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STERN WAVES WITH VORTICITY

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Abstract

Steady two-dimensional free surface flow past a semi-infinite flat plate is considered. The vorticity in the flow is assumed to be constant. For large values of the Froude number F, an analytical relation between F, the vorticity parameter ω and the steepness s of the waves in the far field is derived. In addition numerical solutions are calculated by a boundary integral equation method.

1. Introduction

Over the years important progress has been achieved in the computation of two-dimensional nonlinear free-surface flows past surface piercing obstacles. Such flows are relevant to the modeling of a ship moving at a constant velocity on the free surface of a fluid. These flows are often studied by neglecting viscosity and by seeking steady solutions in a frame of reference moving with the obstacle. Interesting particular flows arise from assuming that the object is semi-infinite. They provide a local description of the flow near the stern or the bow of a very long ship. We refer to these flows as stern flows when there is a train of waves on the free surface and as bow flows when the free surface is waveless in the far field.

Vanden-Broeck and Tuck [13], Vanden-Broeck, Schwartz and Tuck [12] and Vanden-Broeck [10] obtained semi-analytical solutions for the stern flow past a semi-infinite two-dimensional flat-bottomed body. They assumed that the flow rises up along the rear face of the body to a stagnation point at which separation occurs. Vanden-Broeck [9] described analytically and numerically another family of stern flows in which the flow separates at the corner of the body. Further studies involving waveless, time dependent and viscous solutions can be found in [3, 5, 8] and [14].

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Dias and Vanden-Broeck [2] computed solutions for the bow flow past a semi-infinite two-dimensional flat-bottomed body. The free surface is waveless in the far field but there is a spray at the bow. The spray is modeled by a layer of water rising along the bow and falling back as a jet. Bow flows with surface tension were considered in [1].

All the above calculations assume that the flow is irrotational. This is usually a very good assumption but vorticity can be generated near solid boundaries or on the free surface (for example by wind stress).

In this paper, we shall generalize the stern flow of Vanden-Broeck [9] for rotational flows. We assume that the vorticity is constant throughout the fluid. This assumption is convenient mathematically and justified when the lengthscale of the free surface variations is short compared to the lengthscale of the vorticity distribution. The fluid is assumed to be of infinite depth. Results in water of finite depth were obtained by McCue and Forbes [6]. Our results should provide a good approximation for flows in finite depth, when the wavelength of the waves generated is small compared to the depth. The problem is formulated in Section 2. In Section 3, we take advantage of the simplicity of the configuration to derive an exact relation between the amplitude of the waves in the far field and the main parameters of the flow. In Section 4, we compute nonlinear solutions by a numerical procedure involving an integro-differential equation coupled with Newton's iterations. The scheme is similar to the ones used in [9] and [11]. The numerical results are discussed in Section 5. We note that the results of this paper provide an example of a flow in which the waves discussed in [7] and [11] occur.

2. Formulation

In this section we formulate the problem of a steady two-dimensional inviscid flow past a semi-infinite flat-bottomed body (see Figure 1). The effect of surface tension is neglected and the flow is assumed to separate smoothly from the flat bottom. The flow is rotational and characterized by a constant vorticity Ω. We introduce Cartesian coordinates with $X = 0$ at the edge of the plate. The level $Y = 0$ corresponds to the "undisturbed level of the free surface", that is, the level the free surface would have at $X = \infty$ if it reached a constant horizontal level. We denote by C the corresponding constant value of the velocity on the free surface. The draft H is defined as minus the ordinate of the edge of the plate.

The flow is described in terms of a stream function $\Psi(X, Y)$ satisfying

$$\nabla^2\Psi = -\Omega \tag{2.1}$$

in the flow domain. We reduce the problem to one for Laplace's equation by subtracting

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a particular solution of (2.1). Thus if we write

$$\Psi = \psi - \frac{\Omega}{2} Y^2 + CY$$

then $\nabla^2 \psi(X, Y) = 0$. We require that $\psi \to 0$ as $Y \to -\infty$. This is consistent with our definition of C.

We make the variables dimensionless by referring them to the velocity scale C and to a length scale $C^2/(2g)$. Thus we define the dimensionless quantities

$$x = \frac{2g}{C^2} X, \qquad y = \frac{2g}{C^2} Y, \qquad \omega = \frac{\Omega C}{2g}.$$

Here ω is the dimensionless vorticity.

The quantity $w(z) = u - iv = \psi_y + i\psi_x$ is an analytic function of $z = x + iy$, where the fluid velocity vector is $(u - \omega y + 1, v)$. We apply Cauchy's integral formula to the function $w(z)$ on a contour consisting of the free surface, the plate (that is, the surface just under the plate), a horizontal line at $y = -\infty$ and two vertical lines at $x = \pm\infty$. Since $w(z)$ vanishes at $y = -\infty$, there are no contributions from the lines at $y = -\infty$ and at $x = \pm\infty$ and we have

$$w(z) = -\frac{1}{\pi i} \int_L \frac{w(\zeta)}{\zeta - z} d\zeta, \tag{2.2}$$

where z is on L. Here L denotes the free surface and the plate. The integral in (2.2) is a Cauchy principal value.

We parameterize the free surface by $x = x_f(t)$, $y = y_f(t)$ and the plate by $x = x_p(t)$, $y = y_p(t) = y(0)$ where t is the arclength. We choose $t = 0$ at the edge of the plate. Then

$$x_f'(t)^2 + y_f'(t)^2 = 1. \tag{2.3}$$

Since $Y(0) = -H$ and $X(0) = 0$, our choice of dimensionless variables implies

$$x(0) = 0, \qquad y(0) = -2/F^2,$$

where $F = C/\sqrt{gH}$ is the Froude number.

On the plate

$$x_p'(t)^2 = 1, \qquad y_p'(t) = 0.$$

We consider u and v on the free surface and on the plate as functions of t. Thus we write $u = u_f(t)$, $v = v_f(t)$ on the free surface and $u = u_p(t)$, $v = 0$ on the plate.

Taking the real part of (2.2), we obtain after some algebra and letting

$$V1 = u_f(s)y_f'(s) - v_f(s)x_f'(s) \quad \text{and} \quad V2 = u_f(s)x_f'(s) + v_f(s)y_f'(s),$$

$$\pi u_f(t) = -\int_0^\infty \frac{(x_f(s) - x_f(t))\,V1 - (y_f(s) - y_f(t))\,V2}{(x_f(s) - x_f(t))^2 + (y_f(s) - y_f(t))^2}\,ds$$
$$+\int_0^\infty \frac{u_p(s)x_p'(s)(y_f(t) - y(0))}{(x_p(s) - x_f(t))^2 + (y_f(t) - y(0))^2}\,ds, \tag{2.4}$$

when z is on the free surface and

$$\pi u_p(t) = -\int_0^\infty \frac{(x_f(s) - x_p(t))\,V1 - (y_f(s) - y(0))\,V2}{(x_f(s) - x_p(t))^2 + (y_f(s) - y(0))^2}\,ds \tag{2.5}$$

when z is on the plate.

On the free surface the kinematic condition and Bernoulli equation yield

$$(u_f(t) - \omega y_f(t) + 1)y_f'(t) = v_f(t)x_f'(t), \tag{2.6}$$

$$(u_f(t) - \omega y_f(t) + 1)^2 + v_f(t)^2 + y_f - 1 = 0. \tag{2.7}$$

Equation (2.7) expresses the fact that the pressure is constant on the free surface.

For given values of ω and F, we seek the functions u_f, v_f, x_f', y_f' and u_p satisfying (2.3), (2.4)–(2.7).

3. Conservation of momentum

In this section, we show how to use the principle of conservation of momentum to derive an exact relation between the Froude number F, the vorticity parameter ω and the steepness s of the waves in the far field. For $\omega = 0$, this relation reduces to the one derived by Vanden-Broeck [9]. For simplicity we assume that s is small so that the waves in the far field are described by linear theory. The validity of this assumption will be justified by the numerical calculations of Section 5.

The principle of conservation of momentum implies that

$$\int_S \left[\mathbf{V}(\mathbf{V}\cdot\mathbf{n}) + gY\mathbf{n} + \frac{P}{\rho}\mathbf{n}\right] ds = 0. \tag{3.1}$$

Here S is any closed simply connected contour inside the fluid region, $\mathbf{V} = (V_X, V_Y)$ is the vector velocity, P is the pressure, ρ is the density and $\mathbf{n}$ is the exterior normal to the contour. We now choose S to consist of the plate S_p, the free surface S_F, a vertical line S_R at $X = +\infty$, a horizontal line S_H at $Y = -\infty$ and a vertical line S_L at $X = -\infty$. Taking the component of (3.1) along the X-axis, we obtain

$$\int_S \left[V_X(\mathbf{V}\cdot\mathbf{n}) + \frac{P}{\rho}n_X + gYn_X\right] ds = 0. \tag{3.2}$$

Here n_X is the component of $\mathbf{n}$ along the X-axis. It is convenient to replace the line S_H by a horizontal line at $Y = -d$, where d is arbitrarily large. The integrals over S_p and S_H in (3.2) do not contribute since n_X=0 and $\mathbf{V}(\mathbf{V} \cdot \mathbf{n}) = 0$. Along S_F, $\mathbf{V}(\mathbf{V} \cdot \mathbf{n}) = 0$ and $P = 0$, so that the integration over S_F gives

$$-\int_{-H}^{0} gY\,dY = \frac{g}{2} H^2.$$

Here we chose S_R such that $y = 0$ at the intersection of S_R and S_F.

As $X \to \infty$, we assume a train of linear waves of amplitude a. Using linear theory (see for example [3]), we write

$$V_X = -\Omega Y + C - aCke^{kY} \cos kX, \tag{3.3}$$

$$V_Y = -aCke^{kY} \sin kX. \tag{3.4}$$

The choice of S_R implies that $\cos kX = 0$ and $\sin kX = 1$ along S_R. Thus

$$V_X = -\Omega Y + C, \tag{3.5}$$

$$V_Y = -aCke^{kY}. \tag{3.6}$$

To perform the integration over S_R, we need an expression for the pressure P. For this purpose we consider the Y-component of the Euler equation

$$V_X \frac{\partial V_Y}{\partial X} + V_Y \frac{\partial V_Y}{\partial Y} = -\frac{1}{\rho} \frac{\partial P}{\partial Y} - g. \tag{3.7}$$

Differentiating (3.4) with respect to X and using $\cos kX = 0$ implies that $\partial V_Y / \partial X = 0$ along S_R. Therefore integrating (3.7) with respect to Y along S_R gives

$$-\frac{P}{\rho} - gY = \frac{1}{2} V_Y^2 + \alpha, \tag{3.8}$$

where α is a constant of integration.

Substituting (3.8) into (3.2), we get after neglecting terms of order $\exp(-2kd)$

$$\int_{-d}^{0} \left(V_X^2 + \frac{P}{\rho} + gY \right) dY = \frac{\Omega^2 d^3}{3} + \Omega d^2 C + dC^2 - \frac{a^2 C^2 k}{4} - \alpha d.$$

As $X \to -\infty$,

$$V_X = -\Omega Y + T \quad \text{and} \quad V_Y = 0. \tag{3.9}$$

Here T is a constant which tends to C as $d \to \infty$. We obtain an equation for the pressure by substituting (3.9) into (3.7) and integrating with respect to Y. This gives

$$-P/\rho - gY = \beta. \tag{3.10}$$

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Here β is a constant of integration.

Substitution of (3.10) into (3.2) at $X = -\infty$ yields

$$-\int_{-d}^{-H}\left(V_X^2+\frac{p}{\rho}+gY\right)dY=\frac{\Omega^2H^3}{3}+\Omega H^2T+HT^2-\frac{\Omega^2d^3}{3} \\ -\Omega d^2T-dT^2+\beta(d-H).$$

Combining the various contributions over S_F, S_R and S_L in (3.2) gives

$$\frac{gH^2}{2}-\frac{a^2C^2k}{4}-H\beta+\frac{\Omega^2H^3}{3}+\Omega d^2C+\Omega H^2T \\ -\Omega d^2T+HT^2+dC^2-dT^2+\beta d-\alpha d=0. \qquad (3.11)$$

We now derive an equation for $\beta-\alpha$. We first note that the principle of conservation of mass implies

$$\int_{-d}^{-H}(-\Omega Y+T)\,dY=\int_{-d}^{0}(-\Omega Y+C)\,dY. \qquad (3.12)$$

Evaluation of the integrals in (3.12) gives

$$dC^2-dT^2=-\frac{\Omega TH^2}{2}-\frac{\Omega CH^2}{2}-THC-HT^2. \qquad (3.13)$$

Next we write Bernoulli's equation at $Y=-d$ (where d is arbitrary large) as

$$\frac{1}{2}(-\Omega Y+T)^2-\beta=\frac{1}{2}(-\Omega Y+C)^2-\alpha. \qquad (3.14)$$

We then combine (3.13) and (3.14) and obtain

$$d(\beta-\alpha)=\Omega d^2T-\Omega d^2C-\frac{1}{2}(dC^2-dT^2) \\ =\Omega d^2T-\Omega d^2C+\frac{\Omega TH^2}{4}+\frac{\Omega CH^2}{4}+\frac{THC}{2}+\frac{T^2H}{2}. \qquad (3.15)$$

Substitution of (3.13) and (3.15) into (3.11) yields

$$\frac{gH^2}{2}-\frac{a^2C^2k}{4}-\beta H+\frac{\Omega^2H^3}{3}+\frac{3}{4}\Omega TH^2-\frac{THC}{2}-\frac{\Omega CH^2}{4}+\frac{T^2H}{2}=0. \qquad (3.16)$$

Since $T\to C$ as $d\to\infty$, we can simplify (3.16) as

$$\frac{gH^2}{2}=\frac{a^2C^2k}{4}-\frac{1}{2}\Omega CH^2-\frac{\Omega^2H^3}{3}+\beta H. \qquad (3.17)$$

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Using Bernoulli's equation on the streamline consisting of the plate and the free surface at $X = \pm\infty$ and the relations (3.5), (3.6), (3.8) and (3.10) yields

$$\frac{1}{2}(-\Omega Y + T)^2 - \beta = \frac{1}{2}(-\Omega Y + C)^2 + \frac{1}{2}a^2C^2k^2 - \alpha. \tag{3.18}$$

Since $T \to C$ as $d \to \infty$, (3.18) implies $\beta = \Omega CH + \frac{1}{2}\Omega^2H^2 + \alpha + O(a^2)$. Furthermore (3.8) on the free surface (where $P = 0$, $Y = 0$ and V_Y satisfies (3.6)) shows that $\alpha = O(a^2)$. Therefore $\beta = \Omega CH + \frac{1}{2}\Omega^2H^2 + O(a^2)$. Since (3.17) implies $H = O(a)$, we can simplify (3.17) as

$$\frac{gH^2}{2} = \frac{a^2C^2k}{4} + \frac{1}{2}\Omega CH^2 + O(a^3). \tag{3.19}$$

Multiplying both sides of (3.19) by $1/(HC^2)$ gives

$$\frac{1}{2F^2} = \frac{a^2k}{4H} + \frac{\omega}{F^2}. \tag{3.20}$$

The dispersion relation of linear waves $C^2 = (g - \Omega C)/k$ (see [4] for example) can be rewritten as

$$F^2 = (1 - 2\omega)/kH. \tag{3.21}$$

We note that this dispersion relation implies that the linear waves considered here only exist for $g < \Omega C$, that is, $\omega < 0.5$.

Substituting (3.21) into (3.20) gives

$$\frac{1}{2F^2} = \left(\frac{a}{H}\right)^2\left(\frac{1-2\omega}{4F^2}\right) + \frac{\omega}{F^2}. \tag{3.22}$$

Thus

$$a/H = \sqrt{2}.$$

We derive the relation between steepness of the wave and Froude number by noting that $2a$ is the peak-to-trough wave height since $Y = a\cos kX$. Therefore the steepness s of the waves (that is, the peak-to-trough wave height divided by the wavelength) is $2a/\lambda$. Using (3.21) and (3.22), we have

$$s = \frac{(2(1-2\omega)^2)^{1/2}}{\pi F^2}. \tag{3.23}$$

If we set $\omega = 0$, (3.23) reduces to the relation

$$s^2 = 2/(\pi^2F^4)$$

derived by Vanden-Broeck [9]. Next we use (3.21) to derive the following equation for the dimensionless wavelength

$$\frac{\lambda}{H} = \frac{2\pi F^2}{1 - 2\omega}. \tag{3.24}$$

Finally we note that the wave height h (that is, the difference of ordinates between a crest and a trough) is given by

$$h/H = s\lambda/H = 2a/H = 2\sqrt{2}. \tag{3.25}$$

4. Numerical schemes

In this section, a numerical scheme based on the integro-differential equation formulation derived in Section 2 is used to solve the problem in the fully nonlinear case. First, we define N mesh points on the free surface and N mesh points on the plate by specifying values of the arclength parameter $t = S_i$ where $S_i = E(i - 1)$. Here E is the interval of discretization. We shall also make use of the intermediate mesh points $S_{i-1/2} = (S_{i-1} + S_i)/2$. We now define $5N - 1$ corresponding fundamental unknown quantities

$$u_{f,i} = u_f(S_i), \quad v_{f,i} = v_f(S_i), \quad x'_{f,i} = x'_f(S_i), \quad y'_{f,i} = y'_f(S_i) \tag{4.1}$$

$(i = 1, 2, \dots, N)$ on the free surface and

$$u_{p,i} = u_p(S_i), \quad i = 1, 2, \dots, N - 1 \tag{4.2}$$

on the plate. We estimate the values of $x_{f,i} = x_f(S_i)$, $y_{f,i} = y_f(S_i)$ in terms of the fundamental unknowns by the trapezoidal rule, that is, $x_{f,1} = 0$, $y_{f,1} = -2/F^2$ and

$$x_{f,i} = x_{f,i-1} + x'_f(S_{i-1/2})E, \quad y_{f,i} = y_{f,i-1} + y'_f(S_{i-1/2})E, \quad i = 2, 3, \dots, N$$

where $x'_f(S_{i-1/2})$ and $y'_f(S_{i-1/2})$ are evaluated from $x'_{f,i}$ and $y'_{f,i}$ by a four-point interpolation formula.

We satisfy (2.3), (2.6) and (2.7) at the mesh points S_i, $i = 1, 2, \dots, N$. This yields $3N$ nonlinear algebraic equations. Next we evaluate $x_f(S_{i-1/2})$, $y_f(S_{i-1/2})$ by four-point interpolation formulas. We then satisfy (2.4) and (2.5) at the points $t = S_{i-1/2}$, $i = 2, 3, \dots, N$ by applying the trapezoidal rule to (2.4) and (2.5) with a sum over the points $s = S_j$, $j = 1, 2, \dots, N$. The symmetry of the discretization and of the trapezoidal rule with respect to the singularity of the integrand at $s = t$ enables us to evaluate the Cauchy principal value integrals by ignoring the singularity, with an

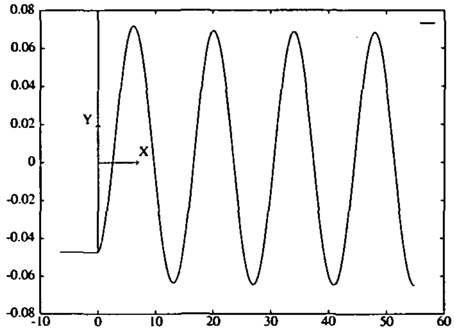

FIGURE 1. Computed free surface profile with $\omega = 0.05$ and $F = 6.5$.

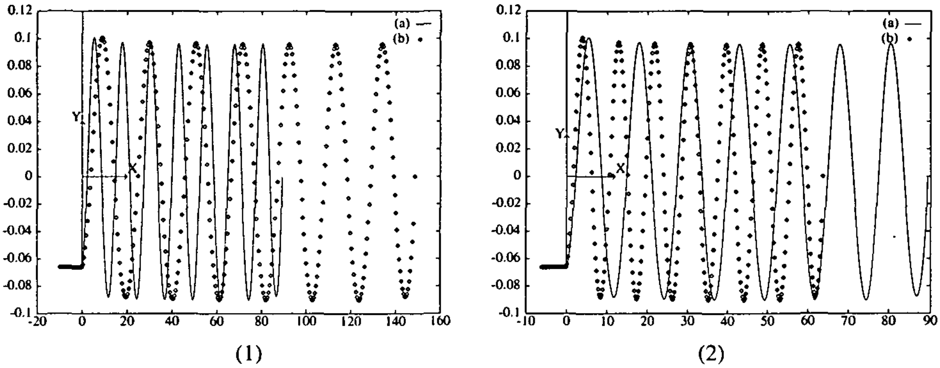

FIGURE 2. (1) Computed free surface profile of wave with $F = 5.5$ and (a): $\omega = 0$, (b): $\omega = 0.2$. (2) Same as (1) with (a): $\omega = 0$, (b): $\omega = -0.2$.

accuracy no less than a non-singular integral. This yields $2N - 2$ extra nonlinear equations. One more equation is obtained by imposing

$$v_{f,1} = 0. \tag{4.3}$$

We now have $5N - 1$ equations for the $5N - 1$ unknowns (4.1)–(4.2). This system is solved by Newton's method for given values of F and ω.

We found that the truncation of the integrals in (2.4) and (2.5) at $s = S_N$ did not affect the accuracy of the results near the plate, provided S_N was sufficiently large. The only noticeable effect of the truncation was a distortion of the free surface profile over the last few computed wavelengths in the far field. This distortion can be moved to larger and larger values of X by increasing S_N. Furthermore we found that this distortion could be minimized by choosing S_N such that $y_{f,N} = 0$. Therefore the results presented in the next section are based on a modified version of the scheme described above in which we impose the extra equation

$$y_{f,N} = 0 \tag{4.4}$$

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TABLE 1. Values of s, h/H and λ/H for various values of ω and $F = 5.5$ with $N = 301$ and $N = 225$.

	ω	s	h/H	λ/H
Analytical solution	−0.2	0.020834	2.828427	135.760152
	−0.1	0.017858	2.828427	158.384310
	0.0	0.014882	2.828427	190.056914
	0.1	0.011905	2.828427	237.574826
	0.2	0.008929	2.828427	316.766434
Numerical solution $N = 301$	−0.2	0.020894	2.820860	135.006413
	−0.1	0.017915	2.821571	157.497852
	0.0	0.014930	2.821375	188.973970
	0.1	0.011939	2.819560	236.169036
	0.2	0.008941	2.814659	314.798733
Numerical solution $N = 225$	−0.2	0.020430	2.813131	137.697517
	−0.1	0.017521	2.814568	160.636548
	0.0	0.014608	2.815461	192.739231
	0.1	0.011686	2.815219	240.873556
	0.2	0.008762	2.813313	321.067336

TABLE 2. Values of s, h/H and λ/H for various values of ω and $F = 2.35$ with $N = 301$ and $N = 225$.

	ω	s	h/H	λ/H
Analytical solution	−0.2	0.114119	2.828427	24.784891
	−0.1	0.097816	2.828427	28.915495
	0.0	0.081513	2.828427	34.698727
	0.1	0.065211	2.828427	43.373408
	0.2	0.048908	2.828427	57.831211
Numerical solution $N = 301$	−0.2	0.137738	3.082258	22.376201
	−0.1	0.117949	3.084950	26.154959
	0.0	0.098071	3.079049	31.396071
	0.1	0.078529	3.078580	39.203178
	0.2	0.058687	3.063495	52.200828
Numerical solution $N = 225$	−0.2	0.134793	3.079198	22.843887
	−0.1	0.119337	3.083014	25.834607
	0.0	0.099302	3.080104	31.017534
	0.1	0.079107	3.064428	38.737748
	0.2	0.057356	3.055341	53.269672

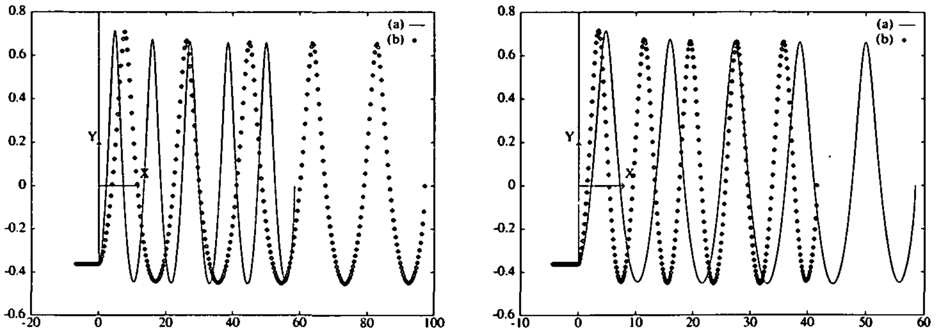

FIGURE 3. (1) Computed free surface profile of wave with $F = 2.35$ and (a): $\omega = 0$, (b): $\omega = 0.2$. (2) Same as (1) with (a): $\omega = 0$, (b): $\omega = -0.2$.

and add E to the list of unknowns. We then have a system of $5N$ nonlinear equations with $5N$ unknowns to be solved by Newton's method for given values of N, F and ω. We note that in this version of the scheme, N is fixed and E is found as part of the solution to satisfy (4.4). There are of course an infinite number of points in the far field for which $y = 0$, each two successive points being separated by half a wavelength of the train of waves. The choice of the particular point at which (4.4) is satisfied depends on the initial guess and defines the point at which the free surface is truncated. A convenient choice for the initial guess for the modified scheme is a converged solution of the first scheme in which (4.4) is approximately satisfied.

5. Discussion of the results

The free surface profile contains a train of waves behind the plate. The highest point of the profile corresponds to the crest nearest the plate and the steepness of the waves decreases away from the plate and reaches a constant value after a few cycles even if the vorticity is not zero. As the vorticity increases, the wavelength lengthens. Most of the computations were performed with $N = 301$.

In Tables 1 and 2, we compare the numerical values of s, h/H and λ/H with the analytical approximations (3.23), (3.25) and (3.24) for $F = 5.5$ and $F = 2.35$. Results for different values of N are listed to illustrate the accuracy of the results. All the calculations were done with the scheme satisfying (4.4). The results show that the agreement between analytical and numerical values improves as F increases. This is consistent with the fact that the amplitude of the waves decreases as F increases. Therefore the waves in the far field are described by the linear theory as $F \to \infty$.

Typical computed nonlinear profiles are shown in Figures 1–3. We note that the waves in Figure 3 are nonlinear gravity waves with sharp crests and broad troughs.

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Finally let us comment on formula (3.23). It implies that for a fixed value of the Froude number F, positive vorticity decreases the steepness of the waves in the far field. This suggests that the generation of vorticity (for example by the boundary layers) might reduce the wave resistance.

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