line is a Voigt profile which is symmetric. If the quasi-
static approximation is assumed to be valid for ions over
the whole profile, the profile derived is asymmetric. Ion
broadening is negligible near line centre so the profiles
only differ in the line wings as can be seen in Figure 2.
However the equivalent widths of the two profiles differ
by only a few per cent so a Voigt fit may be used. This
shows that ion broadening has little effect on the equivalent
widths of the isolated helium lines.

Therefore, in order to carry out a curve of growth
analysis on the weak isolated lines of neutral helium, the
effect of electron scattering must be taken into account
in calculating $S_x$. The model atmosphere may be assumed
not to change with the variation of the helium abundance
and the effect of ion broadening is so small that a Voigt
profile may be used with an error of only a few per cent.

Theories have been developed for the treatment of the
strong overlapping lines of neutral helium by workers
elsewhere and line profiles and curves of growth will be
computed for these lines in the near future. The effects
of non-L.T.E. on the weak isolated lines of neutral helium,
the effect of electron scattering must be taken into account
for two-term recurrence relations, we have an expression
for the mass distribution $q(x)$:

$$q(x) = \left[ a^3 \left\{ a(d + e + f - g) + \mathcal{F} \right\} \right] \left\{ (4a - d) \right.$$

$$\left. - (4(a + c) + e) x^b + (4c - f) x^{2b} \right\}$$

$$+ \frac{1}{2} \left\{ (d + e + f) \left\{ a(4a - d) + \frac{a(x + c + e) x^b}{2} \right. \right.$$

$$\left. + a(4c - f) x^{2b} + f x^{3b} + g x^{4b} \right\}\right]$$

(4)

Now, our condition for a three-term symmetric recurrence
relation to follow from the substitution of (1) into (3) is
that $(2 + z)$ and $(2 + y)$ must belong to the set $(0, b, 2b)$, for
some $b$. Clearly this involves nine possibilities

$$2 + z = 2b \quad 2 + y = 0,$$

As it stands equation (4) for $q(x)$ involves the parameter $\mathcal{F}$
which, through its relationship with $\sigma_z^3$, is dependent
on the particular mode of oscillation. Thus, for possible
density distributions to result, we must cancel the two
brackets involving $\mathcal{F}$ in (4), by a particular choice of coeffi­
cients in each case.

This condition is satisfied when, for instance,

$$2 + z = 2b, \quad 2 + y = 0, \quad d = 4a,$n and $4(a + c) + e = g = 0$,
then

$$q(x) = \frac{x^{2b} \left[ g + (f - 4c) x^b \right]}{(g + f - 4c)}$$

and the pressure

$$p = K(1 - x^b) \left[ \frac{e + g}{9(a - c)} \right] (a - c x^b) \left[ \frac{4c^2 + 4b + ec}{9(a - c)} \right]$$

follows from integrating

$$\frac{x}{p} \frac{dp}{dx} = -\mu,$$

where $K$ is a constant.

The density of the model is given by

$$\rho(x) = \frac{\mu p R_x}{GM q(x)},$$

and in this case

$$\rho(x) = -\frac{R K}{GM} x^{2b-3} (g + f - 4c) (1 - x^b)^6 (a - cx^b),$$

where

$$\theta = [e + f + 4a + b(a - c)]/bc(a - c)$$

and

$$\phi = [4c^2 + 4f + ec - bc(a - c)]/bc(a - c).$$

Moreover, $q(x)$ and $\rho(x)$ above are related by

$$\frac{dq(x)}{dx} = 4x R^3 \rho(x),$$

and the resulting identity in $x$, which will allow us to deter­
mine the possible values of the coefficients in equation (3) is

$$g(3 - b) + 3(f - 4c) x^b = -\frac{4\pi R^3 K}{GM^2} x^{2b-3} (g + f - 4c) (1 - x^b)^6 (a - cx^b).$$

There are now four possibilities that arise, in particular,
considering the case $f = 4c$ when $b = 2/3$ and taking

\[ \text{References:}  
\]
we find that the coefficients in (3) can be expressed as follows in terms of the parameter $a$:

\[
\begin{align*}
  d &= 4a \\
  e &= -\frac{14}{3}a \\
  g &= \frac{2}{3}a.
\end{align*}
\]

The expressions for the density $\rho$, and the pressure $p$, become

\[
\rho = \frac{2RKx^{-2/3}}{\lambda a} \quad p = aK(1 - x^{2/3}),
\]

which satisfies the boundary condition $p = 0$ at $x = 1$.

From a systematic investigation of the other possible cases we found two other models. Our results for the three models are summarized below, where we have assumed a solution of the type

\[
\eta = \sum_{n=0}^{\infty} a_n x^{n+rac{v}{3}}
\]

to yield a symmetric three-term recurrence relation:

(i) Density $\rho = \rho_0/x^{2/3}$

\[
\begin{align*}
  &v = 0, \quad a_0 = a_3 = a_5 = \ldots = 0, \\
  &a_2 = \frac{3\alpha}{11} a_0,
\end{align*}
\]

\[
(2n + 4) (2n + 13) a_{2n+4} - \{2n + 2 + 6(2n + 13) + 9fa_{2n} = 0,
\]

\[
n = 0, 1, 2, \ldots.
\]

(ii) Density $\rho = \rho_0 (1/x^{2/3} - 1)$

\[
\begin{align*}
  &v = 0, \quad a_0 = a_3 = a_5 = \ldots = 0, \\
  &a_2 = \frac{81\alpha}{110} a_0,
\end{align*}
\]

\[
10(2n + 4) (2n + 13) a_{2n+4} - \{2n + 2 + 6(2n + 13) + 162a \} a_{2n+2} + \{14(2n + 15) + 9f \} a_{2n} = 0,
\]

\[
n = 0, 1, 2, \ldots.
\]

(iii) Density $\rho = \rho_0/x^{4/3}$

\[
\begin{align*}
  &v = \frac{1}{2} (-7 + \sqrt{49 + 24\alpha}), \quad a_0 = a_3 = a_5 = \ldots = 0, \\
  &a_2 = \frac{v(v + 9) a_0}{\{(v + 2)(v + 9) - 6\alpha\}}
\]

\[
n = 0, 1, 2, \ldots.
\]

\[
(2n + v + 4) (2n + v + 11) - 6(2n + v + 2) (2n + v + 1) a_{2n+4} + 9f a_{2n+2} = 0
\]

\[
n = 0, 1, 2, \ldots.
\]

Our original purpose in trying to find three-term symmetric recurrence relations was to make use of Prasad’s method of solution. However, during the course of our investigation a paper by Alexander appeared which gave a method for solving the type of three-term recurrence relations we were dealing with. Using this method we derived the eigenvalues for the first five modes of oscillation as listed in Table I. The last line of this table gives the values obtained by Prasad using his continued fraction method.

Resulting from this investigation we have established analytical solutions of Eddington’s equation for density distributions ‘intermediate’ to those of the homogeneous and inverse-square models considered by Sterne.

### Table I

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\rho_0/x^{2/3}$</th>
<th>$\rho_0 (1-x^{2/3} - 1)$</th>
<th>$\rho_0/x^{4/3}$</th>
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<tr>
<td>1</td>
<td>0.4112</td>
<td>14.3603</td>
<td>0.6055</td>
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<td>2</td>
<td>4.7091</td>
<td>37.7056</td>
<td>4.6410</td>
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<td>3</td>
<td>11.2778</td>
<td>74.6608</td>
<td>10.9849</td>
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<td>20.1072</td>
<td>134.601</td>
<td>19.5772</td>
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<td>5</td>
<td>31.1710</td>
<td>236.579</td>
<td>30.3959</td>
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<table>
<thead>
<tr>
<th>Model</th>
<th>$\rho_0 (1-x)$</th>
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</thead>
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<tr>
<td>Prasad</td>
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<tr>
<td></td>
<td>0.9837</td>
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<td></td>
<td>2.2199</td>
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<td></td>
<td>3.8430</td>
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<td></td>
<td>5.8516</td>
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</table>

Companions to RR Lyrae Variables

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A powerful approach to the understanding of any group of stars is the study of any companion stars which can be shown to be physically connected. But RR Lyrae variables, in common with other members of Population II are deficient in physical pairs. Baize’s catalogue of variables with companions contains only one: CSV 1795 = V690 Sco. As far as can be determined no period or light curve has been published for this star and it remains doubtful whether it is an RR Lyrae variable.

Photo-electric photometry exists for 7 companions to RR Lyrae variables. They are listed in Table I where

### Table I

<table>
<thead>
<tr>
<th>Variable</th>
<th>$b^\circ$</th>
<th>$\rho$</th>
<th>$m_{pe}(comp.)$</th>
<th>$n_{12}$</th>
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<tbody>
<tr>
<td>DN Aqr</td>
<td>$-69^\circ$</td>
<td>12$^\circ$</td>
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<tr>
<td>SW Cru</td>
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<td>14.5</td>
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<td>6</td>
<td>14.9</td>
<td>3.2</td>
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<tr>
<td>UZ Scl</td>
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