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Artinian rings, finite principal ideal rings and algebraic error-correcting codes

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DESCRIPTION OF THE THESIS

This thesis contains structure theorems for several special types of Artinian rings, namely the finite rings, commutative Artinian rings with identity, finite commutative rings with identity and semisimple Artinian semigroup-graded rings. Chapter 1 provides an introduction to Artinian rings, semigroup-graded rings and algebraic coding theory. All the theory in Chapters 2 to 5 is new results, some of which have appeared in [**1**], [**2**] and [**3**].

Chapter 2 contains theorems about the generators and weights of certain types of polynomial codes. A class of ideals in polynomial rings is considered which includes all generalized Reed-Muller codes. Necessary and sufficient conditions are obtained for such an ideal to have a single generator. A description is also given of all quotient rings $(\mathbb{Z}/m\mathbb{Z})[x_1, \dots, x_n]/I$ which are commutative principal ideal rings where I is generated by univariate polynomials. Formulas are provided for the minimum Hamming weight of the radical and its powers in the algebra $F[x_1, \dots, x_n]/\big(x_1^{a_1}(1 - x_1^{b_1}), \dots, x_n^{a_n}(1 - x_n^{b_n})\big)$ for an arbitrary field F. Most of Chapter 2 appears in [**1**].

Chapter 3 contains theorems about tensor products and quotient rings of finite commutative rings with identity. Principal ideal rings play a central role in the theory of these finite rings, see [**5**]. For finite commutative rings R and S, necessary conditions are obtained for the tensor product $R \otimes_{\mathbf{Z}} S$ to be a principal ideal ring. These conditions are shown to be sufficient when R and S are principal ideal rings. Conditions are given for the ring $R[x]/\big(f(x)\big)$ to be a principal ideal ring when R is a principal ideal ring and $f(x)$ is a monic polynomial. For a polynomial ring $Q = R[x_1, \dots, x_n]$, and an ideal $I \subset Q$ generated by univariate polynomials, conditions are obtained for Q/I to be a principal ideal ring when R is a principal ideal ring and Q/I is finite. Conditions are also provided

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for Q/I to be a direct sum of finite fields or Galois rings. Part of Chapter 3 appears in [3].

Chapter 4 contains theorems about radicals of finite rings and principal ideal rings. Radicals are basic structural tools of ring theory, see [4]. For a class $\mathcal{R}$ of finite rings, necessary and sufficient conditions are given for $\mathcal{R}$ to be a radical class and also a semisimple class. Hereditary radical classes are characterized. Conditions are obtained when several such classes consist of principal ideal rings.

Chapter 5 contains structure theorems for Artinian semigroup-graded rings. In [6], Zel'manov proved that if a nonzero semigroup ring FS is right Artinian, then the semigroup S is finite. Chapter 5 has several necessary and sufficient conditions for various S-graded rings $R = \bigoplus_{s \in S} R_s$ to be semisimple Artinian under certain finiteness conditions on $\operatorname{supp}(R) \subset S$. Various necessary and sufficient conditions are given for R to be semisimple Artinian when S is a semilattice, an inverse semigroup or a Clifford semigroup. All semigroup varieties, $\mathcal{V}$, are described such that the semigroup algebra FS is semisimple Artinian, for every finite semigroup $S \in \mathcal{V}$, where F is an arbitrary field. Most of Chapter 5 appears in [2].

Soft-bounded copies of this thesis are available from the author at the address given below.

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