Abstract. Angular diameters of $0.0056 \pm 0.0011$ and $0.0049 \pm 0.0008$ have been calculated for the red giants 46 Leo and $\phi$ Aqr respectively from occultation observations. The occultation curves of $\alpha$ Lib and $\epsilon$ Cap show that they are binary systems. The separation of the components of the visual binary $\epsilon$ Ari, as calculated from its occultation curve, is 1.3 times its reported value. The occultation curve of $\alpha$ Leo shows distortion effects which are attributed to irregularities in the lunar limb.

1. Introduction

The photoelectric occultation observations that I wish to report on were started in 1966 (Poss and Kremser, 1967). We have recorded about 20 occultations thus far and I will describe 6 of them: two cases of red giants having measurable angular diameters, two cases of close binaries, one example of a visual binary, and one example, representative of a number of others, in which the pattern does not resemble a simple straight edge diffraction pattern.

2. Instrumentation

The instrumentation makes use of commercially available components for the most part. Observations are currently being made with the photometer that is in use on the 28 in. reflector of the Flower and Cook Observatory of the University of Pennsylvania. It makes use of a 1P21 photomultiplier with standard UBV filters. The photomultiplier output is amplified by an electrometer (Keithley 610C) and then recorded on a frequency modulated tape recorder (Ampex SP 300). In this way, assuming that there is a negligible amount of distortion in the recording process, the original signal with its full information content is preserved as a readily accessible electrical waveform which can be displayed on an oscilloscope or analyzed by computational techniques presently available or which may be developed in the future. The overall frequency response of the system extends from DC to 2500 kHz. The tape recorder has four channels so that observations could be made simultaneously in more than wavelength region if a suitable photometer were available. Radio time signals are recorded on one channel. The time at which the occultation occurs can be established with reference to the received signals (i.e. neglecting transit time delays) with an accuracy ranging from 0.001 sec to 0.01 sec, depending on the quality of radio reception and the signal-to-noise of the occultation curve.
3. Analysis

The determination of angular diameters for the two cases reported below is based on measuring the intensity of the first maximum in the diffraction pattern relative to the free field value. Extensive analytical studies of straight edge diffraction patterns for sources of finite angular diameter with this purpose in mind have been carried out by Wijesinghe (1966). In Figure 1, the intensity of the first maximum of the diffraction pattern relative to the free field value is plotted as a function of $K\theta$. $\theta$ is the angular diameter of the source and the scale factor is given by

$$K = a \sqrt{\frac{1}{2\lambda} \left( \frac{1}{a} + \frac{1}{b} \right)}.$$ 

Here, $\lambda$ is the effective wavelength of the light, $a$ is the distance from the straight edge to the observer, and $b$ is the distance from the straight edge to the source. The calculated curve was checked by laboratory measurements in which values of 1 m were used for $a$ and $b$. In the lunar occultation case, $b$ is essentially infinite compared to the lunar distance $a$ and the corresponding value of $K$ is $1.4 \times 10^4$ times its laboratory value. Thus a star of angular diameter $0'\!005$ can be simulated by a pinhole aperture in the laboratory of angular diameter $70'$. The curve in Figure 1 was calculated on the assumption that we are dealing with circular sources. In the radio case, this as-

![Fig. 1. Intensity at the first maximum of the diffraction pattern relative to the free field value vs. angular diameter of the source. $K$ is a scale factor defined in the text. Curve $a$: intensity ratio for the true maximum. Curve $b$: intensity ratio for the value of the Fresnel parameter corresponding to the first maximum for a point source.](https://www.cambridge.org/core/core/figure/fig-1-intensity-at-the-first-maximum-of-the-diffraction-pattern-relative-to-the-free-field-value-vs-angular-diameter-of-the-source-k-is-a-scale-factor-defined-in-the-text-curve-a-intensity-ratio-for-the-true-maximum-curve-b-intensity-ratio-for-the-value-of-the-fresnel-parameter-corresponding-to-the-first-maximum-for-a-point-source)
sumption cannot be made and a similar curve published by Hazard (1962) is con-
sequently not identical with the curve of Figure 1.

If $\theta$ is expressed in seconds of arc, the corresponding value of the scale factor $K$
for the lunar occultation case is approximately 100. In practice, the smallest departure
from a point source that can be detected probably corresponds to values of $K\theta$ in
the range of 0.1 to 0.2, or values of $0'001$ to $0'002$ for the minimum detectable stellar
diameter by the occultation method. For values of $K\theta$ greater than 1, or stellar dia-
meters greater than $0'01$, the diffraction effects largely disappear and the determina-
tion of the angular diameter becomes a geometrical problem, requiring a knowledge
of the effective velocity of the Moon’s limb at the point where the occultation takes
place.

The bandwidth of the optical filters used has the effect of damping out the higher
order maxima and minima of the diffraction pattern. The amplitude of the first
maximum, however, is reduced by only a negligible amount compared to the mono-
chromatic case, considering the accuracy of our measurements.

The diffraction curves for the two close binaries reported on below have been
analyzed by superimposing two point source curves, varying the amplitude and
spacing of one source relative to the other to obtain the best fit to the data. The more
elaborate restoration procedures are more appropriate for studying the occultation
curves of radio sources for which they were originally devised and where the shape
of the source cannot be assumed in advance. The routine application of these pro-
cedures to the analysis of optical occultations can lead to completely erroneous
results if the distortions in the pattern produced by scintillation or limb irregularities
are not recognized as such.

4. Results

A. 46 LEO

The occultation of 46 Leo (Yale Bright Star Catalog No. 4127) was observed on 26
May 1966 using the 36 in. telescope of the Kitt Peak National Observatory. The
occultation curve shown in Figure 2 is an oscilloscope photograph of the tape play-
back using RC filtering with a time constant of 2.5 msec to reduce high frequency
noise. Only the first maximum and minimum are easily distinguishable above the
background, an indication of the finite angular diameter of the source. Measurements
of different oscilloscope traces made with varying values of playback speed and RC
filtering are in the range

$$I_{\text{max}}/I_0 = 1.22 \pm .05.$$  

$I_{\text{max}}$ is the intensity of the first maximum and $I_0$ is the free field value. Scintillation
fluctuations produce an uncertainty in the free field value at the time of the occul-
tation. In this particular case, the large amplitude fluctuations tend to be of long
period (about 0.1 sec) compared to the occultation time scale as can be determined by
examining a longer section of the recording prior to the occultation than is shown
in Figure 2. The estimate of the free field value at the time of the occultation was made by continuing the slight downward slope of the light curve just prior to the occultation. Because of the quasi-periodic behavior of the low frequency scintillation components over short intervals, this procedure should lead to a more accurate estimate of the free field value as compared to just taking the average value over several seconds prior to the occultation.

The measured value of $I_{\text{max}}/I_0$ and the calculations upon which the curve in Figure 1 are based lead to a value of $K\theta = 0.52 \pm 0.11$. The slope of the curve in Figure 1 is such that a given uncertainty in $I_{\text{max}}/I_0$ leads to a larger uncertainty in the value of $K\theta$. In this observation, an EMI 9502S photomultiplier was used with a yellow filter, the effective wavelength of the combination, allowing for the spectrum of the star, being 5050 Å.

The resulting value for the angular diameter is

$$\theta = 0\:'0056 \pm 0.0011.$$  

This value is uncorrected for limb darkening and refers to a uniformly bright disk. We can estimate the effective temperature of the star most simply by making use of the equation (Russell et al., 1938)

$$\log \theta'' = (5900/T) - 0.20 m_v - 3.05. \quad (1)$$

Using the above value of angular diameter and the tabulated apparent magnitude, $m_v = 5.54$, we obtain

$$T = 3100^\circ \pm 150.$$  

The uncertainty results from that in the angular diameter. More modern treatments do not contain any new observational data in the red giant region and do not lead to significantly different values. For example, the temperatures of the red giants in Table V, p. 267 of Harris (1963) are quite close on the whole to those of Table XXIX,
p. 749 of Russell et al. (1938). The calculated value of the temperature corresponds to a spectral classification M2 III or M2 II-III (Keenan, 1963; Harris, 1963) as compared to the broader classification gM2 in the Yale catalog.

B. **φ Aqr**

The occultation of φ Aqr (Bright Star Catalog No. 8834) was observed on 21 October 1969 using the 28 in. reflector of the Flower and Cook Observatory of the University of Pennsylvania. A 1P21 photomultiplier with a V filter was used and the effective wavelength was taken to be 5540 A. φ Aqr is of magnitude 4.22 and spectral class M2 III.

The measured value of $I_{\text{max}}/I_0$ is $1.26 \pm 0.03$, the error being attributed largely to the uncertainty in the free field value at the time of the occultation. Following the procedure outlined for 46 Leo, we obtain $K\theta = 0.44 \pm 0.07$ and

$$\theta = 0.0049 \pm 0.0008$$

uncorrected for limb darkening. The effective temperature, calculated as before is

$$T = 3700^\circ \pm 170$$

and is higher than the value of $3100^\circ$ quoted previously for spectral class M2 III. In this connection, we can refer to the recent occultation determination of the angular diameter of λ Aqr (Nather et al., 1970), also of spectral class M2 III. Application of Equation (1) leads to a temperature of $3500^\circ$ in this case for no limb darkening. The estimates of temperature by Nather et al. range from $3250^\circ$ for the limb darkened case to $3460^\circ$ for the uniform disk. A discrepancy exists in this case too, although smaller in value. Wesselink (1969) has given empirically determined relations from which the angular diameter of a star can be calculated from its apparent magnitude and B–V index. The value of B–V for 46 Leo is not tabulated in the Bright Star Catalog. For the case of φ Aqr, the calculated diameter is 0.0061 and for λ Aqr, 0.0087. Both of these values are higher than the occultation measurements for a uniform disk. For λ Aqr, the value estimated for the fully limb darkened case is 0.0082 ± 0.0004. If the same percentage correction for limb darkening is applied to φ Aqr, the upper limit on the angular diameter is likewise close to the value calculated from Wesselink’s relations.

C. **α Lib**

The occultation of α² Lib (Bright Star Catalog No. 5531) was observed on 31 May 1966 using the 36 in. reflector at the Kitt Peak National Observatory. The occultation curve (Figure 3) clearly shows that we are dealing with a binary star. The separation of the two components in the direction of advance of the Moon’s limb was 0.01. If we take this figure to be the approximate value for the semi-major axis of the system, then using the listed parallax (0.049) and assuming approximate masses for the components, we can calculate an orbital period of the order of 20 days. The ratio of intensities of the components is found to be 0.7, corresponding to $Δm = 0.4$. The spectral class listed in the Bright Star Catalog is $Am$ and $m_v = 2.75$ (it is one of the
standards in the UBV system). Following a suggestion of W. P. Bidelman, we assume that the primary component is of spectral class A3V. We can then estimate its apparent visual magnitude to be 3.25. Using the above value of $\Delta m$ then leads to a combined magnitude of 2.7 which is in good agreement with the listed value. This calculation assumes that $\Delta m$, which was measured in B light, would have the same value in V light.

D. $\varepsilon$ CAP

The occultation of $\varepsilon$ Cap (Bright Star Catalog No. 8260) was observed on 4 September 1968 using the 28 in. reflector of the Flower and Cook Observatory. It is similar in nature to that of $\alpha$ Lib and shows a binary component, fainter relative to the primary than in the previous case. The intensity ratio of the components is 0.3, corresponding to $\Delta m = 1.3$ (in B light). The separation of the components in the direction of advance of the Moon's limb was 0\'0047. No measured parallax is listed. If we assume that the spectral class of the primary is B3V (the listed spectrum is B3V?p and $m_v = 4.62$), we can calculate a value for its distance and by using approximate mass values, estimate that the system has a period of the order of 100 days.

Neither $\alpha$ Lib or $\varepsilon$ Cap appear to be generally recognized as binaries. The radial velocity of each, however, is listed as variable. I am indebted to W. P. Bidelman for pointing out to me a note by Slipher (1904) in which he lists five stars observed by him to have a variable radial velocity and which he concluded were spectroscopic binaries. By an interesting coincidence, two of the stars are $\alpha$ Lib and $\varepsilon$ Cap.

E. $\varepsilon$ ARI

The occultations of both components of the visual binary $\varepsilon$ Ari (Bright Star Catalog No. 887, 888) were observed on 20 December 1969 using the 28 in. reflector of the Flower and Cook Observatory. The ratio of intensities of the two components was determined to be $1.41 \pm 0.07$, corresponding to $\Delta m = 0.37$ (in B light). This value is
comparable to the tabulated value of 0.3. The measurement of the intensity ratio does not make any use of diffraction theory and depends simply on the drop in light level following each occultation. The observed separation in time of the two occultations was \(2.129 \pm 0.002\) sec. The most recent observation in the U.S. Naval Observatory double star file for \(\epsilon\) Ari is for 1966.050: position angle 204.5°, separation 1"48. Assuming this value of the position angle, we can convert the observed time separation into an angular separation. This calculation does not require any value for the slope of the lunar limb if we use Fresnel diffraction theory to establish the scale of the pattern as it sweeps past the telescope. Although the assumed position angle is in agreement with the observations in that the fainter component was occulted first, the calculated angular separation is 2"06, so there is a discrepancy with the visual observations. A lower elevation with reference to the mean lunar limb at the point where the brighter component was occulted as compared to the fainter component of about 0.4 km is required to account for the discrepancy.

F. \(\alpha\) Leo

The occultation of Regulus (Bright Star Catalog No. 3982) was observed on 13 May 1970 with the 28 in. reflector of the Flower and Cook Observatory in B light. The occultation curve is shown in Figure 4. An angular diameter of 1.33 \(\pm 0.07 \times 10^{-3}\) sec of arc has been determined from the interferometer measurements of HanburyBrown et al. (1967). The expected occultation curve should be essentially that of a point source with perhaps a detectable indication of a finite diameter. The ratio of intensities of the first maximum to the free field value is close to that expected for a point

Fig. 4. Occultation curve of \(\alpha\) Leo. 13 May 1970.
source, but the first minimum is too deep while the second maximum is too high. While scintillation effects may have distorted the third peak, the variations in the rest of the curve are smooth and I do not believe that the departure of the curve from the theoretical pattern can be attributed solely to scintillation. The distortions would then have to be attributed to lunar limb irregularities. The effects of these irregularities were investigated by Wijesinghe (1966) and most recently by Evans (1970). An earlier treatment was given by Diercks and Hunger (1952).

5. Conclusions

The interpretation of a particular occultation curve is limited in accuracy by scintillation effects and possible limb irregularities rather than by any inherent theoretical considerations. Multiple observations of the same event from different locations and also simultaneous observations in several wavelength regions can hopefully reduce the limitations present in a single observation.

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References