

Inference problems for vector linear time series models

D.F. Nicholls

Two linear models which are of fundamental importance in the analysis of time series are the autoregressive model and the moving average model. This thesis is concerned with a number of inference problems associated with models such as these, and models constructed from them.

The first model to be considered is the vector autoregression, for which a goodness of fit test is derived. An outline of the computational procedure to be followed when applying this test is presented and the test is compared with various other goodness of fit tests (for autoregressive models) occurring in the literature, the advantages and disadvantages of each being discussed.

A chapter devoted to moving average models commences with a description of a procedure, due to Hannan [2], for the efficient estimation of the coefficients of such models. After examining the rate of convergence of the iterative procedure (in the case of the first order moving average model) associated with this method, an extension of Hannan's procedure is described. This extension is introduced so as to prevent the estimation procedure from, on some occasions, giving estimates which fail to converge as the iterations proceed. This chapter concludes with a comparison between Hannan's estimation procedure and two procedures due to Durbin [1].

The second part of this thesis is concerned with the derivation of a method for the estimation of the coefficients of mixed autoregressive moving average models with exogenous variables, which may be expressed

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mathematically in the form

$$(1) \quad \sum_{k=0}^q \beta(k)y(n-k) + \sum_{k=1}^r \delta(k)x_k(n) = \sum_{j=0}^p \alpha(j)\varepsilon(n-j),$$

when the $\{\varepsilon(n); n = 1, 2, \dots\}$ represent the residuals, while the $\{x_k(n); n = 1, 2, \dots\}$ represent the exogenous variables. This procedure, which is based on the method of maximum likelihood, gives estimates which are shown to be asymptotically normally distributed and (asymptotically) efficient. After outlining the computational procedure for the application of this method, the results of a number of numerical experiments, using generated data, are reported. These results help to illustrate the theoretical formulae obtained.

By placing constraints on the coefficients of models of the form (1) above it is shown that it is possible to efficiently estimate the coefficients of certain classes of rational distributed lag models. Finally, the estimation procedure applied to (1) is extended to consider the efficient estimation of the coefficients of vector mixed autoregressive moving average models with exogenous variables.

References

- [1] J. Durbin, "Efficient fitting of linear models for continuous stationary time series from discrete data", *Bull. Inst. Internat. Statist.* 38 (1961), 273-282.
- [2] E.J. Hannan, "The estimation of mixed moving average autoregressive systems", *Biometrika* 56 (1969), 579-593.