

## Editorial

The connection between geometry and mechanics goes back a long way perhaps even as far back as Archimedes. In the 19th century, at the height of the industrial revolution, many of the prominent mathematicians of the day, Chebyshev, Darboux and Sylvester to name just a few, were studying the geometry of mechanisms and machines. Some of the foundations of modern geometry date back to this period, although it is difficult to tell this from a reading of modern texts in geometry. For most of the 20th century, however, this connection was largely ignored by mathematicians and engineers. But the past few years have seen an increasing number of mathematicians looking at geometrical problems in robotics. A few of them have published their work in robotics journals but most have been content to publish their work in maths journals or at specialised conferences. This special issue of *Robotica* seeks to expose some of this work to a more general robotics audience and begin a mutually beneficial discussion between the mathematicians, computer scientists and engineers. The idea is to present some of the more advanced material in this area to demonstrate to the wider robotics community the power, range and sophistication of these modern ideas. It is hoped that this special issue will show that modern mathematical techniques have a great deal to offer for the practising engineers. Also that many problems in engineering can give rise to interesting and sophisticated mathematics.

I wanted to invite some of the leading experts in the field to write reviews or survey articles of their particular areas. Writing review articles has become less popular lately, especially in the UK and US. However, I believe that good review articles are essential for transferring knowledge from a small group of initiates to the wider community. It is a definite skill to condense a subject into a short article giving the necessary background and introduction. Such articles are invaluable to students and other newcomers to a field, enabling them to appreciate quickly the extent of the area, learn standard notation and some of the simpler methods of the subject. Hence, I am particularly pleased that four distinguished scientists agreed to contribute review articles on their respective fields of expertise.

The first of these is by Peter Donelan on singularity theory in robot kinematics. It is well known that singularities are important in the design and control of robots. Singularity theory is a relatively new branch of mathematics that deals with the singularities of smooth maps like the forward kinematics of serial manipulators or the inverse kinematics of parallel robots.

Next, Manfred Husty and colleagues describe how simple algebraic geometry can be used, for example, to solve the inverse kinematics of general 6R serial manipulators. The key here is to represent the group of rigid-body motions as an algebraic variety; the Study quadric. The possible motions

allowed by two or three joint structures then correspond to the subvarieties.

The article by Phillippe Wegner studies cuspidal and non-cuspidal robots. In the joint-space of a robot, there are bifurcation surfaces. These surfaces split the joint space into distinct regions with different numbers of inverse kinematic solutions. The bifurcation surfaces are singularities of the robot but these surfaces can have singularities themselves. The existence of cusp singularities here distinguishes cuspidal from non-cuspidal robots, and the presence of cusps on the bifurcation surface has profound implications for the capabilities of the robot.

The article by Lyle Noake and Tomasz Popiel looks at the differential geometry of paths in the group of rigid-body motions and the rotation group. These paths could be used to plan the paths for the end-effector of a robot or to interpolate the motion of a 3D object in computer graphics. The article reviews some of the many methods that have been introduced to generalise classic interpolation algorithms to rigid-body motions. It also studies the paths determined by variational principles such as minimal acceleration. These curves can have a rich variety of complicated shapes.

I am also extremely pleased that so many other leading experts have responded to the call for papers. The next five articles are contributed works, which present some original research.

In some respects, the article by Zein, Wenger and Chablat continues the work described in the previous work of Wenger. Here a particular design of a parallel robot is studied, and an algorithm is described for finding cusps on the bifurcation (or singularity) surface.

The work of Kwon, Choi and Park represent some early steps in a relatively new area of robotics and geometry. The use of probabilistic methods in robotics is well established but how should we proceed when the statistic to be estimated is not just a number but a rigid-body motion? A method based on the generalisation of a standard particle filter is proposed.

Zadarnowska and Tchoń study how performance measures can be defined for mobile robots and systems composed of a traditional manipulator mounted on a mobile platform. The work uses the dynamics of the robot and is based on notions from the control theory.

In the control and simulation of robot dynamics, a major computational bottleneck is the inversion of the system's generalised mass matrix. Lee, Wang and Chirikjian address this problem, giving a semi-numerical algorithm to rapidly perform the inversion. In essence, the method is to embed the system in a larger unconstrained system where the mass matrix is easy to invert. The algorithm seems particularly well suited to long protein chains. The rigid-body dynamics of these molecules is very similar to that of serial robots.

The article by Murphey proposes a method to control second-order systems that have external frictional forces acting on them by performing kinematic reduction. The reduced system has no dependency on the details of the frictional model or the frictional forces. This idea can be useful in mobile robotics and grasping work.

The final article is somewhat different in character. Andrew Lewis works in the area of geometric control, and together with Francesco Bullo published a textbook on geometric control of mechanical systems. The article published here is an overview of this area. The intention is not to teach any detail, but to give readers an idea of the subject and its power and encourage them to delve deeper into the existing literature (which of course includes some of the preceding articles).

It gives me genuine pleasure to be able to thank everyone who have helped with this project: the authors who have

contributed their work, even those whose work did not make it into the special issue; the reviewers whose diligence and hard work helped to improve the quality of the work; and the Editor, Greg Chirikjan, who gave me much good advice and lots of encouragement.

Originally this special issue was to be jointly edited with Eduardo Bayro-Corachano and was to include work on applications of geometry to sensing. However, so many articles were submitted that we decided to split the special issue into two; a separate special issue on geometry and sensing, edited by Eduardo, will appear shortly.

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