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Structure Coefficients for Use in Stellar Analysis

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Abstract. We give new values of the structural coefficients η_j, and related quantities, for realistic models of distorted stars in close binary systems. Our procedure involves numerical integration of Radau's equation for detailed structural data for stellar models taken from the EZWeb compilation of the Department of Astronomy, University of Wisconsin-Madison.

Keywords. Stellar structure, structural coefficients, close binary systems

1. Introduction

The classical approach to finding the shape of a body distorted by rotation and tides utilized equipotential surfaces (Kopal 1959). This approach permits inroads into the solution of the relevant Poisson's equation, if contributory effects can be regarded as additive perturbations upon simpler, more basic forms having spherical symmetry. The perturbations are expressed in terms of suitable harmonic expansions. The equipotentials satisfy Clairaut's equation, and this becomes tractable, due to the orthogonality conditions for products of harmonics in an integral (MacRobert 1927).

2. Equations

We set out the main underlying equations for this work. More background can be found in Kopal (1959). First, we have a series expansion for the potential, thus:

$$V = \Sigma_0^\infty r^{-(n+1)} G \int r'^n P_n(\cos\gamma)\, dm', \tag{2.1}$$

with mass element dm'

$$dm' = \int\int\int \rho r'^2 dr' \sin\theta' d\theta' d\phi'. \tag{2.2}$$

Clairaut's equation for surface perturbation can be set out in first-order form:

$$\frac{G}{(2j+1)a_1^{j+1}} \int_0^{a_1} \left(j a^j Y_j^i + a^{j+1} \frac{\partial Y_j^i}{\partial a} \right) dm' = \tag{2.3}$$

$$= c_{i,j} a_1^j P_j^i(\theta, \phi).$$

We write now, for the perturbation coefficients,

$$\eta_j(a) = \frac{a}{Y_j^i} \frac{\partial Y_j^i}{\partial a}. \tag{2.4}$$

which also satisfy 'Radau's equation':

$$a\frac{d\eta_j}{da} + \frac{6\rho}{\overline{\rho}}(\eta_j + 1) + \eta_j(\eta_j - 1) = j(j+1), \tag{2.5}$$

3. Procedure and Results

We wrote a small FORTRAN program to carry out numerical integration of Radau's equation. That was first combined with a separate program used to integrate polytropic models of stars. This was comparable to the method of Brooker & Olle (1995; hereafter BO), except that, with modern computers, steps can easily be made suitably small to avoid the numerical problems mentioned by BO, and still return reliable results in a short time. The Lane-Emden equation is rearranged as two simultaneous first-order difference equations, while Radau's equation becomes a first-order difference equation for η_j applying to each layer. We confirmed numerical agreement with BO to 8 digits with this program (RADAU).

Replacing the Emden equation integrator with the numerical tables of internal structure downloaded from the EZWeb compilation

http://www.astro.wisc.edu/~townsend/static.php?ref=ez-web,

we could apply RADAU to derive corresponding structural parameters for these more realistic stellar models (of mass M). We computed also representative polytropic indices (n_1, n_2) for such models for comparison with historic treatments. A few examples follow.

Table 1. Zero Age Solar Composition Models

$M = 0.5$; $n_1 = 2.52, n_2 = 2.19$						
j	2	3	4	5	6	7
η_j	2.83417	3.39029	4.77155	5.03932	6.23161	7.37262
Δ_j	1.76418	1.29864	1.15808	1.09570	1.06282	1.04366
k_j	0.38209	0.14932	0.07904	0.04785	0.03141	0.02183
$M = 3.0$; $n_1 = 2.75, n_2 = 2.80$						
j	2	3	4	5	6	7
η_j	2.97626	3.99355	4.99743	5.99875	6.99931	7.99959
Δ_j	1.00478	1.00092	1.00028	1.00012	1.00006	1.00002
k_j	0.00239	0.00046	0.00014	0.00006	0.00003	0.00001
$M = 10.0$; $n_1 = 2.67, n_2 = 2.47$						
j	2	3	4	5	6	7
η_j	2.89750	3.97376	4.98982	5.99509	6.99730	7.99837
Δ_j	1.02092	1.00376	1.00114	1.00044	1.00020	1.00010
k_j	0.01046	0.00188	0.00057	0.00022	0.00010	0.00005

4. Conclusion

This kind of result should have increasing importance with the improved photometric accuracies of the post-Kepler Mission era, i.e. light curves of mmag accuracy or better. Proximity effects associated with ellipticity are typically of order 0.1 mag in the majority of normal close binary light curves. Our results show that stellar type dependent structural variations affecting the principal terms of the ellipticity are significant at the 1% level, i.e. ~0.001 mag, and therefore should receive attention.

References

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