THE BEAT FREQUENCY MODEL FOR QPOs

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1. INTRODUCTORY REMARKS

We have to date reports of Quasi-Periodic-Oscillation (QPO) observations in some twelve X-ray source, of which at least seven are low mass X-ray binaries (van der Klis 1987). They constitute a formidable zoo of phenomenae with so much variety that they, at times, do not at all even seem amenable to a single model. Some of the other QPO talks in these proceedings will try and present observations in the context of various models. My task is to talk about the Beat Frequency Model which, it seems to me, is by far the prime model for at least some of the QPOs.

The Beat-Frequency-Model (BFM) was constructed in order to account for what is, perhaps, the simplest of the QPOs, GX5-1. As you know, by some good fortune (or bad?) GX5-1 happened to have also been discovered first. I would therefore like to open my talk by first describing the GX5-1 story.

To remind everyone of the observations: van der Klis et al. (1985a,b) reported the discovery of quasi-periods between about 25 and 50 msec in GX5-1 with a Q value ($\equiv f/\Delta f$, where f is the corresponding frequency) of at most 4. The quasi periods were correlated with the overall source's 1-18 keV count rate I, such that $\Lambda \equiv d\log f/d\log I$ was about 2.5, while the size of the modulation seemed to decrease steadily with count rate - from a value of 8% for $I = 2.4\times10^3$ cps to zero at $I = 3.1\times10^3$ cps. In addition, low frequency noise (LFN) was observed in GX5-1. Its intensity correlated well with the size of the QPOs modulation.

A periodicity - albeit a quasi-periodicity - so short, has triggered in Ali Alpar and me (and, presumably, in many others) the following immediate reaction: In our previous scenario for the formation of millisecond pulsars (Alpar et al. 1982) we have argued, that they were weak-field neutron stars spun up by accretion from a companion in a galactic-bulge-type binary system. Millisecond periods were therefore expected in these binaries while they were still X-ray-active, and yet none has yet been observed.

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The GX5-1 observation was the first such kind of period reported. Yet, The large variability of the period length ruled out stellar rotation because of the needed large variability in angular momentum, so that another - external - location for the origin of the oscillations had be be considered. Alpar and I concluded it had to be the magnetospheric boundary: We recalled that accepted models for the region just outside the surface of an accreting neutron star (see, e.g. Ghosh and Lamb 1979) talk about a magnetic-field dominated regime of plasma motion (for surface fields $B_s \gtrsim 10^6$ G) which turns over into a gravity-dominated one (a "Keplerian disk") outside of some boundary region at a radius R_o. R_o can be roughly determined by equating the ram pressure of the Keplerian plasma, ρv_K^2, to the magnetic pressure $B^2/8\pi$; it is given, more or less, by

$$R_0 \sim 10^6 B_8^{4/7} R_6^{12/7} \dot{M}_{17}^{-2/7} (M/M_\odot)^{-1/7} \text{ cm} \tag{1}$$

where in (1), B_8 is the surface magnetic dipolar field in units of 10^8G, R_6 is the stellar radius R_S in units of 10^6 cm, M is the neutron stellar mass, $M_\odot$ is the solar mass and $\dot{M}_{17}$ is the total mass accretion rate in units of 10^{17} g/sec. The Keplerian angular frequency at R_0, Ω_0, is

$$\Omega_0 \equiv \left(\frac{GM}{R_0^3}\right)^{1/2} \sim 10^4 B_8^{-6/7} R_6^{-18/7} \dot{M}_{17}^{3/7} (M/M_\odot)^{5/7} \text{ rad/sec} \tag{2}$$

Ω_0 is therefore a prime candidate for variability because of its dependence on $\dot{M}$ hence on the count rate I; however, it only varies weakly with $\dot{M}$ to account for a Λ of 2.5. So, Alpar and I (1985) postulated that the QPO frequency, $\Omega_{QPO} \equiv f/2\pi$, is to be identified with Ω_B, the beat frequency,

$$\Omega_B = \Omega_0 - \Omega_S \equiv A\dot{M}^{3/7} - \Omega_S \tag{3}$$

where A is some constant and Ω_S is the stellar spin. This, provided the stellar rotation axis is perpendicular to the disk. We shall make this assumption throughout this talk, and only modify it towards the end (p. 12).

While we are at it, let us recall that, even though it was hard to see how this might be the source of the GX5-1 period, there are two additional points of interest outside of a rotating, accreting neutron star:

The corotation radius R_c is that radius for which the Keplerian angular velocity equals the stellar spin angular frequency Ω_S:

$$\left(\frac{GM}{R_c}\right)^{3/2} = \Omega_S$$

hence

$$R_c \sim 1.5\times10^6\ (M/M_\odot)^{1/3}\ (P_S/1\ \mathrm{msec})^{2/3}\ \mathrm{cm} \tag{4}$$

where P_S ($\equiv 2\pi/\Omega_S$) is the neutron stellar period. In a rotating star with a magnetic field structure attached, R_c plays a very important role: If any field lines are threading the accretion disk inside of R_c, they develop an azimuthal component B_ϕ downstream which tends to slow down the plasma - hence spin-up the star; the reverse holds outside of R_c. Thus, spin-up and spin-down torques act simultaneously on the star; for some value of R_c their resultant should vanish - that determines the equilibrium period P_{eq} of the accreting star. It is roughly given by (Alpar et al. 1982)

$$P_{eq} \sim (0.33\ \mathrm{msec}) B_8^{6/7} (M/M_\odot)^{-5/7} (\dot{M}/10^{-8} M_\odot \mathrm{yr}^{-1})^{-3/7} R_6^{15/7} \tag{5}$$

Secondly, the velocity-of-light radius R_L is the distance for which corotation with the star would mean moving at the speed of light,

$$R_L \sim 5\times10^6\ \mathrm{cm}\ (P_S/1\ \mathrm{msec}) \tag{6}$$

For millisecond pulsars R_0, R_c and R_L close in on each other, still with $R_0 < R_L$; and, as long as the neutron stars are still spinning up, then, on the average, $R_0 < R_c < R_L$. If the mass accretion rate fluctuates, R_0 fluctuates too, but R_c and R_L do not. Now, in (3), A is some constant which depends on B_S, the surface magnetic dipolar field. Comparing (3) with the correlation data of Ω_{QPO} and the count rate I yielded $2\pi/\Omega_S \sim 7$ msec, $B_S \sim 5\times10^9$ G which were amazingly close to what one may have expected from the above millisecond pulsar model. Also, (3) gives

$$d\log f/d\log I = (3/7)\left(1-\Omega_S/(\Omega_S+\Omega_{QPO})\right) \simeq 2.7 \text{ average,}$$

in close agreement with the GX5-1 observations.

What was the physical mechanism, giving rise to the beat frequency (3), going to be be? We had originally felt that a variety of physical phenomenae at R_0 would cause a modulation of flow onto

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the star at the beat frequency (BF) and, to be specific, suggested one class of phenomenae - a 'magnetic gate' process: Assume that plasma coming down to R_0 is magnetized (for example, after having been temporarily magnetized by the stationary stellar field at R_c or by some random magnetization process). Then, it will cross R_0 preferentially depending on the stellar field at R_0 (e.g., where the stellar field is opposite in direction to the plasma magnetization so that field reconnections could occur); that will give a modulation of the crossing rate at Ω_B.

Naturally, any BF process can only come about if the plasma does not flow into R_0 hydrodynamically. Instead, every region in the plamsa must carry its own magnetization along and remain pretty localized until crossing R_0. Fred Lamb has suggested to us that perhaps matter actually comes down in blobs - evidence for the blobby nature of accretion flow onto neutron stars in some compact X-ray sources has been around for a while. As we shall see below, blobs do make a fine physical scenario for realizing the Ω_{QPO}; however, as we shall also see below, they are not the only one.

It is also important to realize, that the magnetic axes of the neutron stars in QPO sources may well be at an angle to their rotation axes, perhaps even perpendicular to the latter. Therefore, the magnetospheric boundary may not be a circle in the accretion plane at all, but only some closed curve with mirror symmetry through the center, even if the average curve for large scale phenomenae is circular. This, of course, can provide for an Ω_B modulation without the incoming plasma being previously magnetized, and even in the hydrodynamic flow limit there may be an Ω_B component. These things are very hard to discuss in any great detail because of their great mathematical complexity, and we are (nevertheless) presently involved in having a crack at them. In the meantime, however, I shall assume that some kind of function of stellar and material field strengths and directions determines an Ω_B modulation such that any group of particles at R_0 will produce a luminosity signal modulated by the Ω_B clock.

2. THE BFM

There are, essentially, four basic assumptions that go into the BFM:

(1) That the QPO phenomenon, being essentially universal (from having now observed several sources), may actually be independent of the observer's relative geometry and can therefore depend, at most, on internal stellar geometry;

(2) that some special radius exists outside the star, that is the source of the oscillations, possibly R_0;

(3) that Ω_{QPO} is related to the Keplerian frequency Ω_0 - and with the given variety of observed Ω_{QPO} values possibly related to $\Omega_B \equiv \Omega_0 - \Omega_S$; and

(4) that any observed intensity-correlated variability of Ω_{QPO} is related to variability in mass accretion rate $\dot{M}$, even

if intensity changes do not necessarily relate to changes in $\dot{M}$ in any simple way (Priedhorsky 1986).

Given the above, we now describe the QPO modulation as follows:

Let us view everything in a coordinate system corotating with the matter at R_0 and have a density delta function disturbance $\delta(\phi-\phi_i, t-t_i)$ appear on the R_0 circle at azimuth ϕ_i and time t_i. As that disturbance goes down into the neutron star, let it produce a luminosity signal $\ell(\Omega_B t_i-\phi_i,\ t-t_i)$ which depends on the starting position relative to the $\Omega_B \equiv \Omega_0 - \Omega_s$ clock. Note, that an extra phase parameter should come into ℓ due to, say, a random direction of matter field $\Omega_B t_i-\phi_i \rightarrow \Omega_B t_i-\phi_i-\alpha_i$; but we shall, for simplicity, absorb α_i into ϕ_i. Assume now that the luminosity arising from a superposition of many delta-function density disturbances at R_0 is the sum of the individual signals - i.e., no non-linear effects on the way down to the star. Then, if the linear density profile at R_0 is

$$g(\phi,t) = \int d\phi_i dt_i\ g(\phi_i,t_i)\ \delta(\phi-\phi_i,t-t_i) \tag{7}$$

we have

$$L(t) \equiv \int d\phi_i dt_i\ g(\phi_i,t_i)\ \ell(\Omega_B t_i-\phi_i,\ t-t_i) \tag{8}$$

Let us go over to Fourier components,

$$g(\phi,t) = \sum_{\nu} \int d\omega\ e^{i\nu\phi} e^{i\omega t}\ \tilde{\tilde{g}}(\nu,\omega) \tag{9}$$

$$\ell(\psi,\tau) = \sum_{\bar{\nu}} \int d\bar{\omega}\ e^{i\bar{\nu}\psi}\ e^{i\bar{\omega}\tau}\ \tilde{\tilde{f}}(\bar{\nu},\bar{\omega}) \tag{10}$$

Hence

$$L(\omega) = \sum_{\nu} \tilde{\tilde{g}}(\nu,\ \omega - \nu\Omega_B)\ \tilde{\tilde{f}}(\nu,\omega) \tag{11}$$

The simplest BFM (Lamb et al. 1985) utilizes a simple form for $\ell(\psi,\tau)$,

$$\ell(\psi,\tau) = \theta(\tau)\ e^{-\gamma\tau}\ [1 + \beta\cos(\Omega_B\tau + \psi)] \tag{12}$$

where $\theta(\tau)$ is the step function: 1 for $\tau > 0$, 0 for $\tau < 0$. (8) and (12) describe a rather subtle process. One firstly needs, of course, the linearity, namely that various regimes on the R_0 circle do not influence each other's dynamics as they go down the magnetospheric boundary. Secondlay (but also related to the first point), one requires that the specific R_0 bunch of particles retains it coherence during the process; otherwise, if the coherence lifetime is shorter than γ^{-1}, we may need to replace (12) by

$$\ell(\psi,\tau) = e^{-\gamma\tau}\left[1 + \beta e^{-\Gamma\tau}\cos(\Omega_B\tau + \psi)\right]$$

and, for $\Gamma^{-1} << \gamma^{-1}$, the BF essentially disappears from the spectrum. We ask for the process to be, in that sense, not a topologically simple or a hydrodynamical flow process between R_0 and the luminosity-generating location. We do not know yet whether the inner disk dynamics really allows for these physical assumptions to be valid-disk dynamics is still too hard to handle down to such detail. We shall, however, assume that they are valid. Work on testing that is in progress, and it seems that an embedded magnetic field has much to do in securing a small Γ.

Before we proceed, we ought to comment on the possible values for β, since some recent data analysis suggests that $\beta > 1$ while a negative ℓ, which could result from such β, is certainly non-physical.

We note, that $\ell(\psi,\tau)$ could, in principle, also include higher Ω_B harmonics, as long as their magnitude is consistent with the data. It is well known, that cosine or sine Fourier transforms $\tilde{f}(\nu)$ of positive functions $f(\tau)$ have the following property:

$$|\tilde{f}(\nu)| \equiv \frac{1}{\pi}\left|\int f(t)\begin{matrix}\cos & \nu t\\ \sin & \nu t\end{matrix}dt\right| \leq \frac{1}{\pi}\int |f(t)|\left|\begin{matrix}\cos & \nu t\\ \sin & \nu t\end{matrix}\right| dt \leq$$

$$\leq \frac{1}{\pi}\int f(\tau)\ \alpha t \equiv 2\tilde{f}(0)$$

Hence, in (12), the only strict requirement on the coefficient of $\cos\ (\Omega_B t + \psi)$ is really $|\beta| \leq 2$, provided of course, higher harmonics are present in $\ell(\psi,\tau)$ or else ℓ will indeed not be always positive. For example, notice that

$$0 \leq \sin^{2k}(\theta/2) = 2^{-2k}\left[\binom{2k}{k} + 2\sum_{p=0}^{k-1}(-)^{k-p}\binom{2k}{p}\cos\left[(k-p)\theta\right]\right]$$

$$\equiv 2^{-2k}\binom{2k}{k}\left\{1+2\sum_{p=0}^{k-1}(-)^{k-p}\frac{\binom{2k}{p}}{\binom{2k}{k}}\cos\left[(k-p)\theta\right]\right\}$$

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$$[\theta \equiv \Omega_B t + \psi]$$

and the effective β value (i.e., the coefficient of cos θ) here is

$$\beta = \binom{2k}{k-1} \binom{2k}{k}^{-1} \equiv \frac{2k}{k+1} \; (\rightarrow 2 \text{ for large } k)$$

So, β can be above 1 provided the observational upper bound on the relative power at higher harmonics is not below the corresponding value of $\binom{2k}{p}^2 \binom{2k}{k}^{-2}$. If $\beta = 1.35$ is found (Elsner <u>et al.</u> 1986), then we can take k=2, and expect $\ell(\psi,\tau) = e^{-\gamma\tau}(1 - \frac{4}{3}\cos\theta + \frac{1}{3}\cos 2\theta)$ hence a relative power of 2nd to 1st harmonic 1.4%!

After this detour - back to the calculation of $L(\omega)$. Given the expression for ℓ, we find

$$\tilde{\tilde{\ell}}(0,\omega) = (\gamma + i\omega)^{-1}$$

$$\tilde{\tilde{\ell}}(\pm 1,\omega) = \frac{1}{2}\beta\,[\gamma + i(\omega \mp \Omega_B)]^{-1} \qquad (13)$$

hence

$$L(\omega) = \tilde{\tilde{g}}(0,\omega)\,\frac{1}{\gamma+i\omega} + \frac{1}{2}\beta\tilde{\tilde{g}}(1,\omega-\Omega_B)\,\frac{1}{\gamma+i(\omega-\Omega_B)} + \frac{1}{2}\beta\tilde{\tilde{g}}(-1,\omega+\Omega_B)\,\frac{1}{\gamma+i(\omega+\Omega_B)} \qquad (14)$$

(14) is the fundamental equation of the BFM. The fundamental conclusion we draw from it is that <u>to observe the QPO line, g must have power at $e^{\pm i\phi}$; to observe the LFN, g must have a ϕ-independent (i.e. isotropic) component</u>. These are necessary, not sufficient requirements.

Anyone who can imagine processes involving various contributions from an isotropic and/or a cosϕ term in the density distribution on the R_0 circle can explain any corresponding ratio of QPO to LFN intensities. I have heard it said that the BFM cannot cope with a finite-QPO no -LFN situation. This, however, is not the case, as the following example would show:

Consider

$$g(\phi,t) = A + B(t)\cos(\phi + \alpha) \tag{15}$$

where α is a fixed phase. What (15) describes may be a toy model which works as follows: Imagine that at R_c, plasma does not only get partly magnetized with azimuthal dependence but also acquires - again, due to the magnetic field - an azimuthal density variation by magnetostriction. On the way down to R_0 that density variation, like the magnetization, will be partially preserved; that will lead to (15). (15) may actually be a good approximation for any non-noisy accretion flow at the boundary region, outside of R_0. We should keep in mind that the non-steady-state apsect of the BFM is reflected in (12) and is not necessarily needed for (8).

(15) could also describe the following: Remember that we have absorbed the magnetic field angle into ϕ (p. 5). Suppose the azimuth angle is, in fact, random, but that the probability for a chunk of matter to begin entry into the magnetosphere depends on its relative magnetization angle. A situation like that may arise when the process is sufficiently non-linear. This will bring about a probability function of type (15), which can therefore be treated as an effective R_0 density.

We now have

$$\tilde{\tilde{g}}(0,\omega) \propto A\delta(\omega)$$

$$\tilde{\tilde{g}}(\pm 1,\omega) \propto \frac{1}{2}\tilde{B}(\omega)^{\pm i\alpha}$$

hence

$$L(\omega) \propto c\delta(\omega) + \frac{1}{2}\tilde{B}(\omega-\Omega_B)\ \beta\ \frac{e^{i\alpha}}{\gamma+i(\omega-\Omega_B)} + \text{c.c.} \tag{16}$$

where c is some constant. The $c\delta(\omega)$ represents a dc term in the spectrum, not LFN; hence only the QPO lines exist, shaped according to $\tilde{B}(\omega)$ (or rather $|\tilde{B}(\omega)|^2$). It is easy to see how the dc term appears: If the density distribution at R_0 is isotropic, interference from all ϕ regimes will destroy the Ω_B modulation. If, furthermore, the flow has no temporal structure, no frequency except for the zero one will appear. Also note, that B should only have power around $\omega = 0$, but can be a random process with the $\cos\phi$ term turning randomly on and off. If (15) represents the magnetic field angle, then in (16) $\tilde{B}(\omega-\Omega_B)$ should be replaced by $\tilde{B}(\omega)$, so that B should have power around Ω_B.

Naturally, if A were time dependent, LFN would arise, shaped by $\tilde{A}(\omega)$. But, in general, different LFN and QPO lineshapes can be easily understood via different time dependence of A(t) and B(t).

The easiest way of envisaging power at $e^{\pm i\phi}$ is, of course, in a process which has power at every ϕ harmonic (including $\nu = 0$). A blobby flow can accomplish that (Lamb et al. 1985), with

$$g(\phi,t) = \sum_j a_j \delta(\phi-\phi_j)\, \delta(t-t_j) \tag{17}$$

where a_j, ϕ_j and t_j are all random variables. Since independent evidence for the blobby nature of flow in some X-ray sources does exist, this is a natural process to examine. Then

$$\tilde{\tilde{g}}(\nu,\omega) \propto \sum_j a_j e^{-i\omega t_j} e^{-i\nu\phi_j}$$

and the corresponding LFN and QPO powers are

$$\text{LFN} \rightarrow \frac{1}{\gamma^2+\omega^2} \sum_{p,q} \bar{a}_p a_q\, e^{i\omega(t_p-t_q)} \equiv \frac{1}{\gamma^2+\omega^2} F(\omega) \tag{18}$$

$$\text{QPO} \rightarrow \frac{(\frac{1}{4})\,\beta^2}{\gamma^2+(\omega-\Omega_B)^2} \sum_{p,q} \bar{a}_p a_q\, e^{i(\phi_p-\phi_q)}\, e^{i(\omega-\Omega_B)(t_p-t_q)}$$

$$\equiv \frac{(\frac{1}{4})\,\beta^2}{\gamma^2+(\omega-\Omega_B)^2}\, G(\omega-\Omega_B) \tag{19}$$

For uncorrelated a_j, ϕ_j and t_j,

$$F(\omega) = G(\omega-\Omega_B) \equiv \Sigma|a_p|^2$$

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hence the LFN and QPO powers are always correlated here, much as they are found to be in GX5-1 (van der Klis et al. 1985) with power ratio of $(1/2)\ \beta^2$; as we pointed out earlier, under some circumstances that ratio may be close to 2. Processes in which some cross correlations between a_j, ϕ_j and t_j can be imagined, which therefore have various degrees of coherence, can change the LFN/QPO power.

3. PROBLEMS

The preceding discussion represents, probably, all the distance in mathematics that one could dare to cover before more detailed physical computations will be at hand. Let me, therefore, address myself to some of the obvious problems the BFM may have - which may all be attributed to our ignorance, not necessarily to the inadequacy of the BFM.

The most important problems are:

(i) Underlying all is the assumption that Ω_S exists, i.e. that the neutron star rotates at angular frequency Ω_S which, in the case of GX5-1, say, is of order 10 msec. Yet, Ω_S is yet to be detected in the X-ray luminosity of any of the LMXBs, let alone the QPO ones. This, while the very assumption of the existence of $R_0 > R_S$ means that if the magnetic field is dipolar one should see an X-ray modulation across the stellar surface. Some ways of understanding that have been discussed in Lamb et al. (1985). Effects like the low efficiency of channeling of plasma onto polar caps due to the relative closeness of R_0 and R_S, gravitational lensing (but see Chen and Shaham 1987), optical scattering in the disk corona, preferential viewing along the direction perpendicular to the disk (hence the magnetic field?) because the disk is optically thick - all of these may combine to quench the Ω_S modulation substantially, but possibly not below some of the observational upper bounds. A promising possibility is, however, the following:

Eichler and Wang show in these proceedings, that as the magnetic field in neutron stars decays, higher multipoles become relatively stronger on the surface. The resultant multi-peaked surface field will cause a very different X-ray pulsation pattern than one finds in simple dipole fields: With sufficiently large "cap" areas, already an octupolar field component may smear the Ω_S pulsations substantially; higher mutlipoles will produce low level modulation at much higher - and at a wide range - of frequencies. We note, that this complex surface field need not be relevant to R_0 (hence the accretion process) or R_L (hence the pulsar action after the X-ray source became a pulsar): Even for a 1.5 msec pulsar, $(R_L/R_S)^2 \sim 50$, which is roughly by how much the dipole will be amplified over any other multipole at R_L over their surface ratio. The amplification at R_0 will be [see (5)] ~50 for a 10 msec neutron star.

(ii) The simple f vs. I relation in GX5-1 becomes inverted and even bimodal or indefinite in ScoX-1 (Priedhorsky et al. 1986). How to interpret that? We note, that the BFM predicts an f vs. $\dot{M}$ correlation, not directly an f vs. I. I itself is really not defined until one specifies the frequency range, $\Delta\nu_i$. Now, since torques (magnetic? material?) on the disk vary with $\Omega_S/\Omega_0 \equiv (1 + \Omega_B/\Omega)^{-1}$, and since these fine-tune the amount of total enery available (accretion energy onto the neutron stellar surface $\pm$ spin$^{down}_{up}$ energy of the neutron star), a marked difference may develop between dlog Ω_0/dlog $\dot{M}$ and the various dlog Ω_0/dlogI($\Delta\nu_i$), as was first pointed out by Priedhorsky (1986; see also Shaham and Tavani 1987). In fact, Priedhorsky writes

$$I(\text{hard X-rays}) = \dot{M}\left[\frac{GM}{R_S}\left\{1 - \frac{1}{2}\left(\frac{R_S}{R_c}\right)^3\right\} - f(\dot{M},\ \Omega_S)\right]$$

$$I(\text{soft X-rays}) = \dot{M}\ f\ (\dot{M},\ \Omega_S) - N\Omega_S$$

where N is the total torque on the star and f is some function. Priedhorsky suggests

$$f = \frac{GM}{2R_0} + \Omega_S(R_0^2\ \Omega_0 - R_S^2\Omega_S)$$

while Shaham and Tavani (1986) investigated the possibility

$$f = \frac{GM}{2R_0}$$

We find, that for a variety of choices of f, dlog I(hard)/dlog$\dot{M}$ indeed changes sign for some value of $\dot{M}$. Furthermore, an extended region with dlogI(hard)/dlog$\dot{M}$ >> 1 can exist, where f is essentially constant with I. A well defined I(hard) vs. $\dot{M}$ curve can be constructed to correspond to the full extent of the Ω_{QPO} vs. I ScoX-1 observations. Thus, even ScoX-1 can be interpreted on the basis of the BFM.

There is certainly a lot more that we do not know about the magnetospheric boundary, and hence about the detailed physics of the BFM. One may want to see the observations in order to either probe the magnetospheric boundary with the aid of the BFM or come up with a clearly better model for QPOs. A group of us is actively following the first option, mostly because the BFM looks, to date, more promising than other suggested models (see W.H.G. Lewin 1987).

One important physical method of probing an unknown region is to have it subject to some controlled external perturbation. Within the BFM, one important perturbation of the magnetospheric boundary can be neutron stellar free precession (as in Her X-1, see Trumper et al. 1985). Free precession will periodically change the opening angle of the cone, on the surface of which any particular magnetic field line segment rotates, so that the actual beat frequency may change (for details see Shaham 1986).

To fix our ideas, assume that the magnetic gate operates via $(\vec{B}\cdot\vec{r})^n$ where $\vec{r}$ is the radius vector of material orbiting at R_0, $\vec{B}$ is the local field at R_0 and n is some integer. Then $\vec{B}\cdot\vec{r} \propto \vec{\mu}\cdot\vec{r}$ where $\vec{\mu}$ is the stellar dipole. An extreme case is when the stellar rotation axis is in the disk plane with $\vec{\mu}$ perpendicular to it. Then $\vec{\mu}$ crosses the disk twice each $2\pi/\Omega_S$ period, and for n = 1 this gate gives both $\Omega_0 \pm \Omega_S$ freqeuncies at equal powers. Another extreme case for n = 1 is with $\vec{\mu}$ aligned with the rotation axis which is, again, in the plane of the disk. Then only Ω_0 appears, i.e. the BF is replaced by the magnetospheric boundary Keplerian frequency. In the most general orientation for n = 1, one obtains the $\Omega_0 \pm \Omega_S$ and Ω_0 frequencies at different powers, generally modulated by the free precession frequency. For larger n values more frequenices show up, among them Ω_S, $2\Omega_S$ and $2\Omega_0$, and I find it interesting that in CygX-2 there is some evidence for Ω_0 showing up (Hasinger 1987). In CygX-2 these things also correlate with hardness ratio (Hasinger 1986). The relation between the above mentioned cone angle and the hardness ratio is to date a complete unknown, but it seems quite worthwhile to look into free precession of QPOs with the Cyg X-2 observations in mind.

4. CONCLUSION

The QPO phenomenon has grown rapidly from a clean, simple one when only GX5-1 was known to a complex, puzzling one as other sources were discovered. Does it mean that there are many ways to produce QPOs - or that the geometries and environments change from source to source while the fundamental process remains the same? Scientific debate is always the hottest when very little is known. So, one should keep looking to present and future observations for guidance and also try to get a better theoretical handle on the magnetospheric boundary. Hopefully, the next IAU symposium on neutron stars will bring with it cooler temperatures and better answers.

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DISCUSSION

R. Narayan: It seems to me that a natural way to obtain blobs is through an instability in the accretion disk, in which case a modulated density might be more likely than random blobs. However, the azimuthal wave-number m of the instability could be greater than 1. Some of the formulae you showed would then change. How does this affect your estimates of magnetic fields, neutron star period and accretion radius?

J. Shaham: In the mathematical picture that I presented, only m=1 will contribute ($e^{\pm i\phi}$). Higher m values will only contribute when $m(\Omega_B\tau+\psi)$ is present in eq. (12). Then, of course, the real $\Omega_B = \Omega_{QPO}/m$. That will reduce Ω_s by m, amplify B by $m^{7/6}$ and multiply R_0 by $m^{2/3}$.

J. Arons: Comment 1: Every speaker attributes the "natural" frequency for QPO to the phenomenon with which he/she is most familiar. You call the frequencies of orbital mechanics/stellar rotation the most natural. Narayan, who has done lovely work on disk instability, suggests in a preceding question the instability of a disk is most "natural." In the interests of continuity, I suggest a third "natural" means of forming QPO. As I showed you on Tuesday, polar cap accretion is prone to form bubbling and boiling motions whose onset time scale is ~ msec; while the ultimate fate of these motions is still unknown, we think they lead to fluctuations in the flow on the order of 10s to 100s of msec. Therefore, in the interests of "natural" theories, let me suggest QPO reflect "convection" at the stellar surface in the accretion flow, rather than an orbital or a disk phenomenon. Comment 2: On a (slightly) more serious note, any magnetospheric model, blob frequency or not, requires a small magnetosphere, consistent with the commonly held thoughts about "decay" of magnetic field in old neutron stars. The only decay model in the literature that makes any sense is that of Flowers and Ruderman, recently given a first quantitative form by Eichler and Wang (this meeting). Here, the MHD motions of the core, constrained to crustal ohmic rates by the penetration of field lines through the crust, cause the dipole field to disappear, in favor of higher order multipoles (perhaps to stop at field complexity ~ octopole). Thus, one gets a small magnetosphere (needed for spin up) with stronger surface fields, and a complex pattern of accretion flow onto the surface rather than two simple polar caps (good for having a broad QPO line, since now many emission sites on the rotating star contribute). But, a field of this structure doesn't give $f_{QPO} \propto (\text{luminosity})^{3/7}$. For example, a quadrupole field gives a magnetopause radius $R_m \propto L^{-2/11}$ instead of the dipolar $R_m \propto L^{-2/7}$ (Arons & Lea, 1980, Ap. J), which in turn gives $f_{QPO} \propto L^{3/11}$; dominance by higher order structure makes the exponent still smaller. If the observers can show that the power really is 3/7 (modulo the bulk of the data which doesn't fit well anyway), I would regard that as evidence for the stars having been born with weak fields, with the currents in the star sufficiently deep down to give a persistent, dipole component which dominates even as close as several stellar radii. I doubt this is right, because we don't see pulses, and prefer the complex field model, but this is clearly a question which needs observational solution. Question: Do you refer to flux transfer events in your reference to geophysical evidence for mass entry when fields are opposed?

J. Shaham: Yes. Some observational evidence exists to suggest that flux entering the Earth's atmosphere is maximum at locations where fields oppose.

F. Verbunt: Does the beat frequency model also apply to pulsars with a strong magnetic field? If so, what frequencies would one expect?

J. Shaham: The question I have in my mind is what role does the region between R_c and R_0 play in preserving the coherence of the matter flow and what role does it play in the transition region at R_0. In principle, large B's increase the scale of everything and less coherence may ensue. If one can still keep some of it, one might expect frequencies to scale as $B^{6/7}$ (Eq. 5).

M. van der Klis: How do you preserve the pattern of the "slightly modulated Keplerian flow" from the corotation radius down to the magnetospheric radius in the presence of the neutron star's magnetic field, the strength of which increases when you go down?

J. Shaham: When R_c and R_0 are sufficiently close, the stellar field will not increase much. But the question is a relevant one even then, as I mentioned in my talk, because flow must be essentially non-interactive, and at present the theoretical situation is not clear even when blobs exist. As mentioned before, an embedded field will help.

W. Brinkmann: So far only power spectra were used in the data analysis. Generally you lose information in going from Fourier transforms to power spectra. Do you think there is any useful information in the amplitude-phase relations?

J. Shaham: There is importance in the phases of the power spectra. One way of utilizing them is described by Gunther Hasinger in his talk, namely, with cross correlation functions yielding information on time delays. Also, the degree of coherence of processes (blobbiness?) may be revealed.

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