# Optical and Infrared Long-Baseline Interferometry: Application to Binary Star Science

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**Abstract.** Interferometric observations of binary stars have a profound impact on many areas of stellar astrophysics. This article gives a brief review of interferometric techniques applied to binaries, and of orbit determination and binary surveys with optical and infrared interferometers.

#### 1. Introduction

Observations of binary stars have been among the most important goals of optical interferometry ever since the first interferometric determination of a "visual" orbit of a double-lined spectroscopic binary (SB2), namely Capella, by Anderson (1920) and Merrill (1922). Their observations with the 20-foot interferometer mounted on the 100-inch Hooker Telescope on Mt. Wilson clearly showed the potential of this method for measuring stellar masses and distances. The technological advances at the end of the twentieth century were necessary, however, to turn optical and infrared interferometry into productive research tools. The new ground-based and space-borne instruments that will come into operation during the first decade of the new millennium will bring about new capabilities that can revolutionize many aspects of binary star research. This article gives a brief introduction to the principles of interferometry applied to binaries, a short summary of the observations that have been carried out during the past few years, and a preview of some developments expected for the near future.

#### 2. Observations with an Interferometer

Two types of observations are important in the context of binaries: interferometric *imaging* and interferometric *astrometry*. The primary observable of an imaging interferometer is the complex visibility  $\Gamma = Ve^{i\phi}$ , which is related to the sky brightness distribution  $S(\xi, \eta)$  through a Fourier transform (the van Cittert-Zernike Theorem, see e.g. Shao & Colavita 1992, Thompson et al. 1986):

$$\Gamma(u,v) = \int \int S(\xi,\eta)e^{-2\pi i(u\xi+v\eta)}d\xi \,d\eta \quad , \tag{1}$$

where  $\xi$  and  $\eta$  are the coordinates in the tangent plane of the sky, and u and v the two components of the baseline vector in the Fourier plane, measured in units of the observing wavelength. The visibility function of a star can be

approximated by

$$\Gamma(x) = \frac{2J_1(x)}{x} , \quad x \equiv \frac{\pi B \theta_{\rm UD}}{\lambda} , \qquad (2)$$

where  $J_1$  denotes the Bessel function of first order, B is the baseline length,  $\lambda$  the observing wavelength, and  $\theta_{\rm UD}$  is the uniform disk equivalent diameter of the star (for a discussion of limb darkening see Quirrenbach et al. 1996). The squared visibility of a binary is given by

$$V^{2} = \frac{1}{(1+R)^{2}} \left[ \Gamma^{2}(x_{1}) + R^{2}\Gamma^{2}(x_{2}) + 2R\Gamma(x_{1})\Gamma(x_{2}) \cos\left(\frac{2\pi\rho B \cos\psi}{\lambda}\right) \right] ; \quad (3)$$

here  $R \leq 1$  is the brightness ratio of the two stars,  $\rho$  their separation, and  $\psi$  the angle in the uv plane between the interferometer baseline and the line joining the two stars. To determine  $\rho$ ,  $\psi$ , and the stellar parameters uniquely from measurements of  $V^2$ , data have to be obtained for many points in the uv plane. This can be accomplished by observations with multiple baselines, by Earth-rotation synthesis, by taking data at multiple wavelengths (since u and v scale with the wavenumber), or by a combination of these techniques. Repeated observations with good coverage of the orbital phases can then be used to determine a "visual" orbit (Armstrong et al. 1992a). A better method was developed by Hummel et al. (1993) and applied in most subsequent analyses. Here the seven orbital elements and the stellar parameters are fitted directly to the observed visibilities. This "global" approach has the advantage that fast orbital motion of short-period binaries is taken into account properly.

An astrometric interferometer measures the position of stars with respect to a reference frame, which can be defined either locally (narrow-angle astrometry) or over the whole sky (wide-angle astrometry). If the star is an unresolved (i.e.,  $\rho \ll B/\lambda$ ) binary system, the observed quantity is the position of the photocenter. Again, observations covering a full revolution can be used to determine the orbital elements. Wide-angle astrometry, which has to be done from space (for the precision relevant in this context), gives the parallax in addition to the photocenter motion. If the binary is resolved, the positions of both components can be determined separately, but the data analysis can be quite complicated.

## 3. Combination of Interferometry with other Methods

The goal of binary star observations is normally the determination of fundamental parameters such as the masses, radii, and luminosities of the two components. It is usually necessary to combine two or more techniques to achieve this goal, since not all orbital elements can be determined with any single method alone (with the notable exception of wide-angle astrometry of resolved binaries, see entry "Vis + Ast +  $\pi$ " in Table 1). A particularly useful case are double-lined spectroscopic binaries for which the visual orbit can also be obtained. However, there is little overlap between the two classes, since spectroscopic binaries tend to

<sup>&</sup>lt;sup>1</sup>Single-baseline Interferometers normally measure  $V^2$ , not  $\Gamma$ , see Shao & Colavita 1992.

Table 1. Information that can be obtained from observations of binary stars. The abbreviations are as follows. SB1: spectroscopic binary with only the primary spectrum. SB2: spectroscopic binary with both spectra. Ecl: eclipsing binary. Vis: visual orbit. Ast: astrometric orbit of the photocenter.  $\pi$ : Parallax. M:  $m_1$  is assumed to be known,  $m_1 \gg m_2$ , and  $L_1/L_2 \gg m_1/m_2$ . a: orbital semi-major axis in linear units. a'': semi-major axis in angular units. LD: limb darkening. RV: radial velocity.

Input Data	Information	Example
SB1	$a_1 \sin i$ , $m_2^3 \sin^3 i/(m_1 + m_2)^2$	
SB1 + M	$a\sin i,\ m_2\sin i$	RV planet
$\mathrm{SB}2$	$a\sin i,\ m_1\sin^3 i,\ m_2\sin^3 i$	
$\operatorname{Ecl}$	$R_1/a, R_2/a, i, LD$	
Ecl + M	$R_1,R_2,a,i,\mathrm{LD}$	photometric planet
$\operatorname{Vis}$	$a^{\prime\prime},i,L_1/L_2$	${\it interferometer}$
Vis + M	$a,i,L_1,L_2$	TPF/DARWIN planet
$Vis + \pi$	$a,i,m_{1}+m_{2},L_{1},L_{2}$	
$\operatorname{Ast}$	$i, rac{m_1 L_2 - m_2 L_1}{(m_1 + m_2)(L_1 + L_2)} a''$	
$Ast + M + \pi$	$a, i, m_2$	$\operatorname{SIM}$ planet
Vis + Ast	$a'',i,m_1/m_2,L_1/L_2$	VLTI / Keck binary
$Vis + Ast + \pi$	$a,i,m_1,m_2,L_1,L_2$	SIM binary
SB1 + Vis	$a_1,i,m_2^3/(m_1+m_2)^2$	RV + interferometer
$SB1 + Vis + \pi$	$a,i,m_1,m_2,L_1,L_2$	
SB1 + Ast	$a_1,i,m_2^3/(m_1+m_2)^2$	SIM black-hole binary
SB2 + Vis	$a,i,m_1,m_2,\pi,L_1,L_2$	RV + interferometer
SB2 + Ast	$a,i,m_1,m_2$	SIM X-ray pulsar
Ecl + SB1 + M	$a,i,m_2,R_1,R_2$	$\mathrm{HD}209458$
Ecl + SB2	$a,i,m_1,m_2,R_1,R_2$	
$Ecl + SB2 + \pi$	$a,i,m_1,m_2,R_{1},R_{2},L_{1},L_{2}$	
$Ecl + Vis + \pi$	$a,i,m_1+m_2,R_1,R_2,L_1,L_2$	
Ecl + SB2 + Vis	$a,i,m_1,m_2,R_1,R_2,\pi,L_1,L_2$	

have small orbits that are difficult to resolve. Even orbits of SB2s obtained with adaptive optics or speckle techniques are rarely precise enough to give masses better than  $\sim 10\%$  (Pourbaix 2000). This is one of the reasons why interferometric observations of SB2s are of great importance for the determination of fundamental stellar parameters.

Table 1 gives a compilation of the information that can be obtained in principle from the most important combinations of observational methods. The period P and eccentricity e can be determined with any technique. (For eclipsing binaries the determination of e is possible only if the secondary eclipse can also be observed, and the separate determination of e and  $\omega$  is difficult.) The table lists other parameters that can be obtained from first principles. The only exception are the cases labeled "M", which are applicable mostly to observations of planets around main-sequence stars, when it can be assumed that the mass of the parent star is known a priori with sufficient precision.

System	Types	a'' [mas]	$M_1 [M_{\odot}]$	$M_2~[M_{\odot}]$	Instr.	Ref.
α Vir	B1III-IV+B3V				Narrabri	H71
heta Aql	B9.5III +B9.5	5III 3.2	$3.6 \pm 0.8$	$2.9 \pm 0.6$	MkIII	H95
$oldsymbol{eta}$ Aur	A2V + A2V	7 3.3	$2.41 \pm 0.03$	$2.32 \pm 0.03$	MkIII	H95
12 Boo	F9IV +F9I	V 3.4	$1.435 \pm 0.023$	$1.408 \pm 0.020$	PTI	B00
64 Psc	F8V + F8V	6.5	$1.223 \pm 0.021$	$1.170\pm0.018$	PTI	B99b
93 Leo	G5III +A7V	7.5	$2.25 \pm 0.29$	$1.97 \pm 0.15$	MkIII	H95
$\zeta^1$ UMa	A2V + A2V	9.6	$2.51 \pm 0.08$	$2.55 \pm 0.07$	MkIII	H95
-		9.8	$2.43 \pm 0.07$	$2.50 \pm 0.07$	NPOI	H98
$\iota$ Peg	F5V +G8V	7 10.3	$1.326 \pm 0.016$	$0.819 \pm 0.009$	PTI	B99a
$\eta$ And	G8III +G8I	II 10.4	$2.59 \pm 0.30$	$2.34 \pm 0.22$	MkIII	H93
α Equ	G2III +A5V	7 12.0	$2.13 \pm 0.29$	$1.86 \pm 0.21$	MkIII	A92b
ζ Aur	K4Ib +B5V	7 16.2	$5.8 \pm 0.2$	$4.8 \pm 0.2$	MkIII	B96
$\theta^2$ Tau	A7III $+A$ :	18.6	$2.1 \pm 0.3$	$1.6 \pm 0.2$	MkIII	T95
$\phi$ Cyg	K0III +K0I	II 23.7	$2.536 \pm 0.086$	$2.437 \pm 0.082$	MkIII	A92a
$\alpha$ And	B8IV $+A$ :	25.2	5.5:	2.3:	MkIII	P92,T95
$\beta$ Ari	A5V +G0\	7: <b>36.1</b>	$2.34 \pm 0.10$	$1.34 \pm 0.07$	MkIII	P90
α Aur	G1III +G8I	II 55.7	$2.56 \pm 0.04$	$2.69 \pm 0.06$	MkIII	H94a

Table 2. Interferometric determinations of orbits and component masses for double-lined spectroscopic binaries.

References: A92a: Armstrong et al. 1992a; A92b: Armstrong et al. 1992b; B96: Bennett et al. 1996; B99a: Boden et al. 1999a; B99b: Boden et al. 1999b; B00: Boden et al. 2000; H71: Herbison-Evans et al. 1971; H93: Hummel et al. 1993; H94: Hummel et al. 1994; H95: Hummel et al. 1995; H98: Hummel et al. 1998; P90: Pan et al. 1990; P92: Pan et al. 1992; T95: Tomkin et al. 1995

It is clear that the precision with which the parameters listed in Table 1 can be determined varies widely between different classes of objects and different observational techniques. In particular, orbital parallaxes from interferometric observations of SB2s offer a significant improvement over currently available parallaxes from ground-based astrometry or HIPPARCOS. A good example is Spica, for which Herbison-Evans et al. (1971) derived a distance of  $(84\pm4)$  pc at a time when trigonometric parallax estimates ranged from 34 pc to 111 pc. The HIPPARCOS measurement of  $(80\pm6)$  pc has now confirmed the orbital parallax. On the other hand, while the visibility function of binaries depends on the component radii according to Equation 3, interferometrically determined radii are in most cases not precise enough for critical tests of stellar models. Therefore the radii are not listed as observables in the "Vis" entry in Table 1.

### 4. Stellar Masses and Tests of Evolutionary Models

As pointed out above, adding the inclination from the interferometric orbit to the spectroscopic elements allows computation of the component masses, and combining the angular diameter of the orbit with the physical scale set by the spectroscopy yields the distance, or "orbital parallax". Because of the fundamental importance of these data, extensive observations of SB2s have been carried out with the Mk III Interferometer, the Navy Prototype Optical Interferometer (NPOI), and the Palomar Testbed Interferometer (PTI). They are summarized in Table 2 (adapted from Quirrenbach 2001). The orbital solutions and error estimates are taken from the references cited, and are therefore not uniform. The error bars refer formally to  $1\,\sigma$ , but some authors may be more conservative than others in assessing systematics in the data or in dealing with discrepancies between different subsets of the data (e.g., different eccentricities from the spectroscopic and interferometric orbits). It should also be pointed

out that determining the scale of the orbit (in angular units), and the subsequent computation of the orbital parallax, requires knowledge of the effective central wavelength of the interferometric observations, which depends on the stellar color (Hummel et al. 1994). Systematic errors in this quantity may easily go unnoticed since they do not affect the  $\chi^2$  of the orbit fit. In many cases, however, the precision of the mass determination is limited by the spectroscopic, not by the interferometric orbit.

It is instructive to compare Table 2 to the masses of eclipsing binaries compiled by Andersen (1991). Only a handful of the interferometrically determined masses meet Andersen's accuracy criterion for being useful for critical tests of main-sequence stellar models, which he set at 2%. Furthermore, the baselines used in the observations compiled in the table are too short to give good stellar radii (with the exception of Capella). On the other hand, the agreement for the component masses of  $\beta$  Aur, the only system in common between the two samples, is encouraging. Furthermore, analyses of pairs with evolved components such as Capella,  $\phi$  Cyg, and  $\alpha$  Equ provide useful tests of post-main-sequence evolutionary models (e.g. Armstrong et al. 1992b). The availability of orbital parallaxes giving good luminosities is a clear advantage in this respect. Further improvements can be expected from the next generation of ground-based interferometers (NPOI, CHARA array, VLT Interferometer, Keck Interferometer), which will provide long baselines ( $\gtrsim 100 \,\mathrm{m}$ ) in the visible and increased sensitivity compared to the Mk III. This will make many more SB2s amenable to precise interferometric orbit determination. Revisiting binaries with visible orbits from speckle or adaptive optics observations can improve the derived masses considerably (e.g., G1570 BC, see Ségransan et al. 1999). The key to noticeable progress will be observations of stars with well-determined spectroscopic elements and state-of-the-art determination of the metal abundance. Comprehensive tests of stellar models require covering all regions of the HR diagram. Many of the eclipsing systems in Andersen (1991) are also accessible to the new instruments, which could provide improved distances and better luminosity ratios for partially eclipsing systems. The good instantaneous coverage of the uv plane afforded by the multiple baselines and wavelength channels of the new arrays will allow determination of orbits from snapshot observations, making them very efficient instruments for binary programs.

## 5. Low-Mass and Sub-Stellar Companions

Observations of low-mass companions with imaging interferometers are difficult because a large magnitude difference between the primary and secondary requires excellent calibration of the visibility. The calibration uncertainty induced by atmospheric turbulence can be mitigated by spatial filtering of the wavefront with single-mode fibers (see Quirrenbach 2001 and references therein). Optimistic extrapolations of the current state of the art imply that it might be feasible to detect companions as faint as "hot Jupiters" (i.e., about  $10^{-3}$  to  $10^{-4}$  fainter than the parent star in the near-IR) with this method (Coudé du Foresto 2000). It may be easier, however, to use the phase rather than the amplitude of the visibility for the detection of faint companions. This may sound surprising in view of the phase fluctuations introduced by atmospheric turbulence, but

these variations are largely achromatic and can thus be eliminated by differential measurements between two wavelengths. For example, the photocenter of a star + planet system shifts with wavelength across molecular absorption bands in the planetary atmosphere. The visibility phase can be used as a proxy for the brightness ratio; measurements with  $\sim 0.1$  milliradian accuracy could therefore be used to perform infrared spectroscopy of "hot Jupiters" (Akeson & Swain 1999, Quirrenbach 2000a).

The steep dependence of luminosity on mass suggests looking for dynamic effects of low-mass companions rather than for the photons emitted by them. This is the basis for radial-velocity and astrometric searches for extrasolar planets (see Marcy & Butler 1998 and Quirrenbach 2000b for recent reviews). Astrometry is complementary to the successful radial velocity technique in many respects. Interferometric observations of stars with planets known from the radial velocity surveys can yield the planet's mass m without the  $\sin i$  ambiguity. This is important in the context of binary stars because it clarifies the incidence of objects with masses above the deuterium burning limit ( $\sim 0.013\,M_\odot$ ), which appear to be rare as companions of  $\sim 1\,M_\odot$  stars. This "brown dwarf desert" is clearly important for our understanding of planet formation as well as binary star formation. An astrometric survey with the VLTI or Keck Interferometer could also perform a census of brown dwarfs in orbit around M dwarfs (Quirrenbach 2000a).

For stars with multiple planets, astrometric measurements can determine whether their orbits are co-planar, an important clue to the dynamical history of the system. The different detection biases of the radial velocity method (signal  $\propto a^{-1/2}$ ) and astrometry (signal  $\propto a$ ) imply sensitivity to different architectures of the planetary system. The radial velocity surveys favor systems with masses increasing with orbital semi-major axis such as v And (Butler et al. 1999), whereas astrometric searches should preferentially detect systems in which the masses decrease with orbital radius. Interferometric astrometry will also enable a census of planets around stars of all spectral types, including pre-main-sequence objects, an important step towards the understanding of planetary system formation.

## 6. Pre-Main Sequence Binaries

The VLTI and Keck Interferometer will provide sufficient sensitivity and angular resolution for studies of binaries in star-forming regions. They will complement the current surveys for pre-main-sequence binaries by providing access to smaller orbital separations and thus giving a more complete picture about the binary frequency (e.g., Richichi & Leinert 2000). A significant first step in this direction has already been done with the Infrared Optical Telescope Array (IOTA); a new binary (MWC 361-A) was found in a sample of 15 Herbig Ae/Be stars observed with this instrument (Millan-Gabet et al. 2001).

Interferometry at mid-infrared wavelengths will allow the determination of the binary frequency at an even earlier evolutionary stage, thus shedding completely new light on the process of binary star formation.  $10 \,\mu\mathrm{m}$  surveys of embedded sources in Orion with the VLTI or the Keck Interferometer will detect binaries down to separations of  $\sim 10 \,\mathrm{AU}$ , comparable to the limit of near-IR

surveys in nearby star-forming regions with speckle methods or adaptive optics. Mid-infrared interferometry will further help to clarify the relation between binaries and circumstellar disks.

Determining masses of pre-main-sequence stars is important for constraining evolutionary models of these objects. Unfortunately most known double-lined pre-main-sequence binaries have orbital radii ( $\lesssim 0.1\,\mathrm{AU}$ ) too small to be resolvable with current interferometers, although that situation may change with new spectroscopic surveys sensitive to periods of up to several hundred days (Guenther et al. 2001, these proceedings). A major improvement is expected from the astrometric capabilities of the VLTI and Keck Interferometer, which will enable measurements of the inclination and thus the component masses of many pre-main-sequence SB2s with orbits as small as  $\sim 0.01\,\mathrm{AU}$  at the distance of the Taurus-Auriga region. The only caveat is that the astrometric signal is proportional to  $m_1L_2-m_2L_1$  (see entry "Ast" in Table 1), which makes this technique insensitive to pairs of nearly equal stars.

#### 7. Conclusions

Optical and infrared interferometry can make important contributions to a large variety of important questions about binary stars. In conjunction with other techniques, interferometry can supply the data necessary to determine stellar masses with a precision sufficient to test evolutionary models in many regions of the HR diagram. In the next few years, interferometry will help establish a precise mass-luminosity relation for the whole main sequence, and enable critical tests of pre-main-sequence evolution. By pushing binary surveys to smaller separations and younger ages, infrared interferometry will enhance our understanding of the processes that govern binary star formation. Interferometric astrometry will enable dynamical studies of planetary systems and the detection of binaries with extremely large luminosity ratios. Many other applications of interferometry to binary stars can be foreseen, from imaging of interacting systems to measuring the masses of black holes and neutron stars in X-ray binaries. The unique capabilities of optical and infrared interferometry for observations with milliarcsecond resolution and microarcsecond precision will have a profound impact on virtually all aspects of binary star research, and thus on many areas at the forefront of stellar astrophysics.

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