability and originality. His son, the subject of this memoir, received his early education at the parish school of Old Meldrum, some two miles distant from his home. From an early age he gave marked promise of intellectual distinction, though physically he was far from strong, and was hampered by a lameness which he afterwards outgrew, but which precluded him from joining in some of the more boisterous activities of boyhood. Early in the 'sixties the family removed to Aberdeen, where in 1863 he entered the Grammar School. Of this period few memorials survive, beyond a number of medals which show that he maintained the early promise of his childhood. In 1866 he gained the Williamson Scholarship, and in 1867 he passed, in his seventeenth year, into the University. Among his teachers here, he was accustomed to refer to Bain as having perhaps had the greatest influence on his whole intellectual development. But he also acknowledged his deep indebtedness to Geddes, Fuller, Nicol, Thomson, and others, of whom some account, inspired by himself, will be found in the recently published *Life* of his elder contemporary, William Robertson Smith, who had graduated with the highest distinction in 1866, and whose brilliant career is known to have been a pattern and an incentive to so many of the ablest men of the younger generation. Chrystal, who was the friend and contemporary at Aberdeen of Sir William M. Ramsay, seems at first to have been much attracted to classical scholarship, and his interest in the Literæ Humaniores continued strong and keen to the end of his life; but ultimately the study of the mathematical sciences became the absorbing pursuit of his academic years. In this he undoubtedly was following the natural bent of his genius; and a happy determining

circumstance was his connection with Dr David Rennet, the talented and stimulating extra-mural teacher to whose instruction he resorted in his summer vacations, and to whom many years afterwards he dedicated his *Introduction to Algebra*, "in memory of happy hours spent in his classroom in days of old." That he did not specialise thus early to the exclusion of all other intellectual pursuits may be judged from the fact that, when he graduated, in 1871, he received the Town's Gold Medal, "awarded annually to the most distinguished scholar at the termination of the Arts curriculum." By this time, however, he had found a clear vocation in the sphere of the mathematical sciences, as is strikingly shown by the fact that within less than a year he gained all the honours accessible to students in this department: the Simpson Prize in Mathematics, the Arnott Prize in Natural Philosophy, the Fullerton Scholarship in Mathematics and Physics, the Ferguson Mathematical Scholarship open to recent graduates from any of the four Universities of Scotland, and, finally, an open scholarship at Peterhouse, Cambridge.

At Cambridge he commenced residence in 1872, and in 1875 he graduated as second wrangler and second Smith's prizeman. Of his teachers during these three years the most influential and formative undoubtedly was Clerk Maxwell, who had delivered his inaugural lecture as Cavendish Professor of Experimental Physics in October 1871, had published his *Treatise on Electricity and Magnetism* in 1873, and had opened the Cavendish Laboratory in June 1874. Clerk Maxwell was not one of those whose instruction came within the ordinary scope of work for the Mathematical Tripos, and Chrystal, who from an early date threw himself with great ardour into the work of the laboratory, seems to have been regarded by some of his friends as having "wasted" a good deal of time there. But this was not his own opinion, nor was it borne out by the decision of the examiners. He found himself so well abreast of his proper tripos work that he had ample leisure not only for this but for other parerga, such as that of writing an essay on "Wit and Humour in English Poetry," which won the Members' Prize in 1873, and also for full enjoyment of undergraduate companionship and the characteristic recreations of the place. Writing long afterwards to the late Dr Adam of Emmanuel, who was engaged on a memoir of that eminent scholar Mr R. A. Neil of Pembroke, he said: "The happiest days of my life were my undergraduate days at Peterhouse, and the chief joy of that time was my friendship with Neil." He rowed a little, and was for a time an energetic volunteer. Some sentences from his own retrospect of the three undergraduate years at Cambridge as compared with the four undergraduate years he passed in

Aberdeen are worth quoting here. Speaking of Aberdeen, he says: "The work in all the ordinary classes was very elementary. The course was the same for everyone, viz. the old seven subjects, plus a course of natural history, which included both zoology and a little geology. Yet there was a great variety, and if we did not get much of any one thing, all that we got was highly digestible, and men who went conscientiously through that course carried with them in after-life, for the most part, an intellectual mark that was unmistakable." "When I went to the University of Cambridge . . . I found that the course there for the ordinary degree in Arts was greatly inferior in educational quality to the Scottish one. On the other hand, the courses in Honours were on a very much higher standard, although they suffered greatly from the chaotic organisation of the English universities." He goes on to say: "I have frequently been tempted to think that the three years I spent as an undergraduate at Cambridge were wasted years of my life: if they were to be valued merely by the amount of new knowledge acquired, they were largely wasted; but, on the other hand, they were of great advantage to me in other respects. I made the acquaintance of a large number of the ablest young men of my generation; and it was no small matter to come even within view of such men as Cayley, Adams, Stokes, and Maxwell, and to have lived for a time within the College walls which had sheltered Tait and Kelvin. Cambridge at that time presented strange contrasts. Although almost decadent as an educational institution, it numbered among its members, as the names I have just quoted prove, perhaps the greatest galaxy of intellectual stars that ever illustrated any period of the history of a university."

Shortly after his graduation in 1875, Chrystal was elected Fellow and Lecturer of Corpus Christi College, with which society he retained a life-long connection, having been subsequently chosen an Honorary Fellow. For some two years he lectured in mathematics and physics to the students of a group of colleges which included both Corpus and Peterhouse; in this period of his life, besides his activity and success as a teacher, a noteworthy feature was the part he took in promoting certain measures of University and College reform. It was during these years, too, that he carried out his important work in connection with the experimental verification of Ohm's law, and began the article "Electricity" for the ninth edition of the *Encyclopædia Britannica*, which will be dealt with more fully in another part of this notice.

In the summer of 1877 a vacancy occurred in the chair of Mathematics in the University of St Andrews, and Chrystal made application for the

post, fortified by testimonials of exceptional strength and cordiality from all the most eminent of the Cambridge men of science with whom he had been brought into contact: Clerk Maxwell, Sir William Thomson, Professor Tait, Sir George Stokes, and Mr Routh (pre-eminent in the annals of Cambridge coaches) united in praising what he had already accomplished and in forming the happiest auguries for his future. But, in view of his youth—it will be remembered that he was little over twenty-six,—he had not much expectation of success, and he used to tell afterwards with glee how much he was surprised on a certain Saturday morning to receive from the Home Secretary a telegram, followed by a letter, informing him that he had received the appointment and was expected to enter upon its duties at the earliest possible date, which proved to be the following Monday. At St Andrews, apart from his professorial work, he found strenuous employment in the completion of the "Electricity" article already referred to.

In June 1879 he married Miss Margaret Ann Balfour, whom he had known from his childhood, and in the following month he was elected by the Curators to the chair of Mathematics in the University of Edinburgh in succession to Professor Kelland. His inaugural lecture, delivered on October 30th, opened a career of uninterrupted professorial activity, which extended over thirty-two years, and which was destined to be memorable in the history of Scottish education.

If Professor Chrystal was far from agreeing with Poisson in the saying which he used to quote, that "La vie n'est bonne qu'à deux choses—à faire les mathématiques et à les professer," he none the less found great happiness in being able all through life to surrender himself with love and devotion to the twofold task of a discoverer and of a teacher in the subject he professed.

Professor Chrystal was in fact not merely the brilliant Professor of Mathematics during his thirty-two years' occupancy of the Edinburgh chair: he created an epoch in the study of that subject not only within the University, but throughout Scotland at large. Nay, more, of the great and far-reaching changes made in the whole educational system of the country during the last forty years he might well have said without the slightest shadow of boasting: *quorum magna pars fui.* We may here recapitulate, in a few sentences, the principal stages in that great movement.

In 1878, while he was still in St Andrews, the Royal Commission appointed in 1876, with Lord President Inglis as chairman, to inquire into the Universities of Scotland, had issued their Report; and, though

the consequent executive changes did not take place until eleven years later, after the passage of the Universities (Scotland) Act, 1889, the problems raised by the Report were from the first eagerly and anxiously discussed in academic circles and had a prominent place in the thoughts of Professor Chrystal. The Universities, however, were only a part, though no doubt a highly important part, of the problems of national education to the solution of which so many of the best years of his life were devoted. In 1882 the Educational Endowments (Scotland) Act was passed, establishing a Commission with compulsory powers, Lord Balfour of Burleigh being chairman, and Mr Alexander Gibson, advocate, secretary. Professor Chrystal was not a Commissioner, but he was on terms of intimate friendship with the secretary, and there can now be no impropriety in saying that the many questions the Commission had to deal with were frequently discussed by Mr Gibson with Professor Chrystal and their common friend Professor Robertson Smith, who about that time was much in Edinburgh, and that Mr Gibson found their opinion always helpful and generally such as might profitably be suggested for the consideration of his Commission. These discussions served to deepen in Chrystal's mind the interest he had long felt in educational reform as it ought to be regarded by a statesman. In 1872 Lord Young's Act had revolutionised primary education in Scotland, but on his return to his native country in 1877 Chrystal remarked with concern that secondary education had not only not kept pace with primary education, but had, on the whole, retrograded. Secondary schools were dying, or, even if apparently prosperous, far from efficient. The Universities, he said, were "unwholesomely prosperous"; their standard, like that of the secondary schools, was "below the level of the cultivated nations of Europe." His reflections soon led him to become more and more the advocate of extending the policy of state aid to secondary schools.

In 1886 he was appointed to represent the University on the newly constituted governing body of the Heriot Trust, and he continued to hold office until 1902. The task was a congenial one. He took the greatest interest in the affairs of the Trust, and in his capacity of Governor he was delighted to be able to play an influential part in laying the foundations of the new organisation both of George Heriot's School and of the Heriot-Watt Technical College. Meanwhile, another important development of his educational activities had taken place. In 1885 the Scotch Education Department had been reconstituted, the Secretary for Scotland being made Vice-President of the Committee of Council, and one result of this important administrative change was that for several years it fell to Professor Chrystal,

along with some other Scottish professors, to take part in the inspection of secondary schools. It was in the course of his labours in this connection that the idea occurred to him of a simultaneous written examination so arranged that he might be able to report on all the schools he visited in a uniform manner; and when he came to write his report on the schools he had visited, he sketched a complete scheme for the Leaving Certificate examination which was immediately taken up, and in most of its essential features ultimately adopted by the Department. The importance of this step can be more fully appreciated now, after the lapse of a quarter of a century, than was possible at the time.

The institution of the Leaving Certificate examination was almost immediately followed by the Universities (Scotland) Act, 1889, by which another Commission, this time armed with executive powers, was brought into being. The result, as is well known, was a fundamental change in the Scottish University system. A series of ordinances were issued, which reorganised the finance and internal management of the Universities, greatly widened the curriculum, by introducing into it an elaborate system of options, set up a new system of Honours degrees, and admitted women to lectures and graduation.

The administrative work involved in bringing all these changes into operation required much time, labour, and executive capacity; and when Professor Chrystal was appointed Dean of the Faculty of Arts on the resignation of Professor Campbell Fraser in 1891, he had a formidable duty to face. It is the simple truth to say that, so far as Edinburgh was concerned, the task of carrying out the reforms embodied in the new ordinances fell very largely on the new Dean, and, as the Minute of Senatus drawn up at the close of his twenty years' term of office records: "To his knowledge of public opinion, to his mastery of the educational problems of the day, and to his unwearying zeal and administrative capacity, it was mainly due that these changes were successfully accomplished."

It was not long before some even of the fundamental alterations made by the Commissioners called urgently for revision, in view of the rapid developments that were taking place. In 1907, as the result of much consideration and many years' toilsome experience, the Edinburgh University Court formulated a very important new ordinance giving power to establish a three-term session and to overhaul completely the scheme for graduation in Arts. The reforms foreshadowed in that ordinance did not become effective until the beginning of the session 1909–10, the details having had to be worked out in the interval through the Senatus. On Chrystal, as Dean of the Faculty of Arts, rested the

main burden of devising the new regulations and of piloting them into harbour after the leading principles had been decided on. How much strenuous effort this cost, only those who were in intimate touch with him at the time can ever know. In his Promoter's Address of 1908 he thus referred to the impending changes in words in which those nearest to him were concerned to detect a premonitory note of weariness:—

"I am keenly interested in the developments that lie before us; but I must confess that I shrink from the labour that they will involve. Yet the whole of my career has been a turmoil of University reform, beginning at Cambridge; and it may as well end as it began, if it be decreed that it is to continue any longer."

It was natural, and indeed inevitable, that Professor Chrystal, who had so great a share in the remodelling of the curricula of the Universities of Scotland, should be called upon to take part in the inception and execution of the further reforms in the primary and secondary schools of Scotland which had been rendered necessary by this radical change. In obedience to this call he took a leading part in the business of framing a new system for the training of teachers. When the first Edinburgh Provincial Committee charged with the administration of this system entered upon its duties, he joined it as a representative of the University and was elected chairman. Dr George Macdonald, the Assistant Secretary to the Scotch Education Department, has allowed us to transcribe the following appreciation of Professor Chrystal's public services in this connection:—

"The immediate tasks that confronted the Committee were three in number. In the first place, the Department's new Regulations for the Training of Teachers—a code that involved a veritable revolution in the educational system of the country—were submitted to them in draft for consideration and discussion. Many of the problems were new to Chrystal, who had never taken any active part in primary school administration. But he mastered the whole subject in an astonishingly brief space of time, and made himself the guiding spirit in the Committee's deliberations—deliberations which resulted in certain important modifications being made.

"In the second place, negotiations of an exceedingly delicate character had almost at once to be entered upon with representatives of the two great Presbyterian Churches as well as with certain prominent members of the Episcopalian community, the object in view being to arrange for the transference of the existing Training Colleges to the new Committee. These negotiations were of a very protracted and difficult kind, and it was largely due to Chrystal's tact and fairmindedness, and to his success

in securing the complete confidence of everyone concerned, that matters were brought to a satisfactory issue. It is true that in the case of the Episcopalian Training College no transfer took place. But even there the Committee and the Church parted company on the friendliest terms. As far as the two Presbyterian bodies were concerned, much of the work which had to be done had to be done in common with the Provincial Committees of Glasgow and Aberdeen. Chrystal was chairman of a Joint Committee representing the three, so that his labours in this matter secured appreciation from a circle much wider than his position as chairman of the Edinburgh Committee alone might have implied.

"In the third place, the whole of the business arrangements for the new body had to be organised. How complex an undertaking this was may be gathered from the fact that an expenditure of somewhere about £30,000 a year of public funds was involved. Innumerable individual susceptibilities had to be taken account of and considered. All sorts of contingencies had to be provided for. Yet the whole machinery was brought into working order without friction of any kind in a comparatively brief period. If mistakes were made in this matter of detail or in that, Chrystal himself made none. And he was, as I know from the personal testimony of those who worked under him, most unselfish in taking upon his own shoulders, ungrudgingly and uncomplainingly, the burden of rectifying any error for which anyone else was responsible.

"His work in these and in many other ways met with comparatively scant recognition from the public. I daresay the average man might have thought more of him if he had accepted the knighthood which the Government is understood to have offered him. But, as you know, he cared for none of these things, and was content with the consciousness of having done his duty. If, however, the circle of those who learned, through his connection with the Provincial Committee, to appreciate his worth and to care for his personality was small, the measure of that appreciation, and of the liking that was engendered, was large indeed. I do not know any instance of a public man whose labours and whose personality have been spoken of with greater or more uniform cordiality by all of those who were privileged to be in touch with what he was doing."

Another important activity of the last ten years of his life was his membership of the Committee which, as one of the results of the South African War, had been appointed by the War Office to advise in regard to the education of officers. This gave him yet another opportunity of placing at the disposal of his country his administrative gifts and his ripe experience as a teacher and a master of educational methods. The deliberations

of the Committee may not perhaps have yet borne fruit in the establishment of an ideal system of military education; but Chrystal's services were recognised and appreciated by successive Secretaries of State.

The foregoing paragraphs will serve to indicate in some degree how strenuous was Professor Chrystal's life as investigator, as author and teacher, and as administrator, from his twenty-sixth to his fifty-ninth year. But in his holiday employments and recreations also he showed the same characteristic zeal and thoroughness. At Cambridge, as we have said, he was for some time a zealous volunteer and as good a shot as his myopic blue eyes permitted; all his life he was a keen angler and for many years an energetic cyclist. He was deeply interested in both the science and the art of photography, in the pursuit of which he was indefatigable and most successful. In travel also he found great pleasure and refreshment. His first visit to the Continent was in 1874, when he studied for some months in Tübingen; in the summer of 1876 he took a Cambridge reading party (which, as it happened, included Sir Martin Conway and Prof. F. O. Bower, then undergraduates of Trinity) to Sterzing, Tyrol, and found a new recreation in mountaineering; in later life he frequently visited France, Germany, Norway, and Italy, and in 1892 he spent some weeks in the Western States of America. His "literature," to use the term in the eighteenth-century sense, was remarkably extensive, and was founded on a wide knowledge of the classics, for which, as we have said, he retained an early taste. He did not lose hold even of Greek, which in the case of men engrossed in other pursuits is apt to become rather a faint memory in middle life. German was the first modern foreign language which he mastered, and German books, especially poetry books, were long one of his favourite relaxations. He spoke German fluently, and had much of Heine by heart. In French he had read very widely after his predilections; but in his maturity it was perhaps to Norse and Italian that he turned with the greatest enthusiasm. He professed to be a desultory reader and to have forgotten much, if not most, of what he had read, but, in Bacon's phrase, he was both "a full man" and "a ready man"; and, as might be expected in one of such wide and varied knowledge and experience, his talk was always interesting and informing; and this combined with his genial kindliness to make him a delightful host, guest, travelling companion, comrade, and friend. Amid all his many interests his home life was the greatest. Mrs Chrystal died in 1903, leaving a family of six sons and daughters to the care of a devoted father, who was also the most enthusiastic and the most intimate of their friends.

It was towards the autumn of 1909 that Professor Chrystal's friends

began to notice in him symptoms of impaired health, and some loss of the indefatigable vigour that had hitherto characterised him. Indeed, he began, to their dismay, to speak sometimes of withdrawing from his many activities. The first actual step in this direction was taken in October 1909, when, at the end of the first four years' term of office as chairman of the Edinburgh Provincial Committee, he intimated his inability to accede to a request which had been urgently addressed to him that he should consent to accept nomination as a member of the new Committee. "I have been medically advised," he wrote, "that for some time to come I must diminish the amount of business for which I am responsible, if I am not to court final unfitness for all business whatsoever. As the work of your Committee was the last faggot added to the bundle, it must be the first removed." After touching upon the arduous work of the Committee during the preceding four years, he went on to allude to his personal relations with his colleagues. "These relations, I am happy to say, are not clouded by a single unpleasant recollection. The name of the local administrator is 'writ in water.' He must look for his reward in the approbation of his own conscience, and in the keen sense of friendly comradeship which is generated by sharing a common enterprise for what is believed to be the public good. Such reward is enough, in my opinion, for any man; certainly enough for me. During the four years that I have worked with you I have learned to know and like many men with whom I should otherwise never have become intimate. I hope the friends I have thus made will remember me as long as I shall remember them, and with equal pleasure. . . . I thank you for the uniform kindness and courtesy with which you have treated me during my term of office. The best I can wish for my successor is that his Committee may show him the same consideration as you have always given to me."

The Committee, in recording their grateful sense of his eminent services, spoke of his public work in connection with education in Scotland, in what had really been a fresh chapter of its record, as having merited and as having received "the warmest appreciation of all who are conversant with the subject"; and added: "He has been an ideal chairman, and his resignation has been received by all with a profound sense of personal loss. He has justly earned their sincere and affectionate regard. They have admired his great business capacity and the singular thoroughness and devotion with which he carried out the duties of his responsible office. They appreciate no less his courtesy, his fairness of mind, his personal kindness, and his unselfish readiness to credit to others the success of work which he had himself inspired."

The incipient breakdown in health, hinted at in the letter just quoted, was not destined to be arrested, and the obscure illness gradually became more marked in its character as the session advanced. At its close he was still able, however, to undertake a tour in Italy; and the energy and cheerfulness he displayed throughout those weeks will never be forgotten by those who were privileged to be his companions. Milan, Perugia, Assisi, and (chiefly) Rome were the cities visited; and the brief diary he kept indicates unflagging delight in the glories of nature and the miracles of art. Neither this visit to Italy, however, nor treatment at Harrogate and a visit to Northumberland in the following summer sufficed to effect any lasting improvement in his health. He continued to fight bravely on, and proved equal to the discharge of his professorial duties to the end of the winter session 1910–11. When the spring had far advanced there remained no room for doubt in the minds of those who were nearest him that his trouble was incurable, and that all the highest professional skill could now do was to mitigate the inevitable suffering incident to a distressing and mortal illness. Yet he continued to find pleasure and refreshment in his work; and though at the beginning of the winter session of 1911–12 the University Court had granted him extended leave of absence, his enthusiasm and strength of purpose enabled him to attend at the University and award the bursaries as late as 21st October. The end came on the morning of Friday, 3rd November, in the eighth month of his sixty-first year. He was laid to rest on 8th November in the churchyard of Foveran, Aberdeenshire, where his parents are buried, and at the same hour an impressive service, attended by a large congregation, which included many students as well as representatives of the various public bodies with which he had been associated, was held in St Giles' Cathedral, Edinburgh.

A letter from the Secretary of the Royal Society of London, announcing that the King had been pleased to approve of the recommendation of his name for the award of a Royal medal "on account of his contributions to mathematical and physical science, especially, of late years, on the seiches of lakes," arrived in Edinburgh only two hours after his death. It was felt by the Council of the Royal Society that the award thus made should not be cancelled, but that the medal should be transmitted to his family as a visible token of the admiration with which the Society regarded his work. In giving his sanction to this proposal, the Royal donor caused also the following message to be sent: "The King trusts that you will be so good as to convey to the family the assurance of His Majesty's sincere sympathy in the terrible loss that they have sustained, through which so distinguished a career has been brought to a close."

Professor Chrystal's close connection with this Society began with his Edinburgh career and continued without intermission to the end. He joined it early in 1880, and in November of that year was elected to the Council. He served three terms of office as Councillor (1880–3, 1884–7, 1893–5), and two terms as Vice-President (1887–93, 1895–1901), finally succeeding Professor Tait as General Secretary in 1901. Of Professor Tait as General Secretary it is recorded in the minutes that "the Council always felt that in his hands the affairs of the Society were safe, that nothing would be forgotten, and that everything that ought to be done would be brought before it at the right time and in the right way." The ideal thus set up was certainly taken up and realised in a very notable manner by his successor. In the Council of the Society as the chairman's right-hand man he impressed one afresh at every meeting with a sense of extraordinary alertness and resourcefulness, tactfulness and courtesy, good sense and sagacity, so that when the meeting came to an end, however anxious the deliberations had been, one felt that it had been a privilege, a pleasure, and an education to be present.

Part II.—Scientific Work. By Professor C. G. Knott.

Chrystal's cast of mind was fundamentally physical. This was clearly shown during his student days at Aberdeen; but it was not till he went to Cambridge that he found opportunity for real scientific work. In the year in which he took his degree the British Association had appointed a small committee consisting of Clerk Maxwell, Everett, and Schuster to test experimentally the validity of Ohm's law in electricity. Clerk Maxwell undertook to have the test made in the Cavendish Laboratory, and to Chrystal was entrusted the task of making the experiments. Two forms of experiment were devised by Maxwell, the second proving to be the more satisfactory. The general idea of this experiment was to balance against each other in a Wheatstone bridge a thin and thick wire of the same material, and then pass through the system in alternation a strong and weak current. When the galvanometer showed no deflection under those conditions, the weak current was reversed in direction. Since this reversal of the weaker currents did not affect the equilibrium, it followed that Ohm's law was true within the limits of error of the experiment. The experiments, which involved considerable difficulties of manipulation, were carried out with great success by Chrystal, who wrote the report which was presented to the Glasgow meeting of the British Association in 1876.

A third form of experiment was devised by Chrystal himself, being a modification of one already tried by Schuster. It was based upon the fact that in an induction coil the induced current at break of the primary has a higher maximum intensity than the induced current at make. If, then, the induction currents from the secondary circuit of an induction coil, whose primary is made and broken by a tuning-fork, are passed through a galvanometer, the induced currents will not balance in their effects if resistance depends on strength of current. Certain effects, which at first (as in Schuster's experiments) seemed to indicate a departure from Ohm's law, were traced by Chrystal to the galvanometer. The explanation of these peculiar effects was given by Chrystal in a paper on "Bi- and Uni-lateral Galvanometer Deflections," which was published in the *Philosophical Magazine* for December 1876. Maxwell, writing to Tait on 5th February 1876, put the results obtained by Chrystal in these words: "Ohm's law has now been tested with currents that make the wire swag and swelter, and it is now at least 10^5 to 1 that if Schuster observed anything it was not an error of Ohm's law." As indicating the impression which Chrystal's personality had made on Maxwell, the following quotation from a later letter to Tait is of interest. In the summer of 1878 Tait had evidently asked Maxwell for some help in conduction of heat calculations, and Maxwell replied: "If you mean that I am, by the aid of Fourier, to get up the theory of a square box, and let you have it before the Edinburgh University Library opens, then in that case also you will not bother me, for I will not do it. Nevertheless, I have heard Chrystal say that the variable state of a parallelepiped was more tolerable than that of a cylinder, and he therefore cut his paraffin into a square prism. He also said that in this matter Poisson was of more use than Fourier." The most direct expression we have of Maxwell's opinion of Chrystal's capacity as an experimentalist is contained in the testimonial with which, on 10th July 1877, he supported Chrystal's application for the chair of Mathematics in St Andrews. "Of Mr Chrystal's papers," he wrote, "the most important is that on the 'Testing and Verification of Ohm's Law.' . . . The difficulties which he encountered and overcame in the course of this work can be appreciated only by one who, like myself, has had opportunity of watching his progress through all its stages." The testimonial ends with a reference to his "extensive and thorough culture, his original and penetrating intellect, and his untiring energy."

No doubt it was on Maxwell's recommendation that Chrystal was asked to contribute the electrical articles to the ninth edition of the *Encyclopædia Britannica*; and before he left Cambridge to take up his new duties in

St Andrews the manuscript of the article "Electricity" must have been in the printer's hands. The volume containing this article appeared in 1878. When in 1879 Chrystal became a candidate for the chair of Mathematics in the University of Edinburgh, Maxwell strengthened his former testimonial by adding these words: "I think it is of the greatest importance that, in a university in which the time that the majority of students can give to mathematics is so limited, their attention should be specially directed to those branches which will be most useful to them in their subsequent study of natural philosophy. This has always been kept in view in the University of Edinburgh. . . . That Professor Chrystal is well qualified to maintain the old reputation of the University is amply shown by the article 'Electricity.' . . . I have reason to know something of the amount of matter which must be gone through in order to write such an article, and of the difficulty of co-ordinating it, and I can confidently assert that the manner in which Professor Chrystal has made use of this mass of matter shows that he has the power, so invaluable in a professor, of giving such an account of what has been done in any subject as will give his students the greatest advantage in dealing with it themselves."

The article "Electricity" was followed in due course by the supplementary article "Magnetism," and the two are best considered together.

In gathering material for these and other contributions to the *Encyclopædia Britannica*, Chrystal spared no pains in getting access to original sources. With unerring discrimination he sifted out from the mass of accumulated and rapidly accumulating experimental results those which were essential in the progress of our knowledge of electrical science. Not only are the articles compact history, but the varied experience of all types of scientific investigator is woven into a unity under the formative influence of Faraday's conceptions and Maxwell's fruitful methods. Theory and experiment go hand in hand. Where necessary, mathematics of a high order are introduced; but the student not familiar with higher mathematics has no difficulty following the general argument and appropriating for his own purposes both the methods and the best results of experiment. In short, as an exposition of the development of the sciences of electricity and magnetism down to the date of publication, these two articles, concise and clear cut in their literary form, have never been surpassed for thoroughness of treatment, clearness of vision, unity of plan, and lucidity of expression. They at once became the English text-book for all real students of electricity and magnetism.

In addition to these two great articles, Chrystal also wrote for the *Encyclopædia Britannica* the articles "Electrometer," "Galvanometer," "Gonio-

meter," "Mathematics," "Parallels," "Perpetual Motion"; and the biographies of J. von Lamont, Mascheroni, Michell, Montucla, R. Murphy, Musschenbroek, Oughtred, Pascal (part), G. Peacock, Pell, Pfaff, Playfair, Plücker, Poggendorf, Poisson, Recorde, Rheticus, Riemann, Robins, Sturm. Of these, "Pascal" and "Poisson" are of particular interest.

"Mathematics" is a brief exposition of the historical development of the fundamental ideas of the various branches of the science, and in "Parallels" there is a clear account of the rise of non-Euclidian geometry. An address on this subject was given by Chrystal before the Royal Society of Edinburgh during his first year here—indeed, before he was formally elected a Fellow. In the same year he contributed along with Professor Tait an obituary notice of Professor Kelland, his predecessor in the Edinburgh chair. A peculiar interest attaches to this obituary notice, inasmuch as Chrystal gives in it a remarkably appreciative account of Kelland's investigations in wave motion—a subject which, towards the close of his own life, Chrystal was destined still further to elucidate in his masterly papers on seiches.

An important part of the scientific labours of a Scottish University Professor of Mathematics is the practical work of teaching. When Chrystal came to Edinburgh every Arts student had to study mathematics as one of the seven compulsory subjects. There were no options. These conditions were not conducive to a high standard in mathematical study; but even in these early days many a man of classical or philosophical attainments trembled as he entered the examination hall and sat down to tackle the algebra or the Euclidean geometry paper. The first year of Chrystal's professoriate struck terror to the hearts of those unfortunates to whom the *pons asinorum* was a bridge of sighs. Keen, rapid, logical, full of suggestions as to wider realms of mathematical delights, Chrystal transformed the whole atmosphere of the class-room. "Principles," "symmetry," "form"—not an endless wrestling with examples—were his watchwords; yet his exercises were splendid training. Eagerly the mathematical minds followed his fascinating lead; despondingly and despairingly those not so gifted fell hopelessly behind, perceiving faintly, if at all, the finely knit sequence of ideas which formed the thread of his discussions. Nevertheless, when the time of testing came, the really intelligent, hard-working student got full credit for his limited mathematical powers; for, with all his strenuous and successful labours to raise the standard of mathematical teaching, Chrystal was essentially just. With the close of the winter session of 1879–80 the University and Academic world of Edinburgh knew that a fresh force had come into their midst.

In these days in the Scottish Universities, there were during summer no regularly constituted classes in the Arts Faculty. Consequently, freed from the trammels of class work by the beginning of April, Chrystal found opportunity to resume with eagerness his experimental investigations, from which his St Andrews career had completely divorced him. Tait invited him to work in the Physical Laboratory and to utilise to the full all its appliances. It was my first year as Tait's assistant, and the incursion of this young professor of twenty-eight years into our midst gave all our minds a new orientation. His constant presence in the laboratory during the summer months and his ready accessibility at all times gave a great impetus to the experimental study of electricity and magnetism. Tait himself was at the time fully occupied with the corrections to be applied to the *Challenger* thermometers and with the related work on high pressures. This work was being done in the basement by a few of the senior students working directly under Tait's supervision; and Tait was rarely seen in the upper rooms where most of the other laboratory work was going on. Summer after summer Chrystal flitted through these laboratories, busy with his own researches, but not too busy to take a keen interest in all that was being done. Many a helpful suggestion he gave for new lines of work, and many an eager student did he encourage by inviting his co-operation in some special bit of investigation. The advanced students of these years came into more direct contact with him than with Tait, and owed much of their scientific progress to his sympathetic help. My own research work in magnetism, which has continued over many years, had its origin in a conversation over a passage in the article "Magnetism."

The first work to which Chrystal devoted himself was the comparison of inductances and capacities according to the methods soon to be expounded in his important paper on the differential telephone, for which he was awarded the Keith prize. This paper, indeed, contains for the first time the complete theory of the Wheatstone bridge through which a periodic current is passing, when coefficients of mutual and self-induction are given their full significance. In the course of his investigations he constructed many forms of apparatus, which were the pioneers of the more elaborate and refined methods of the present day. Side issues of enticing interest often led him away for a time from the main trend of his researches. On one of these, the wire telephone, he made two communications to our Society, and showed some of the experiments. The accounts of these in our *Proceedings* are mere abstracts; a much fuller description will be found in *Nature* of 29th July 1880.

During this first strenuous year at Edinburgh, Chrystal also presented

a paper on Minding's theorem, which Tait had shortly before discussed by quaternion methods. Chrystal developed the subject in illustration of Plücker's complexes and congruences.

After a few years Chrystal found himself compelled to give up experimental work, in large measure, no doubt, on account of increasing demands on his time. The tercentenary celebrations of 1884 were followed by an awakened interest in University reform: the mathematics department was being rapidly developed, and much of Chrystal's leisure must have been spent in preparing his text-book on *Algebra*. Also, as he himself once said, he found that he was monopolising all the best pieces of apparatus in the Physical Laboratory, so that the senior students were seriously handicapped.

The publication of the first volume of Chrystal's *Algebra* marked an epoch in the teaching of mathematics in our schools and colleges. Already in 1885, as President of Section A of the British Association at its meeting in Aberdeen, Chrystal had pointed out in clear, unequivocal terms the need of a revolution in the presentment of algebraic theory. The following quotations are not altogether inapplicable even in these days:—

"In the higher teaching, which interests me most, I have to complain of the utter neglect of the all-important notion of algebraic form. I found, when I first tried to teach University students co-ordinate geometry, that I had to go back and teach them algebra over again. . . . I found that their notion of higher algebra was the solution of harder and harder equations. . . . Many examination candidates, who show great facility in reducing exceptional equations to quadratics, appear not to have the remotest idea before hand of the number of solutions to be expected. . . . The whole training consists in example grinding. What should be merely the help to attain the end has become the end itself. The result is that algebra, as we teach it, is neither an art nor a science, but an ill-digested farrago of rules whose object is the solution of examination problems. . . . The end of all education nowadays is to fit the student to be examined; the end of every examination not to be an educational instrument, but to be an *examination* which a creditable number of men, however badly taught, shall pass. We reap, but we omit to sow. Consequently our examinations, to be what is called fair—that is, beyond criticism in the newspapers—must contain nothing that is not to be found in the most miserable text-book that anyone can cite bearing on the subject. One of my students, for example, who was plucked in his M.A. examination—and justly so, if ever man was—by the unanimous verdict of three examiners, wrote me an indignant letter because he believed, or was assured, that the paper set could not

have been answered out of Todhunter's *Elementary Algebra*. . . . The problem for the writer of a text-book has come now, in fact, to be this—to write a booklet so neatly trimmed and compacted that no coach, on looking through it, can mark a single passage which the candidate for a minimum pass can safely omit. Some of these text-books I have seen, where the scientific matter has been, like the lady's waist in the nursery song, compressed so 'gent and sma',' that the thickness of it barely, if at all, surpasses what is devoted to the publisher's advertisements.

"The cure for all this evil is simply to give effect to a higher ideal of education in general, and of scientific education in particular. . . . Science cannot live among the people . . . unless we have living contact with the working minds of living men. It takes the hand of God to make a great mind, but contact with a great mind will make a little mind greater. . . . In the future we must look to men and to ideas, and trust less to systems. Systems of examination have been tested and found wanting in nearly every civilised country on the face of the earth. . . . The University of London . . . has for many years pursued its career as a mere examining body. It has done so with rare advantages in the way of Government aid, efficient organisation, and an unsurpassed staff of examiners. Yet it has been a failure as an instrument for promoting the higher education—foredoomed to be so, because, as I have said, you must sow before you can reap."

Within a year of uttering these stirring words, Chrystal presented to the mathematical world of teachers his solution of part of the difficulty in the form of the first volume of his well-known *Algebra*. Its merits were immediately recognised. The first object he set before him was "to develop algebra as a science, and thereby to increase its usefulness as an educational discipline." The introduction into an elementary text-book of such subjects as the complex variable, the equivalence of systems of equations, the use of graphical methods, and the discussion of problems of maxima and minima, was a new feature; but subsequent developments have fully vindicated the innovations. Graphical methods of representing simple functions, and the transition to the solution of equations, are now the stock-in-trade of every modern *Algebra*. From these methods, the principles of co-ordinate geometry flow naturally and simply; and Chrystal himself, somewhat later, made a further valuable contribution to the nomenclature of the subject by his introduction of the expressive terms, "the constraint equation" and "the freedom equations of a curve." The second volume followed in 1889, and was marked by the same lucid and logical treatment of the more advanced parts. In 1898 Chrystal further enriched the

literature of the subject by bringing out his *Introduction to Algebra* for the use of secondary schools and colleges.

Although he did not for many years take up any experimental work, he was always ready when occasion offered to advise and help others engaged in such work. For example, he took a keen interest in the Ben Nevis Observatory, and devised special forms of hygrometer and anemometer for use in these altitudes, especially during the winter season. He also turned his attention to the invention of an electrical method for reversing deep-sea thermometers.

In 1883, through the initiative of the late Mr Yule Fraser, at that time mathematical master in George Watson's College, the Edinburgh Mathematical Society was founded. As its first secretary I was brought into still closer touch with Chrystal, who took a warm interest in its welfare from the first. In these early years he contributed a number of notes on mathematical subjects, and greatly encouraged the members by his presence at the meetings. His paper on certain inverse roulette problems contains elegant solutions of particular cases of the general problem: Given the body centrode and the roulette for one point of a plane figure moving in its plane, to find the space centrode. The most elaborate paper which he communicated to the Mathematical Society was entitled "On the Theory of Refraction of approximately Axial Pencils of Light through a Series of Lenses, more especially with regard to Photographic Doublets and Triplets." He was led to take up the subject in connection with his own photographic work. The theory is worked out in a very simple form, and instructions are given the student of photography how most easily to determine the constants of his system of lenses.

Between 1891 and 1896 Chrystal communicated three fairly elaborate papers on differential equations. There was a rumour at one time that he purposed writing a book on this subject; whether this was so or not, the character of these papers shows that he had given careful consideration to the logical foundations of the theory of certain parts. For example, in the first paper he proves the defective nature of Lagrange's demonstration of the rule for the solution of the partial differential equation of the first order, and supplies a demonstration free from the fallacy which had been generally current since Lagrange's days. Similarly, in the second paper he aims at establishing rigorously a fundamental theorem regarding the equivalence of systems of ordinary linear differential equations. This leads to a systematic way for solving systems of this kind without the introduction of superfluous arbitrary constants; and the paper ends with illustrations of the practical use of the method. In the third paper, that

on the p-discriminant of a differential equation of the first order, Chrystal begins by pointing out that Cayley, in his early discussions of singular solutions, was led to a proposition which was erroneous, or at least very misleading. Cayley stated that when the differential equation had no singular solution it did not admit of an algebraic primitive. Chrystal, on the other hand, showed that although it is true that, when a singular solution exists, the primitive is algebraic, yet it is the exception and not the rule that there is a singular solution when the primitive is algebraic. It is interesting to note that there has been a steady demand for copies of this particular paper on the p-discriminant and the connected theory of envelopes.

The outstanding characteristic of these mathematical papers is the endeavour after rigorous demonstration. They are clearly written, and are put together in a way which shows a keen perception of the essential nature of the problem contemplated and the inborn power of the teacher in presenting his demonstrations so as to be easily followed by the intelligent student. Chrystal, indeed, possessed the intuitive art of the born teacher. Wide and deeply read in all that was best in mathematical literature, possessing at the same time an artistic appreciation of the beauty of form and logical sequence, he never failed to impart to his presentation of a mathematical argument the distinctive personal touch which appeals to the real student. The following sentences drafted by his eldest son, Mr George Chrystal of the Home Office, give what may be regarded as Chrystal's own estimate of himself in the ranks of mathematicians:—

"My father in familiar conversation with me always declined the title of a great original mathematician. How far this was justified, I have no means of judging; but his real bent seemed to be towards physical science —towards the concrete rather than the abstract. With this, however, he had a keen appreciation, a great knowledge, and a thorough understanding of what had been achieved by the giants of the mathematical world—the Cayleys and the Riemanns, whose results, as he used to tell me, were sometimes reached by stages and processes which even these great men themselves could not always thoroughly explain or account for.

"What he regarded (I believe, though he never told me so in terms) as his special service to mathematics was that, by study and diligence and the exercise of the intellectual power which he possessed, he had been able to consolidate some of the conquests made by the great mathematicians, his predecessors and contemporaries, and had evolved and excogitated a method by which the diligent student of average ability could retread the path which had conducted the man of genius to his discoveries.

"This method required two things: *in the first place*, the abandonment of the traditional practice of occupying, as it were, isolated points in the terrain to be conquered by science, from which isolated forays or raids were conducted under the guise of problem-solving and other virtuosities. Henceforth the pupil was to be conducted by an orderly series of reasonings up a sort of inclined plane from one well-defined conception to another, to the higher levels of the science—morphology, in the words of Sylvester, was to be introduced into algebra and mathematical analysis in general.

"*Secondly*: even in its elementary stages the science of algebra required setting in order, and the morphological method required a new, a precise, and to some minds a 'forbidding' terminology. This was the 'raising of the standard' playfully and ruefully described by Mr J. M. Barrie in *An Edinburgh Eleven*."

All who had any acquaintance with Chrystal's work will endorse these impressions, which spring from a sympathetic intercourse between a father and a son who had many intellectual interests in common, save perhaps that of mathematics. It is interesting in this connection to establish a correspondence between Chrystal and his great colleague, Peter Guthrie Tait. Tait was one of the examiners when Chrystal took his degree at Cambridge. The similarity of their mental gifts was brought out by the fact, often referred to by Tait, that Chrystal was distinctly the best candidate from Tait's standpoint. Tait used to say that Chrystal easily outdistanced the other Wranglers on the problems which demanded real thought! Another link between Tait and Chrystal was the preparation of the second edition of Thomson and Tait's *Natural Philosophy*, vol. i., the proofs of Part I. having been read by Chrystal and Burnside. Chrystal's appointment to the Edinburgh chair was hailed with intense delight by Tait, who knew he had gained a colleague who could think in physical lines. The full facilities he offered Chrystal to work in the laboratory have been already referred to. Each had for the other the greatest admiration, and much mutual help was given before their communications took final form in the printed page of the Royal Society *Proceedings*. In mathematics, and especially in geometry, Tait's was, no doubt, the more original mind; but Chrystal's knowledge of recent developments was wider, and probably he had a juster view of the perspective of things. Tait revelled in the direct and powerful methods of quaternions; but Chrystal used to say that it was Tait the man, and not quaternions the method, which was the real factor in any investigation which Tait made. Chrystal took a passing interest in Hamilton's calculus during his early years at Edinburgh, and gave a course of lectures on vector methods. But

they never gripped his mind—chiefly, as he expressed it, because they did not fall in line with the general symmetrical methods of modern analysis which specially appealed to him. Chrystal and Tait were at one in their love for physical investigation, and both were able to utilise to the full their mastery of mathematical symbolism.

I have heard Tait express the hope that when he retired from the chair of Natural Philosophy Chrystal would be his successor; but when the time of retirement came the situation had altered. Had it been Chrystal's fortune early in his professorial career to have had official control of a physical laboratory, he would certainly have founded a strong experimental school.

On Tait's death in 1901, our Society, looking around for a General Secretary, naturally turned their eyes to Chrystal, whose administrative powers and driving force had become widely recognised. It is not necessary again to speak at length of the exceptional ability shown by Chrystal in carrying on the work of the Royal Society. It seems important, however, to bring to light the valuable work he did in connection with the change of abode from the Royal Institution in Princes Street to the present location in George Street.

It had been apparent for many years that the west wing of the Royal Institution, which had been assigned to the Royal Society from the beginning, was becoming far too limited to accommodate the growing activities of the Society. Accordingly in 1903, largely through the initiative of Sir John Murray, a scheme was set on foot for securing for science the whole of the building. The Royal Society would have thus been able to find accommodation for its invaluable library, and other scientific societies might have found shelter under the same roof. A representative committee was formed, and the Secretary for Scotland approached on the subject. Mr Graham Murray, now Lord Dunedin, met the deputation, and spoke very sympathetically in favour of the movement. The difficulty was how to provide accommodation for the schools of art which occupied other parts of the building.

Meanwhile the Liberal Government came into power; and one of its earliest acts was to introduce a Bill handing over to a specially appointed board the whole of the building for the purposes of art. There was at first no provision in this Bill for securing in any way the vested rights of the Royal Society of Edinburgh. The situation demanded supreme vigilance; and fortunately in the person of the General Secretary we possessed the very man for the emergency. As early as 1885, in his British Association address already quoted from, Chrystal, who at that time had had no

direct experience of the wonderful way in which our legislators acquire knowledge and draft Bills, wrote these words:—

"We all have a great respect for the integrity of our British legislators, whatever doubts may haunt us occasionally as to their capacity in practical affairs. The ignorance of many of them regarding some of the most elementary facts that bear on everyday life is very surprising. Scientifically speaking, uneducated themselves, they seem to think that they will catch the echo of a fact or the solution of an arithmetical problem by putting their ears to the sounding-shell of uneducated public opinion."

His experience during the early stages of the fight which our Society had to make for recognition must have brought back to his mind more than once the memory of these sentences. In his reply to the first deputation which was received in Edinburgh, the Secretary for Scotland, while sympathising with the object, was of opinion that it was not supported by a body of public opinion. Before the second deputation was received in London, Chrystal had collected a vast array of facts and had fostered in the body of the Scottish representatives a public opinion which there was no gainsaying. By the sympathy and support of the Scottish Members of Parliament, as well as of the Fellows of the Royal Society of London and other eminent scientific men, more was achieved than was at first hoped for. Only a man of Chrystal's alertness of mind, clearness of vision, knowledge of affairs, fairmindedness, and yet determination to have the Society's rights recognised, could have successfully manœuvred the Society through this time of strain and stress when its status and efficiency were threatened. Through all the cross currents of opinion, while many acted valiantly and worked effectively, it was Chrystal who was the man at the helm. He was the prompter, supplying needed information at every stage to those who came forward in the interests of the Society. In such work success is its own reward, and Chrystal never grudged the time and energy he was called upon to devote to the cause of science in securing for all time from the Government of the day a generous recognition of its claims.

In this, as in other similar cases, Chrystal's labours were wholly disinterested. There was never anything personal or selfish in his aims. It was this detachment from self-interest that added to the strength of his appeals and secured the recognition of the principles for which he fought.

Meanwhile, before the introduction of the Government's Galleries Bill (Scotland), the progress of which through its various stages will be found discussed by Sir William Turner in his Presidential Address on the occasion of the opening of the new rooms (8th November 1909), Chrystal's advice had been

sought by Sir John Murray in connection with the observations of seiches on the Scottish lakes. This Sir John proposed to make part of his general survey of the fresh-water lochs of Scotland, and to this end he set up forms of limnometer which had been designed by Forel and others in Switzerland. The problem was just the kind to awaken Chrystal's keenest interest. It involved hydrodynamical problems of great difficulty, which could be surmounted only by use of mathematics of a high order. It demanded experimentation of an enticing nature, full of difficulties, and ever presenting new conditions to be considered. Into this work Chrystal accordingly threw himself wholeheartedly. Not only so, but he drew round him a devoted band of helpers, who accumulated new data by patient observation along the shores of several Scottish lochs, and who also arranged under his supervision ingenious experimental models of seiche phenomena in lakes. His papers on this subject, which will be found enumerated below, are recognised as among the most important bearing upon the subject. For this work he was awarded the Gunning Victoria Prize by our Society and a Royal Medal by the Royal Society of London.

His seiche investigations brought him into close touch with F. A. Forel, the veteran naturalist of Lake Geneva, whom he advised the Council to invite to deliver an address to the Society. This Professor Forel did in the summer of 1911. Unfortunately Chrystal was at the time too ill to receive the genial Swiss or to listen to his interesting lecture on the Fata Morgana.

To our Society Professor Chrystal's family have gifted his unique collection of books and papers bearing on the subject of seiches; these, with copies of his own valuable contributions, are now arranged, partly chronologically, partly according to size and country, in several volumes, which cannot fail to be for the future student a most important compendium of literature.

Chrystal's last published paper, "On the Theory of the Leaking Microbarograph," is, in a sense, a continuation of his investigations into the causes of seiches. It has, however, a much wider application, and is an important contribution to the theory and method of observation of small and rapid barometric fluctuations.

The list of papers appended is for the most part arranged in chronological order.

List of Contributions by George Chrystal to Scientific Literature.

1. Report of the Committee for testing experimentally Ohm's Law. *B.A. Reports, Glasgow Meeting*, 1876.
2. On Bi- and Uni-lateral Galvanometer Deflections. *Phil. Mag.*, December 1876.
3. Articles "Electricity" and "Electrometer." *Ency. Brit.*, 1878.
4. Articles "Galvanometer" and "Goniometer." *Ency. Brit.*, 1879.
5. Obituary Notice of Professor Kelland (in conjunction with Professor Tait). 1879. *Proc. R.S.E.*, vol. x.
6. On Minding's System of Forces. 1880. *Trans. R.S.E.*, vol. xxix. Abstract in the *Proc.*, vol x.
7. Address on Non-Euclidean Geometry. 1880. *Proc. R.S.E.*, vol. x.
8. On a New Telephone Receiver. 1880. *Proc. R.S.E.*, vol. x.; also in *Nature*, vol. xxii., 24th June 1880.
9. On the Differential Telephone. 1880. *Trans. R.S.E.*, vol. xxix. Abstract in *Proc.*, vol. x.
10. On the Wire Telephone and its Application to the Study of the Properties of strongly Magnetic Metals. *Proc. R.S.E.*, vol. x. See also *Nature*, vol. xxii., 29th July 1880, for a fuller account.
11. Note on Thomas Muir's Transformation of a Determinant into a Continuant. 1881. *Trans. R.S.E.*, vol. xxx.
12. On a Special Class of Sturmians. 1881. *Trans. R.S.E.*, vol. xxx.
13. Article "J. von Lamont" in *Ency. Brit.*, 1882.
14. Remarks on Dielectric Strength. 1882. *Proc. R.S.E.*, vol. xi.
15. Articles "Magnetism," "Mascheroni, "Mathematics," "Michell," "Montucla," in *Ency. Brit.*, 1883.
16. Present Fields of Mathematical Research. 1883. Opening Address to Edinburgh Mathematical Society. Never published.
17. Articles "R. Murphy," "Musschenbroek," in *Ency. Brit.*, 1884.
18. Mathematical Models chiefly of the Surfaces of the Second Degree. 1884. *Edin. Math. Soc.*, vol. ii.
19. Application of the Multiplication of Matrices to prove a Theorem in Spherical Trigonometry. 1884. *Edin. Math. Soc.*, vol. ii.
20. On the Discrimination of Conics enveloped by the Rays joining the Corresponding Points of Two Projective Ranges. 1884. *Edin. Math. Soc.*, vol. ii.
21. On a Problem in the Partition of Numbers. 1884. *Edin. Math. Soc.*, vol. ii.

22. Articles "Oughtred," "Parallels," "Pascal" (part), "G. Peacock," "Pell," "Perpetual Motion," "Pfaff," "Playfair," "Plücker," "Poggendorf," "Poisson," in *Ency. Brit.*, 1885.
23. On the Problem to construct the Minimum Circle enclosing Given Points in a Plane. 1885. *Edin. Math. Soc.*, vol. iii.
24. On Certain Formulæ for Repeated Differentiation. 1885. *Edin. Math. Soc.*, vol. iii.
25. On a Method for obtaining the Differential Equation to an Algebraic Curve. 1885. *Edin. Math. Soc.*, vol. iii.
26. On the Hessian. 1885. *Trans. R.S.E.*, vol. xxxii.
27. Address as President of Section A of the British Association at the Aberdeen Meeting of 1885. *B.A. Reports.*
28. Articles "Recorde," "Rheticus," "Riemann," "Robins," in the *Ency. Brit.*, 1886.
29. Article "Sturm" in the *Ency. Brit.*, 1887.
30. On certain Inverse Roulette Problems. 1887. *Edin. Math. Soc.*, vol. v.
31. On the Inequality $mx^{m-1}(x-1) \gtrless x^m - 1 \gtrless m(x-1)$ and its Consequences. 1888. *Edin. Math. Soc.*, vol. vi.
32. An Electrical Method of reversing Deep-Sea Thermometers. 1888. *Proc. R.S.E.*, vol. xv. Title only.
33. An Elementary Discussion of the Closeness of the Approximation in Stirling's Theorem. 1889. *Edin. Math. Soc.*, vol. vii. Title only.
34. A Demonstration of Lagrange's Rule for the Solution of a Linear Differential Equation, with some Historical Remarks on Defective Demonstrations hitherto current. 1891. *Trans. R.S.E.*, vol. xxxvi.
35. A Fundamental Theorem regarding the Equivalence of Systems of Ordinary Linear Differential Equations and its Applications to the Determination of the Order and the Systematic Solution of a Determinate System of such Equations. 1895. *Trans. R.S.E.*, vol. xxxviii.
36. A Summary of the Theory of the Refraction of Thin approximately Axial Pencils through a Series of Media bounded by Spherical Surfaces, with Application to a Photographic Triplet, etc. 1896. *Edin. Math. Soc.*, vol. xiv.
37. On the p-Discriminant of a Differential Equation of the First Order, and on Certain Points in the General Theory of Envelopes connected therewith. 1896. *Trans. R.S.E.*, vol. xxxviii.
38. Some Elementary Theorems regarding Surds. 1901. *Edin. Math. Soc.*, vol. xix.

39. Note on the Mathematical Theory of Miller's Trisector, and its Relation to other Solutions of the Problem of Trisection. 1901. *Proc. R.S.E.*, vol. xxiv.
40. On the Relation of Miller's Trisector to the Quartic Trisectrix, with a Description of a Seven-bar Limaçonograph. 1901. *Proc. R.S.E.*, vol. xxiv.
41. Obituary Notice of Professor Tait, *Nature*, 25th July 1901.
42. On the Theory of Seiches. With experimental illustrations by E. M. Wedderburn. 1903. *Proc. R.S.E.* Title only.
43. Obituary Notice of Luigi Cremona. 1904. *Proc. R.S.E.*, vol. xxiv.
44. Some Results in the Mathematical Theory of Seiches. 1904. *Proc. R.S.E.*, vol. xxv.
45. Some Further Results in the Mathematical Theory of Seiches. 1905. *Proc. R.S.E.*, vol. xxv.
46. On the Hydrodynamical Theory of Seiches. 1905. *Trans. R.S.E.*, vol. xli.
47. Calculations of the Periods and Nodes of Lochs Earn and Treig from the Bathymetric Data of the Scottish Lake Survey. In conjunction with E. M. Wedderburn. 1905. *Trans. R.S.E.*, vol. xli.
48. An Investigation of the Seiches of Loch Earn by the Scottish Lake Survey. Part I.—Limnographic Instruments and Methods of Observation. Part II. (by James Murray)—Limnographic Observations. 1906. *Trans. R.S.E.*, vol. xlv.
49. An Investigation of the Seiches of Loch Earn by the Scottish Lake Survey. Part III.—Observations to determine the Periods and Nodes. Part IV.—Effect of Meteorological Conditions upon the Denivellation of Lakes. Part V.—Mathematical Appendix on the Effect of Pressure Disturbances upon the Seiches in a Symmetric Parabolic Lake. 1908. *Trans. R.S.E.*, vol. xlvi.
50. On the Theory of the Leaking Microbarograph, and on some Observations made with a Triad of Dines-Shaw Instruments. 1908. *Proc. R.S.E.*, vol. xxviii.

Books.

1. Algebra, an Elementary Text-Book for the Higher Classes of Secondary Schools and for Colleges. A. & C. Black, Edinburgh. Part I., 1886; Part II., 1889.
2. Introduction to Algebra for the use of Secondary Schools and Technical Colleges. A. & C. Black, London. 1902.