Abstract. I describe three key moments in the rotational evolution of three single stars, chosen to illustrate the richness and complexity of the interaction between rotation and magnetic fields. I then piece together two general scenarii for the rotational evolution of early-type and late-type stars, in which the "fossil" magnetic field inherited by a star from its formative years basically sets its subsequent rotational fate. Since no one has yet offered a convincing scenario according to which a star could somehow get rid of its primordial fossil field, the inescapable conclusion is that magnetic fields cannot be neglected in modelling rotation, and that "rotational evolution" should really be thought of as "magnetorotational evolution".

1. Introduction

Rotation and magnetism are the evil twins of stellar structure and evolution. Like so many twins, they have evolved their own language, and that language is magnetohydrodynamics (MHD). MHD finds its quantitative expression in the so-called induction equation:

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) , \tag{1} \]

describing the inductive action of a flow \( \mathbf{U} \) on a magnetic field \( \mathbf{B} \) in the presence of resistive dissipation, \( \eta \) being the magnetic diffusivity. Although linear in \( \mathbf{B} \), Eq. (1) couples to the momentum equation for \( \mathbf{U} \) which also inherits an additional term on its RHS, nonlinear in \( \mathbf{B} \) and corresponding to the Lorentz force exerted by the magnetic field on the embedding plasma.

A truly unmagnetized star is as physically unimaginable as a truly non-rotating star\(^1\). Magnetic fields and angular momentum abound in the interstellar clouds from which stars form, and indeed stellar formation can only proceed if a magnetic field is present to extract excess angular momentum from protostellar cores (see, e.g., Mouschovias 2001). Measurement of remnant magnetism in meteorites offer one reference point: when the protosolar nebula reached a radius of \( \sim 20 \) AU, it was pervaded by a \( \sim 100 \) mG magnetic field (Levy 1988, and

\[ \text{I say so fully aware of the existence of extremely slowly rotating white dwarfs, which I'm happy to consecrate as the exception confirming the rule.} \]
Even if only as little as 1% of this field survives the final contraction phase to the ZAMS, internal fields of $\sim 1$ G are expected; such fields are dynamically significant for rotational evolution, and their Ohmic decay time exceeds the age of the Universe. Like it or not, stars are magnetized.

Magnetic fields interact with rotation in complex ways. They can stabilize otherwise unstable hydrodynamical flows; they can destabilize otherwise stable flows. They can couple to magnetized outflows or accretion disk to extract angular momentum (AM) from stars; and they can efficiently suppress internal differential rotation, on timescales much shorter than any relevant evolutionary or hydrodynamical timescales —convection excepted— even for relatively weak fields, e.g. of order $\sim 1$ G. While magnetic fields contribute insignificantly to total pressure in the bulk of stellar interiors, they can still influence structural evolution, by channelling and/or regulating mass loss and/or accretion, especially in the pre- and post-main sequence evolutionary phases. Finally, magnetic field amplification by dynamo action in convective regions is very likely under typical stellar interior conditions, adding a host of possible feedback mechanisms and timescales. Magnetic fields are present in stars, and their story is forever coupled to the story of rotation, from day one and ever after.

Since I cannot hope to cover comprehensively these parallel stories in this short contribution, I chose instead to focus on a few key moments in the magnetorotational lives of three single stars (§2). The chosen moments are “key” in two senses. First, they are crucial for the subsequent rotational evolution of these stars; second, they appropriately illustrate the richness and complexity of the interaction between magnetic fields and rotation. In §3 I piece together these key moments to draw two general magnetorotational scenarii applicable respectively to early-type and late-type stars.

This paper is not meant as a comprehensive review of the recent relevant literature, but rather as a deliberately polemical contribution to these proceedings, touching on a number of topics where neglecting magnetic fields amounts to an ostrich strategy bound to lead to conclusions that are at best incomplete, and at worst plain wrong.

2. Key Moments in the Lives of Three Rotating Stars

2.1. The Solar Spin-Down

Through mechanisms not yet properly understood but almost certainly involving magnetic fields, rotating magnetized stars like the sun manage to sustain million-degree coronae, which in turn power wind-like outflows. The stresses building up in rotating, magnetized winds are extremely efficient at extracting angular momentum from the outer stellar layers, resulting in rotational deceleration, or spin-down, on timescales much shorter than main-sequence (MS) lifetimes. Here modeling seems to stand on pretty firm grounds: measurements of angular momentum loss rates in the solar wind match well wind model calculations (Pizzo et al. 1983); observations of rotational velocities of solar-type stars in young clusters have amply demonstrated the reality —and swiftness— of main-sequence spin-down (see Stauffer, this volume); models of angular momentum loss and evolution offer a reasonable match to data (see Charbonneau, Schrijver & MacGregor 1997, and references therein), with the notable excep-
tion of slow ZAMS rotators (more on these in §2.2 below). One important aspect of AM loss in MS solar-type stars—as modeled, say, by the Weber-Davis formalism (Belcher & MacGregor 1976) is the fact that AM loss is a rapidly increasing function of surface rotation rate; fast rotators spin down faster, so that the surface rotation of middle-age stars like the sun has lost all memory of the initial conditions, and typically cannot be used to discriminate between competing rotational evolutionary scenarii in the PMS or early-MS phases (see, e.g., MacGregor & Brenner 1991).

Arguably the single most constraining recent piece of observational evidence for rotational evolution modelling has been the demonstration by helioseismology that the solar radiative core rotates very nearly rigidly, at a rate comparable to that of the solar surface (Tomczyk, Schou & Thompson 1995; Charbonneau et al. 1998; Christensen-Dalsgaard, this volume). This finding stands in stark contrast to the predictions of purely hydrodynamical models of internal angular momentum redistribution, which invariably predict a rapidly rotating core in the present-day sun (e.g., Chaboyer, Pinsonneault & Demarque 1995)—as well as significant beryllium depletion, contrary to the most recent observational determinations (see Balachandran, this volume).

It has long been realized that magnetic fields offer a sure means of enforcing rigid rotation in radiative interiors of stars (see, e.g., Mestel & Weiss 1987, and references therein). Numerical studies of the solar spin-down in the presence of internal magnetic fields (e.g., Charbonneau & MacGregor 1993) have revealed a number of remarkable properties, not the least of which being the near complete insensitivity of internal AM redistribution to the assumed strength of the internal poloidal field. As solar-type stars arrive on the MS, a radial shear develops at the core-envelope interface in response to the torque \( (\frac{dJ}{dt}) \) exerted by the wind on the outer convective envelope. For a time, the core and envelope remain rotationally decoupled, with the envelope spinning down and the core retaining its higher ZAMS rotation. However, a toroidal magnetic component \( B_\phi \) builds up inexorably as the shear acts on the pre-existing poloidal component, leading to the appearance of a Lorentz force tending to oppose the shear. This continues until sufficient magnetic stresses have built up in the outer portions of the core to balance the wind-mediated torque:

\[
  r^3 \langle B_r B_\phi \rangle \simeq \frac{dJ}{dt},
\]

where the angular bracket represents a surface integral over a spherical surface of radius \( r \). At this point, core-envelope rotational recoupling takes place. Starting with a weaker poloidal field (i.e., \( B_r \) in Eq. (2)) simply requires a larger induced \( B_\phi \)—and proportionally more time—for this recoupling to occur (see Charbonneau & MacGregor 1993, Fig. 7, panels A and C).

In the case of the solar spin-down with a \( \sim 1 \text{ G} \) internal initially purely poloidal field, \( B_\phi \) reaches a strength of a few \( 10^4 \text{ G} \) relatively early in MS evolution; for a \( \sim 0.01 \text{ G} \) poloidal field, a \( \sim 10^6 \text{ G} \) peak \( B_\phi \) results. One starts to wonder about the stability of such strong internal magnetic fields (cf. Pitts & Tayler, 1985; Balbus & Hawley 1994; Spruit, this volume). If stability is lost before Eq. (2) can be satisfied, then core-envelope recoupling may never occur, and the star could be left with a rapidly rotating core throughout MS evolution.
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Figure 1. Schematic representation of a PMS star magnetically “locked” to a surrounding Keplerian accretion disk. Magnetic coupling is assumed to take place between the inner disk radius \( r_i \) and some outer radius \( r_o \). The corotation radius \( r_c \) is the locus in the disk where the local Keplerian angular velocity \( \Omega_K(r) \) is equal to the star’s angular velocity \( \Omega_* \). As the star spins-up due to contraction, magnetic stresses can enforce \( \Omega_* \approx \Omega_K(r_c) \) provided AM can be evacuated outwards from the \( r_c \leq r \leq r_o \) disk region fast enough, either by magneto/viscous torques within the disk, or perhaps by winds emanating from the disk.

The stability of the induced \( B_\phi \) then leads to a bimodal sorting mechanism based on the strength \( B_p \) of the primordial internal poloidal field. If \( B_p \) is below some threshold \( B_{\text{crit}}(1) \), \( B_\phi \) becomes unstable before Eq. (2) can be satisfied, and core-envelope recoupling fails. If on the other hand \( B_p > B_{\text{crit}}(1) \), \( B_\phi \) remains stable and core-envelope recoupling occurs. The latter situation must have prevailed in the sun.

2.2. Star-Disk Locking in the T Tauri Phase

The same magnetic equilibration of applied torques by magnetic stresses may well characterize a totally different astrophysical system, namely a pre-main sequence (PMS) star magnetically coupled to a surrounding accretion disk (see Figure 1). The idea that a contracting star could transfer AM to a disk, and in so doing “lock” its rotation rate to that of the outer portions of the said disk, has been invoked as a means of explaining both the observed rotation rate distributions of T Tauri stars, as well as the large number of slowly rotating stars observed in young clusters (on observations see Bouvier et al. 1993; Rebull 2001; Mathieu, this volume; Stauffer, this volume; on models Collier Cameron & Campbell 1993; Keppens et al. 1995; Hartmann 2002; Barnes & MacGregor 2002). In particular, Barnes & MacGregor (2002) have recently presented a set of calculations exploring the nature of this dynamical balance. One noteworthy aspect of their solution is the large toroidal field building up in the “magneto-spheric” region threaded by the initially poloidal magnetic field (strength \( B_p \))
coupling the star to the disk. As in the solar interior case discussed above, this field builds up in order to satisfy something like Eq. (2), where the RHS is now the torque generated by the contracting star spinning up in order to conserve AM. I am ready to postulate that there exists a critical lower-bound poloidal field strength, $B_{\text{crit}}(2)$, below which the induced magnetospheric toroidal component disrupts the magnetosphere, and disk-locking fails. This is another bimodal sorting mechanism that segregates PMS stars, crossing the birthline with a range of rotation rates and magnetic field strengths, into two groups: a slowly-rotating disk-locked group, for $B_p > B_{\text{crit}}(2)$, and a freely spinning group (with or without disks) for $B_p < B_{\text{crit}}(2)$.

2.3. Flux Expulsion in MS Early-Type Stars

I tried really hard to come up with a mechanism through which a star could somehow get rid of whatever magnetic field it may have inherited from its formative years. The following is the best I could come up with. For reasons outlined below, I doubt that it can succeed in completely eliminating the magnetic field, but it might reduce its amplitude significantly.

Magnetic flux expulsion is a well-known MHD process (see, e.g., Weiss 1966). The basic idea is that in an electrically conducting fluid, a magnetic field threading a closed, steady flow cell gets expelled from the cell interior; unless the field is somehow maintained at the boundaries, this greatly accelerates the decay of the field, as compared to the slower decay due to good old Ohmic resistive dissipation. The mechanism is a robust one, in the sense that it does not depend on details of the assumed cellular flow, and operates efficiently whenever the magnetic Reynolds number $Rm = U\ell/\eta$ exceeds $\sim 10^2$ ($U$ and $\ell$ being characteristic values for the flow speed and cell size).

Consider then (Fig. 2, panel A) an axisymmetric configuration where an initially purely poloidal magnetic field $B_p$ threads the radiative envelope of a rotating early-type stars in which a so-called Eddington-Sweet thermally driven meridionally circulation $U_p$ is also present (see Tassoul 1978, chap. 8). What flux expulsion really does is to accelerate the decay of the field component perpendicular to the flow. It cannot do a thing to the component parallel to the flow, as even a quick glance at Eq. (1) will quickly reveal. This means that flux expulsion can at best reduce the poloidal field strength by a fraction $f$ given by something like

$$f = \frac{\int_V \frac{U_p \cdot B_p}{|U_p||B_p|} \, dV}{V}$$

on the circulation turnover timescale, with the remaining field decaying on the much slower Ohmic dissipative timescale.

And it’s not even that simple. The magnetic field will resist the flow-induced distortion imposed by the flow in the early phases of flux expulsion (cf. Fig 2, panels B and C). For a given rotation rate, there will exist a critical poloidal field strength $B_{\text{crit}}(3)$ above which magnetic flux expulsion will fail altogether. This leads to yet another bimodal sorting mechanism. If $B_p < B_{\text{crit}}(3)$ and $f$ is close to zero, the radiative envelope may evolve in a nearly field-free state, perhaps then free to develop internal differential rotation. If on the other hand $B_p > B_{\text{crit}}(3)$, flux expulsion fails and rigid rotation inexorably will prevail.
Figure 2. Flux expulsion in the radiative envelope of an early-type star. This illustrative 2D calculation is purely kinematic, with a given, steady axisymmetric circulatory flow (streamlines in gray) acting on an initially dipolar magnetic field (fieldlines in black). Results are plotted in a meridional quadrant, with the polar axis coinciding with the left boundary. The flow is clockwise, the magnetic Reynolds number is $Rm = 1000$, and a freely-decaying boundary condition is used at the core-boundary (i.e., no core dynamo action). Note how the field tends to become streamline-aligned in later evolutionary phases (panel D). A proper calculation of flux expulsion in this context should also include (a) the differential rotation caused by AM advection by the circulation, (b) the response of the circulatory flow to the growing differential rotation, (c) the induction of a toroidal component by the differential rotation, and (d) the magnetic backreaction on both the circulation and differential rotation. This may seem like a tall order, but in the laminar regime this problem is currently tractable.
3. Two Magnetorotational Scenarii

A thousand plausible speculations are not worth a single good, solid calculation. But then, having started to speculate, I might as well go all the way and offer two grand magnetorotational scenarii that piece together the various “key moments” discussed above for late-type (Fig. 3) and early-type (Fig. 4) stars. Time runs downwards, and the acronyms along the LHS refer to the usual evolutionary phases of Pre-Main Sequence, Main-Sequence, Giant Branch, and Horizontal Branch. The diagrams are fairly self-explanatory, so I will only here call the reader’s attention to a few noteworthy features. First, in both scenarii the magnetorotational evolution of stars is essentially set by the strength of the “fossil” magnetic fields they inherit from their formative years. Which of the possible flowchart paths ends up being followed is then a matter of whether the fossil field strength exceeds or not the various $B_{\text{crit}(n)}$'s introduced in §2. Second, in the case of early-type MS stars (Fig. 4), the presence of slow surface rotation and/or strong magnetic fields allows an identification of classes of chemically peculiar stars with some of the possible magnetorotational evolutionary paths. Third, Fig. 3 offers, in principle, the means of producing rapidly rotating HB stars from solar-type progenitors (the “No+No” path), something that is hard to accomplish in the absence of internal DR (e.g., Sills & Pinsonneault 2000).

Amazingly enough, qualitative predictions can actually be extracted from Figs. 3 and 4. Here are a few (1) In the T Tauri phase, fast rotators should be found with or without disks; stars with disks should exhibit a range of rotation rates; but slowly rotating stars without disks and still on the convective tracks should not exist; (2) Because of interference by their strong, non-oscillating internal fossil field, slowly rotating early-MS solar-type stars should exhibit more asymmetric activity cycles than their rapidly rotating cousins; (3) At a given mass, rapidly rotating horizontal branch stars should have weaker magnetic fields than their slowly rotating cousins; (4) Chemically “normal” early-type stars — if there is really such a thing— should have stronger magnetic fields than their chemically peculiar “non-magnetic” cousins (AmFm and HgMn stars) — but not as large as the officially magnetic Ap and Bp stars.

There is one major complicating factor that may muddle things up: dynamo action. If this takes place during the PMS phase, “memory” of the primordial field may well be lost by the MS, although disk locking may still have regulated rotation earlier on. If it occurs in the core of early-type MS stars, then any possible correlation between magnetic field and rotation on the HB may be lost.

4. Conclusion

We have collectively only begun to scratch the surface of stellar magnetorotational evolution. There are major hurdles lurking ahead: dynamo action is arguably the tallest one, as it is not well understood even in the well-constrained case of the sun. A proper understanding of the stability of magnetic fields under stellar interior conditions is still lacking, and I don’t quite see it as forthcoming. Nevertheless progress has been made, and there are areas where important issues can be clarified with extant computational methods and hardware. One such tractable problem is the calculation of angular momentum loss rates in...
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Figure 3. A magnetorotational scenario for a late-type star, i.e., a star with a convective envelope and radiative core on the MS. Time runs downwards, and protostars are assumed to reach the PMS phase with a range of rotational rates and internal magnetic field strengths. "DR" stands for internal differential rotation (core-to-envelope, i.e., primarily radial), as opposed e.g. to the latitudinal differential rotation characterizing the solar convective envelope.
Figure 4. A magnetorotational scenario for an early-type star, i.e., a star without an outer convective envelope on the MS but possibly with a convective core. Recall that a “No” on “Flux expulsion” can happen either for slow rotation or moderate internal fields, while a “Yes” can mean either fast rotation or very weak internal fields. Possible links with classes of common chemically peculiar MS stars are indicated in parentheses.
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rotating magnetized winds including realistic, topologically complex field geometry. Computing the efficiency of magnetic flux expulsion in radiative envelopes of early-type stars is a tractable problem in laminar MHD. The degree of magnetic coupling—or lack thereof—between the contracting core and expanding envelope of a star ascending the giant branch is also a tractable problem.

This is by far the most speculative piece I have ever written, and although I have no intention of letting this become a habit I do wish to add a few final comments regarding the magnetorotational scenarios outlined in §3. They rely on one key assumption: there exists an upper limit to the induced magnetic field strength that can be sustained in a magnetized stellar system subjected to torques. Although physically plausible, this remains an ad hoc hypothesis. This being said, it does not strike me as particularly more ad hoc than other physical assumptions on which other popular rotational evolution models are built—extremely efficient horizontal AM transport by anisotropic hydrodynamical turbulence springs immediately to mind! One thing should be clear: in the rotational evolution business, if you ignore magnetic fields, you will be sorry...

References

Pitts, & Tayler, R.J. 1985, MNRAS 216, 139