Now from the nature of the eccentric circle, a_1b_1, d_1e_1 meet in S, that is, the point g_1 is S.

Similarly „ „ h_2 is S

and „ „ k_3 is S.

Hence the three straight lines, $g_1h_1k_1$, $g_2h_2k_2$, $g_3h_3k_3$ have one point S common and they are parallel, because the figures are similar and similarly situated;

∴ the three lines are coincident.

Now taking the three triangles $O_1a_1g_1$, $O_2a_2g_2$, $O_3a_3g_3$ which are similar, we have
$$\frac{O_1g_1}{O_1a_1} = \frac{O_2g_2}{O_2a_2} = \frac{O_3g_3}{O_3a_3}$$
and if O_1m_1, O_2m_2, O_3m_3 are the perpendiculars from O_1, O_2, O_3 to the directrix, we have
$$\frac{O_1a_1}{O_1m_1} = \frac{O_2a_2}{O_2m_2} = \frac{O_3a_3}{O_3m_3} = e\,;$$
it therefore follows that
$$\frac{O_1g_1}{O_1m_1} = \frac{O_2g_2}{O_2m_2} = \frac{O_3g_3}{O_3m_3}\,;$$
∴ O_1, O_2, O_3 are in a straight line, and it passes through the point in which the Pascal line of the cyclic hexagons meets the directrix.

On Newton's Theorem in the Calculus of Variations.

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