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CHAIN DECOMPOSITIONS OF GRAPHS

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A *chain* is essentially a continuous route in a graph which does not repeat any edge. A chain can be either finite or infinite, and can have 0, 1 or 2 end vertices. A *chain decomposition* of a graph is a set of chains which partition the edge set of the graph. This thesis is primarily concerned with chain decompositions of countably infinite graphs. We consider both abstract graphs and graphs embedded in surfaces.

The introductory chapters are largely the result of joint work of Eggleton and Skilton [*1*]. We begin by laying a foundation of definitions and results for the study of chain decompositions. Next we consider small chain decompositions of graphs. Small decompositions of finite graphs have been fully treated by two fundamental theorems of graph theory, due to Euler [*4*] and Listing [*5*]. We introduce three kinds of small chain decomposition of infinite graphs, each of which generalizes certain features of the small decompositions of finite graphs. Some results are derived about these new decompositions. Next we use the framework provided by these new decompositions to briefly survey existing chain decomposition results, and formulate what we consider to be the main unsolved problems in this area.

One class of chain decomposition results is primarily concerned with *the number of chains in a decomposition*. In particular, we discuss the characterization of these infinite graphs which are decomposable into just

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a single chain, or which are decomposable into finitely many chains. These characterizations are due to Erdös, Grünwald, and Vázsonyi [3]. We also treat the problem of finding the smallest number of chains into which a given graph can be decomposed. This problem was posed by Ore [8], and solved by Rothschild [9], using work of Nash-Williams [7]. We simplify the existing solution. Next we deal with a problem concerned with the numbers of each kind of chain in the small decompositions of a given graph.

In another class of chain decomposition results, interest focuses on the kinds of chain present. Proofs of the few results existing in this area are outlined. These results are due to Nash-Williams [6]. Attention is then drawn to the relevant outstanding problems, one of which is solved. Small decompositions into a restricted class of chains are also of interest, and these are discussed.

For a graph embedded in a surface, it makes sense to speak of a chain crossing, that is *transecting*, itself or some other chain at a vertex. We discuss transection-free chain decompositions of surface embeddings of graphs, both finite and infinite. A number of results for transection-free decompositions are established; most are analogues of chain decomposition results for abstract graphs, but some involve complications or limitations imposed by the embedding. This section is essentially joint work of Eggleton and Skilton [2].

The thesis concludes with some remarks about future research, followed by three of the author's relevant papers, as Appendices.

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