IMAGING TECHNIQUES

Limitations to the quality of images

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1. Introduction

Our ideas of how to image objects have progressed by leaps and bounds in the last twenty to thirty years. We now have a sophisticated understanding of many clever and subtle approaches to imaging. There are a few outstanding contributions which have provoked flurries of development and acheivement in many different areas: top of my list would be the principle of aperture synthesis, Jennison's closure phase, Högbom's CLEAN algorithm and Labeyrie's speckles. In addition, we have benefited tremendously from developments in computing hardware, software and algorithms (the most spectacular being the Cooley-Tukey Fast Fourier Transform). Each one of these contributions did not so much spur development in existing areas as open up entirely new vistas of possibilities. For example, Jennison's closure phase is rarely directly used by radio-interferometrists now but it did show, particularly when developed in VLBI, that imaging in the presence of severe phase errors is possible. This success then encouraged the two separate developments. First in the more flexible selfcalibration routines in which closure is implicit and, second, in pushing imaging interferometric arrays to shorter and shorter wavelengths, now ending up in the infra-red and optical regimes. Following on from these great works, many people have made lesser but still vital contributions. The example of speckle comes to mind as one where many people have had a hand in determining what is now standard technique.

The goal of this talk is to discuss the limitations which currently exist on the quality of imaging. I see four key factors in determining the quality of imaging:

- *The Measurement Equation* describes how an instrument responds to an object,
- *Inversion methods* allow estimation of an object, thus forming an *image*,
- *Calibration methods* derive unknown parameters in the *Measurement Equation*,
- *Measurement Strategies* determine methods for collecting data to be used in *Inversion Methods.*

To make an image, one must address all of these factors and any one can limit the image quality. Hence our advances in imaging over the years have come from pushing in one or more of these directions.

The plan of this talk can now be laid out. First I will discuss briefly just what we mean by imaging quality. Then I will go on to discuss these four factors and

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describe what recent developments have been relevant to each one. Finally, I will conclude with some guesses for interesting areas to investigate.

To fit this review into the finite amount of time and space available, I will concentrate upon interferometric imaging. I will, however, pause to briefly acknowledge other areas such as speckle. I will also concentrate upon describing areas where progress has been possible rather than fundamental limits such as due to detector noise, atmospheric coherence times, etc, all of which differ considerably from one instrument to another.

2. Image Quality

As the name suggests, it is hard to quantify image quality. Part of this is simply due to the vagueness of the scientific question which is to be addressed using an image. When the question is specific, such as photometry, quantification of image quality takes on more meaning and, furthermore, the imaging can often be improved considerably.

That this is a serious matter can be seen from a very common experience in imaging: trying to demonstrate the superiority of a new procedure/telescope/algorithm over competitors. Often one has to resort to simulation of a target object. Since the object is known, one can then judge the deficiencies of the image. However, what metric should one use? the rms deviation of image from object? over the whole field or part of the field? at full resolution or smoothed? The answer, of course, depends upon the science that is to be done, something that is often unknown beforehand, and so one can argue that all of the above metrics are important. The conclusion is that one should use a number of complementary metrics. A corollary is that imaging systems should be designed for flexibility of goals. The NRAO-proposed Millimeter Array (MMA) is an example of a telescope designed with such flexibility. It will be able to provide excellent good images of arcminute-scale regions with sub-arcsecond resolution. Cornwell, Holdaway and Uson (1993) describe some of the factors affecting the design of the MMA. They used a number of quality metrics:

Fidelity is a measure of the on-source signal to noise of an image. By Monte Carlo methods, one can derive the ratio of the expected brightness at any given pixel to the typical or rms error in estimating that brightness. This constitutes an image of the fidelity. Since much of the processing was very time-consuming, Cornwell, Holdaway and Uson (1993) often took the median value for all pixels from a single trial. It is sobering that the on-source SNR as estimated in this way lies only in the range 10 to 100 for good VLA and MMA images.

Dynamic range measures the ratio of the peak brightness to the rms rumble in a region of the image which is believed to be free of intrinsic emission.

Visibility SNR curves plot the ratio of the observed visibility to the reconstruction error plotted as a function of spatial frequency. A single trial can be useful if azimuthal averaging is performed.

In real observations, only the dynamic range can be estimated and so only the dynamic range is quoted. For simulated data, one has more flexibility. Both fidelity and dynamic range are single numbers and so one is tempted to rely upon one of

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these in a shoot-out based on simulated data. Often, however, a curve of both as a function of smoothing scale is more informative and then a decision becomes more complicated.

3. The Measurement Equation

The *Measurement Equation* relates the measured quantity such as fringe visibility to the sky brightness. This relationship may be simple and straightforward as in interferometry or complicated as in the various forms of speckle imaging such as the bi-spectrum methods, where the true observable is the triple correlation of the image (see e.g. Weigelt, 1989). There may also be varying levels of approximation to the true measurement equation such as occurs in interferometry going from monochromatic to broadband radiation, coplanar to non-coplanar arrays, phase-stable to phase-unstable arrays, etc. (see e.g. Thompson, Moran and Swenson, 1986).

Advancing to a more complicated but presumably more faithful *Measurement Equation* can bring two benefits. First, the imaging quality may be improved, and second, a new insight may come. Examples of the former abound and form the staple of many papers on imaging. New insights are harder to come by. One of the nicest examples is that of self-calibration where rewriting the measurement equation explicitly in terms of antenna phases yields a whole new realm of possibilities beyond the nominally equivalent closure phase methods (see e.g. Pearson and Readhead, 1984). For example, adding derivatives with respect to time and frequency to the standard selfcalibration methods delivers global fringe fitting (Schwab and Cotton, 1984). Non-isoplanatism can potentially also be treating using the insight in selfcalibration (Schwab 1984, Subrahmanya 1991). Insights therefore also lead to enhancements in imaging quality, but only indirectly.

The use of a more accurate *Measurement Equation* is often dependent upon improvements in computing. In the case of the VLA observing at long wavelengths, the non-coplanar geometry of the array leads to aberrations in images, essentially because the array has a different shape as viewed from various parts of the field of view. Computing has now advanced sufficiently that we can now correct this aberration in software and improve the noise level by about an order of magnitude (Cornwell and Perley, 1992, Cornwell, 1993). The aberrations of the Hubble Space Telescope are also being treated by the use of a more complicated *Measurement Equation* (see the talk by Hanisch in this volume). It is worth saying that in the case of the HST, the *Measurement Equation* can be very complicated indeed. On top of the well-known spherical aberration, there are other severe obstacles, such as undersampling of the image plane, shift-variance of the PSF, detector saturation and bleeding, photon and readout noise, cosmic ray hits, flat-fielding. Correcting many of these effects can be extremely expensive in computing.

4. Inversion

Inverting the *Measurement Equation* yields an estimate of the object which we traditionally call the *image*. Inversion has been very rich in producing advances in imaging. In interferometry, these have ranged from better weighting schemes for the Fourier inversion (see e.g. Fomalont 1973) to ever increasing sophistication in non-linear deconvolution algorithms (e.g. Högbom 1974, Narayan and Nityananda, 1986). In early forms of speckle, the necessity for working only from the magnitude of the Fourier transform of the object led to the Phase Problem on which a lot of ingenuity has been expended. The more modern version of the Phase Problem is that of reconstructing an object from estimates of the bi-spectrum (see the talk by Weigelt). In optical astronomy, sophisticated inversion schemes only became popular with the unfortunate problems with the HST primary mirror. The Lucy algorithm (Lucy, 1974) seems to have matured into a relatively well-understood and effective tool, and the MEM also seems to have played a very useful role.

4.1. Linear methods

There has existed a curious state of affairs whereby although simple linear methods could work in some circumstances (e.g. VLBI imaging of small fields), non-linear methods such as CLEAN were substantially faster and were also more familiar to the astronomical community. Understanding of linear methods like Singular Value Decomposition, with or without regularization, should be required before proceeding to non-linear methods such as CLEAN and the Maximum Entropy Method. The advances in computing allow the examination of the singular values for quite interesting sizes of imaging operator (a high-end workstation can reasonably do 64 by 64 pixels). This has turned what used to be just an academic exercise (thinking about the ill-conditioned nature of imaging) into a valid and useful approach for small images.

4.2. Failure modes

Systematic explorations of imaging algorithms are easy to do but hard to interpret since the results tend to be strongly tied to the context of the tests. Some general properties of both CLEAN and MEM are known, however. The failure modes of MEM are relatively easy to understand since the image is defined by an equation (Narayan and Nityananda, 1986). From this we know, for example, that MEM fails in the presence of a background of emission. The properties of CLEAN in the underconstrained case, not studied by Schwarz (1978), are rather less well understood. Although the CLEAN algorithm looks simple, in reality it has some rather bizarre aspects. In particular, CLEAN images are prone to striping and mottling of uncertain origin. Furthermore, the paper by Tan (1986) shows what appears to be chaotic behavior of CLEAN in even very simple cases. A valiant attempt by Marsh and Richardson (1987) to place CLEAN on a firmer theoretical basis by postulating that it actually formed a minimum L1 norm solution does not seem compatible with these unexpected properties.

It is widely believed that errors in the inversion are proportional to the brightness so that bright regions suffer large errors and weak regions small errors. Surprisingly even this seems to be incorrect. Briggs and Cornwell (these proceedings) show examples of deconvolved images from the VLA where off-source ripple that would normally be attributed to calibration errors is, in fact, due to invisible distributions generated by the deconvolution algorithm. The only reliable countermeasure is to smooth the image, decreasing the resolution by 30 to 40%.

4.3. New Methods

The development of new methods for inversion is fraught with potential for error. Although CLEAN is ostensibly simple and easy to understand, it is tied to a particular model for an image whereby only spatial frequencies present in the point spread function are present in the dirty image (Schwarz, 1978). With this in mind, it is curious that CLEAN does actually appear to work reasonably well for optical data where this condition is violated (see e.g. Keel, 1991). One possible explanation is that in such cases CLEAN is not run sufficiently close to convergence for problems to arise.

One advantage of MEM over CLEAN is the flexibility it affords in dealing with a complicated *Measurement Equation*. Measurements are incorporated as constraints on the final image and an optimization algorithm is designed to find a solution obeying all known constraints. In this way, many different types of *Measurement Equation* can be incorporated straightforwardly (see e.g. Cornwell 1988). This discipline of writing down an explicit measurement equation for an instrument is very useful in ensuring that the instrumental model matches the algorithm, something that can be missed in other, less transparent methods.

4.4. Overview

The poor state of inverse theory can best be summarized by noting that it is impossible as yet to give error bars for deconvolved images. This unsatisfactory state of affairs is nearly always viewed as scandalous by neophytes to the field and somehow becomes less and less troublesome as one gains experience. Lannes (1987) has started a program to produce such error bars, or in fact an upper bound on the error. While his approach looks quite plausible, his actual methods for image processing have yet to survive wider exposure.

Two steps can help alleviate some of the difficulties arising from our poor understanding of inverse theory. First, a number of complementary inversion methods should always be available. We should look around for inversion methods that are widely used in fields outside of astronomy. Second, defects of the various inversion methods should be described as completely and objectively as possible. While this can probably be done in a systematic way for certain instruments such as the HST and the VLA using incomplete and complete datasets, there is also an argument for providing simulation capabilities at the heart of any modern image processing system. This would ensure that an algorithm can be easily tested in the correct context.

5. Calibration

A *Measurement Equation* usually has a number of unknown parameters which must be derived in order for the inversion to proceed. Estimation of the calibration parameters can be done in two different ways. First, one may use a reference signal of known or assumed properties. Examples of this include astronomical point sources for interferometer calibration (see the paper by Holdaway for an advanced investigation of such calibration), stars for the HST (see the paper by Hanisch), and laser guide stars for adaptive optics (see the papers by Max and by Fugate in these proceedings). Second, one can use the object itself to calibrate the system. Self-calibration does this by allowing the amplitude and phase calibration of the array elements in an interferometric array to float (e.g. Cornwell and Fomalont 1989). The freedom thus introduced can be counteracted by constraints on the final image such as positivity and finite support and by any internal redundancy of the data.

One sign of the success of selfcalibration methods in radio-interferometry is the poor understanding we have of the statistics of the atmosphere. While this is being remedied as we push to imaging of weaker objects at shorter, millimeter wavelengths, it was true that the use of self-calibration at centimeter wavelengths enabled lots of astronomy to be done ignoring the atmosphere. Almost the opposite situation hold for speckle imaging where knowledge of the statistics of the atmosphere is vital since for simple averaging of the power spectrum, all the information comes from the ratio of the target Fourier transform to reference Fourier transform. This was alleviated considerably by the advent of the Knox-Thompson and Bi-Spectrum methods in which a phase could be measured.

The reference signal approach would clearly be preferable if one were sure that it was relevant to the target source. For many cases this is quite doubtful because the calibration measurements are taken at a different time and in different context. So for sufficiently bright objects observed with interferometric techniques, one usually gets higher dynamic range using selfcalibration methods. Similarly, for bright objects observed at optical wavelengths, it is preferable to bypass the question of calibration of the atmosphere by using non-redundant masks to eliminate the decorrelation effect altogether (see talk by Haniff).

Inaccuracies in describing parameters in the *Measurement Equation* can have subtle side-effects leading indirectly to limitations in image quality. The software used for polarization calibration at the VLA has always neglected second-order terms due to the instrumental polarization. This explains the observed limit on the dynamic range of VLA total intensity images of about 10^5 (Holdaway, 1992).

6. Measurement Strategies

Measurement Strategies determine how and data is to be collected. For interferometers, more data usually bring an improvement in image quality. An interferometric array can be reconfigured in many different ways: actually moving the elements (Ryle and Hewish 1960), Earth rotation synthesis (see e.g. Fomalont 1973), spaceborne orbiting elements (Levy *et al.*, 1986) and hypersynthesis (Vivekanand and Downes 1989). All of these can improve the Fourier plane coverage and thereby substantially improve the image quality.

Phase diversity methods (Gonsalves, 1982) are a particularly sweet way of measuring more information. For an aberrated fully filled aperture like the HST, zeroes in the modulation transfer function can be moved around in spatial frequency by changing the position of the focus (this introduces a quadratic phase term which changes the MTF). One can then synthesize a fully sampled Fourier plane by observing at a couple or more focus settings. (Hunt, 1991). Unfortunately for this nice scheme, the HST team did not want to change the focus that often.

It often pays to repose the scientific question. In the case of interferometric imaging of gravitational lenses, one can invert both the interferometric measurement equation and the gravitational lens measurement equation at the same time (Kochanek and Narayan, 1992). This then provides some redundancy which aids both inverse problems. From the view point of the source, it is observed by multiple copies of the interferometric array, each stretched and distorted in a different way, thus forming a new type of synthesis.

Another example of re-posing the scientific question is to exploit prior information about an object such as a close relation between the brightness at adjacent frequencies. This allows one to fill in the Fourier plane by changing the observing wavelength over a small range, say 3-25% (e.g. Conway, Cornwell and Wilkinson, 1991). It is then important to correct for any spectral effects but for a small bandwidth, this can be done using a modified version of the CLEAN algorithm. This technique will also work for optical interferometers and will aid in improving the image quality (e.g. Simon *et al.*, 1991).

7. Overview

It would be a mistake to over-emphasize any one of the four factors as being key in determining image quality. Advances in image quality have come from all four. However, I do think that a more systematic approach to the four factors is worthwhile. This means different things for the different areas:

- *Measurement Equation*: Write down the measurement equation in as general a form as possible. Then explicitly go through any limiting cases to see that all have been handled adequately.
- *Inversion Methods*: Try to really understand the inverse methods, perhaps from simulation of their behavior. Investigate whether inverse methods in other fields are applicable.
- *Calibration Methods*: Does the data have some internal redundancy similar to closure?
- *Measurement Strategies*: Try to repose the scientific question in some slightly different way.

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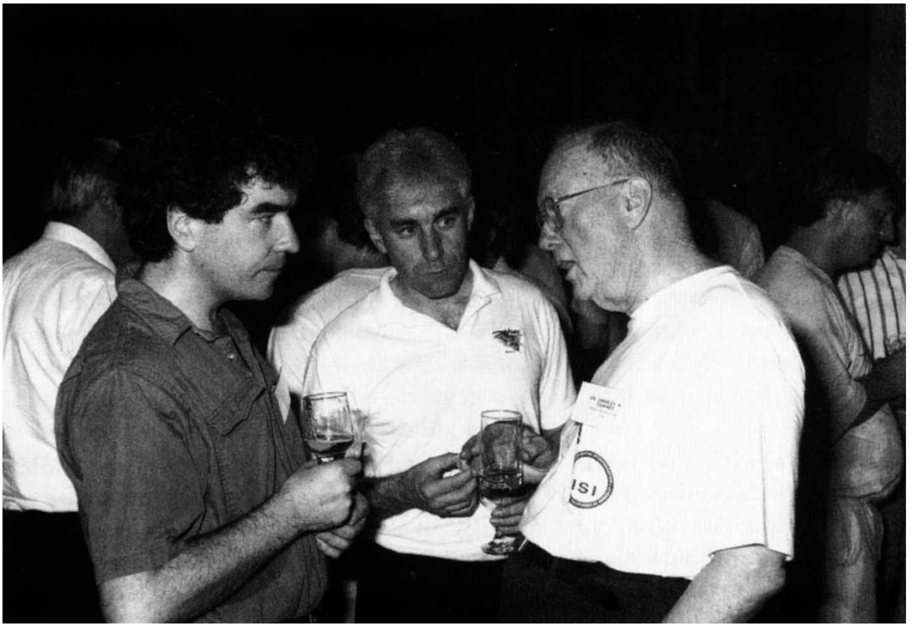
Discussion:

Baldwin:
Did your analysis of the mm array lead to changes in its design?
Cornwell:
Yes, in several ways. First, after much discussion, we eliminated the big single dish which was to be used for short spacing measurements. We instead recover short spacings via mosaicing. Second, a number of the antenna specifications are driven by our study of mosaicing. Examples of affected parameters are pointing accuracy and surface accuracy.

Ekers:
The time scale for designing and building large instruments seems to be longer than the time scale for new imaging algorithms to evolve, consequently design choices based on current algorithms may be inappropriate by the time it is built. I believe this occurred for both the VLA and AT synthesis telescopes.
Cornwell:
In the case of the MMA, we have relied upon general principles, such as the desire to sample the Fourier plane fully, rather than a configuration highly tuned to a particular deconvolution algorithm. Nevertheless, for detailed specification such as pointing precision, we have used very specific algorithms. This is one argument for eradicating 'magic' in algorithms.

Perley:
Why are error estimates for the deconvolved images impossible to determine? Will it be possible for approximate rules to be determined which will permit users to assign errors to their resultant images?
Cornwell:
My pessimism applies especially to the CLEAN algorithm which is so highly non-linear that the effects of noise can easily cause a complete re-arrangement of the CLEANed pixels. In the case of MEM, something approaching an error can be calculated but it is probably better thought of as a measure of stability of the solution.

Simon:
Have you neglected one key approach for improving image quality, namely, building a new instrument?
Cornwell:
One can build a better instrument in a number of different ways. The four factors apply to this aspect as well as to the reduction of data from an existing instrument.



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