Proof.-Draw $\mathrm{F}^{\prime} \mathrm{M}^{\prime}$ perpendicular to OB , draw through $\mathbf{F}$ a parallel CP to $\mathrm{C}^{\prime} \mathrm{P}^{\prime}$, and join OP', producing it to cut CF in $\mathbf{P}$.

Then LQ:QS::OL :FM : : OK : GH:: C'O : $\mathrm{C}^{\prime} \mathrm{G}$ : : $\mathbf{C}^{\prime} \mathrm{O}: \mathbf{C}^{\prime} \mathrm{P}^{\prime}$
$\therefore$ the $\triangle$ 's LQS and $O C^{\prime} P^{\prime}$ have an angle common and the sides about that angle proportional.
$\therefore \quad \angle \mathrm{SLQ}=\angle \mathrm{P}^{\prime} \mathrm{OC}^{\prime}$
$\therefore \quad$ LF is parallel to OP.
Hence
OL : FP : : CL : CF
$:: Q L: Q S$, since the $\triangle$ 's QSL, CFL
are similar
: : OL : FM
$\therefore \quad F P=F M$.
But $\quad \mathrm{F}^{\prime} \mathrm{P}^{\prime}: \mathrm{FP}:: \mathrm{OF}^{\prime}$ : OF
:: $\mathrm{F}^{\prime} \mathrm{M}^{\prime}: \mathrm{FM}$
But $\quad \mathbf{F P}=\mathbf{F M}$
$\therefore \quad \mathbf{F}^{\prime} \mathbf{P}^{\prime}=\mathbf{F}^{\prime} \mathbf{M}^{\prime}$
and $\mathrm{C}^{\prime}$ is the centre of circle AGB
$\therefore$ a circle with centre $\mathrm{F}^{\prime}$ and radius $\mathrm{F}^{\prime} \mathrm{M}^{\prime}$ will touch both OE and OB and will touch the circle AGB at $\mathrm{P}^{\prime}$.

The Algebraic Solution of the Cubic and Quartic in $x$ by means of the Substitution

$$
\frac{\lambda x_{1}+\mu}{1+x}
$$

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