Miss C. M. HAMILL

The death, at the early age of 32, of Christine Hamill has saddened many, not least those who met her at the Society's colloquia at St Andrews in 1951 and 1955.

Miss Hamill graduated at Cambridge in 1945, a Wrangler in the Mathematical Tripos. Twelve months later she began research under Dr J. A. Todd and was a Research Fellow of Newnham College by 1950, in which year she submitted her thesis *The finite primitive collineation groups which contain homologies of period two*. She took the doctorate in 1951. Her first teaching appointment was a lectureship at Sheffield, her second a lectureship at University College, Ibadan, Nigeria. During the long vacation of 1955 she was home in Britain and attended, in addition to the St Andrews colloquium in July, the British Mathematical Colloquium at Exeter in September. Six months later she had a fatal attack of poliomyelitis and died at Ibadan after two days' illness on 24th March, 1956—the day Sir Edmund Whittaker died at Edinburgh. She was to have been married in July.

Miss Hamill's published work culminated in two papers:

A. On a finite group of order 6531840:
Proc. London Math. Soc. (2) 52, 1951;

B. A collineation group of order $2^{13}\cdot 3^5\cdot 5^2\cdot 7$:
Proc. London Math. Soc. (3) 3, 1953;

and it is pertinent, before summarising their contents and appraising their achievement, to sketch their pedigree.

In 1890 H. Burkhardt, whose researches were based on lectures by Felix Klein, published the second part of a long memoir on hyperelliptic modular functions, and in it examined the group of trisection of the periods. This group, of order 25920, had been found 20 years before by Camille Jordan; but Burkhardt constructed 5 theta-functions that were linearly transformed by it and so was able to display it as a group G of linear substitutions on 5 variables. He thereupon worked out the complete set of invariants of G, finding the one, J say, of lowest order to be a quartic. When the variables

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which undergo substitution are taken as homogeneous coordinates in [4], $J = 0$ is the equation of a quartic primal. This primal was encountered by Coble, in 1906, who saw that it has 45 nodes, and gave some details of their configuration, Now H. F. Baker, on a visit to Göttingen to study under Klein, had there met Burkhardt who gave him offprints of his papers; these Baker studied and copiously annotated and when, nearly 50 years on, retirement from his Cambridge chair had brought comparative leisure he set out to describe, without any dependence on theta-functions, the geometry of the 45-nodal primal, calling it Burkhardt's primal. The outcome was the Cambridge tract: *A locus with* 25920 *linear self-transformations* published in 1946. These linear self-transformations, or projectivities, include 45 projections (the homologies of period 2 of Miss Hamill's thesis) whose vertices are the nodes of the Burkhardt primal. Todd, who had found in 1936 a representation of this primal on [3], read the proof-sheets of the tract and noticed that the geometry afforded a means of partitioning the 25920 projectivities into 15 classes such that operations conjugate in G necessarily belonged to the same class. All operations in a class have the same period, and equivalent sets of invariant subspaces, but closer scrutiny may be called for to see whether all operations in the class are conjugate or whether the class is a union of different conjugate sets in G. Todd's results tally in every detail with the separation of G into its 20 conjugate sets accomplished by J. S. Frame in 1936 by different methods. (Frame had represented G as a group of unitary 4-rowed matrices over a Galois Field of 4 marks.) Todd shows incidentally that every operation of G is the product of 5 or fewer projections.

It was at this juncture that Miss Hamill became Dr Todd's pupil, and he set her to analyse certain larger groups wherein the occurrence of projections had been known since 1914. Fortunate she may well have been, but she soon showed in no uncertain fashion that she could exploit the opportunity that fortune gave her. Before the end of June 1948 she had finished in fair copy the paper **A** in which, in the space of 50 pages, the separation into 34 conjugate sets of a group of order 6531840 is set out in full detail, This group includes 126 projections whose vertices lie in a [5]; each has a [4] "opposite to" its vertex and each such [4] contains 45 vertices forming therein the configuration of nodes of a Burkhardt primal. Miss Hamill first describes the whole figure, the various spaces which occur and their mutual relations, and then considers products of projections. The group is partitioned according to the least number of

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projections of which its operations are products and the type of space spanned by the vertices of these factors: for instance, an operation may be the product of 4 projections whose vertices span a solid, and there are 7 types of solid in the figure. All operations expressible as products of 5 or fewer projections can be handled thus; they fall into 28 classes and account for 31 conjugate sets. Some operations however demand 6 factors; these provide 3 more conjugate sets. The whole paper impresses by the steady forward march of the discussion, and by the exercise of insight and imagination that would do credit to any geometer. At the close the subgroup, of index 2, constituted by those operations that are products of even numbers of projections, is mentioned and its separation into 20 conjugate sets deduced.

This mass of new information is all Miss Hamill's own discovery and none of it has ever been published elsewhere. But it so happens that this subgroup, of order 3265920, is isomorphic to one of what Dickson called the hyperorthogonal groups; it is listed in the left hand column on the last page of the text of his *Linear Groups* (Leipzig, 1901). This offers an alternative line of attack, and although the separation into conjugate sets so obtained is unpublished I am given to understand that it is in complete accord with Miss Hamill's. The larger group, which provides the title of **A**, can be got by imposing a certain outer automorphism of period 2 on the smaller one, and Miss Hamill's results for it thus corroborated.

The paper **B** treats of groups G^5, G^6, G^7 of projectivities in spaces of dimensions shown by the superscripts; G^5 includes 36 projections and is a subgroup of G^6 which includes 63 projections and is, in its turn, a subgroup of G^7, the group of the title and including 120 projections. Miss Hamill separates G^7 into 67 conjugate sets and deduces the separations of G^6 and G^5 and then, from that of G^5, the separation of the group of order 25920 which is a subgroup of index 2 in G^5.

Dr Todd suggested these groups as subject-matter for the thesis because they include projections. But they are known in other contexts; not only are they groups of symmetries of regular polytopes in Euclidean space, but they are also the groups of automorphisms of

the 120 tritangent planes of a canonical curve of genus 4,

the 28 bitangent lines of a non-singular plane quartic (i.e. a canonical curve of genus 3),

the 27 lines of a cubic surface.

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Miss Hamill was well aware of this before submitting her paper early in February 1952; but she did not then know that G^5 and G^6 had, in their representations as groups of symmetries of regular polytopes, just been investigated by Frame who published their separations into conjugate sets in Italy in 1951. Indeed, although it had not apparently come to the notice either of Frame or of Miss Hamill, G^6 had been separated into conjugate sets as long ago as 1904 by Dickson, who used its representation as a symplectic group of 6-rowed matrices over the Galois Field of 2 marks. But these anticipations in no way detract from Miss Hamill's spectacular success. Her results for G^7 were not anticipated and those for G^5 and G^6, which were, are mere corollaries once the geometry in [7] has been set out in detail. Table III on pages 76 and 77 of **B**, displaying as it does a huge mass of information that is all her own original contribution, is perhaps the summit of her achievement. It is also her legacy, and two benefactions that may flow from it are already discernible. A mention of them should end this notice.

Those operations of G^7 that are products of an even number of projections form a subgroup whose order, half that of G^7, is 174182400. This number appears, on the page of Dickson's text-book already cited, as the order of a group there symbolised as $FH(8, 2)$ which, being interpreted, denotes a subgroup of index 2 of the group of non-singular linear transformations, of 8 variables over the Galois Field of 2 marks, for which

$$x_1y_1 + x_2y_2 + x_3y_3 + x_4y_4$$

is invariant. There are reasons to suspect this group to be isomorphic to that which provides the title of **B**, so that it can also be investigated as a group of projectivities in the finite space and of the 8-rowed matrices that impose them. This is the first of the benefactions; the second may be the reopening of an old question. The geometry of the 27 lines of a cubic surface is classical, so is that of the 28 bitangents of a quartic curve. The possession of this knowledge has sometimes lured geometers to probe the configuration of 120 tritangent planes of a canonical curve of genus 4; they would be the first to confess their feeling of frustration and disappointment. Miss Hamill's table has now disclosed many numbers that must have some interpretation in the configuration; will their publication impel us at last to discover more about this elusive curve?

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