

WATSON, G. L., *Integral Quadratic Forms* (Cambridge, 1960), pp. viii+143, 30s.

This book presents a good account of those parts of the theory of quadratic forms which do not require analytical methods. The real substance begins at Chapter 3, where the Hilbert and Hasse symbols are introduced and it is proved that a form represents zero if and only if it is a  $p$ -adic form for every  $p$ . Chapter 4 deals with  $p$ -adic equivalence and  $p$ -adic representation. The  $p$ -adic numbers are not defined in the book, but the algebra is so well organised that it is difficult to argue that this is a disadvantage. In Chapter 5 the genus is introduced and conditions established for an integer to be representable by some form of a given genus. There are further chapters on rational transformations, rational automorphisms and the spinor genus.

It is clear that an effort has been made to keep the exposition elementary and brief. This has been very successful, except possibly in the first two chapters, which seem to be a little too condensed. The proof of the theorem that any form is rationally equivalent to a diagonal form, for instance, is wrapped up in a matrix notation which seems both obscure and unnecessary. Fortunately, most readers will already know this result.

The preface indicates that "The bibliography makes no attempt at completeness", but even this disclaimer hardly prepares us for its rather eclectic character. The name of Minkowski is absent, for instance, though some readers might wish to refer to the original work (I don't know where else) to find proofs of the assertions about Minkowski reduction in Theorem 18. Perhaps a sketchy bibliography is a modern trend in books on Quadratic Forms—the list of papers given here is completely disjoint from the list given by B. W. Jones in his book on the same subject.

However, these are unessential criticisms of a fine addition to the literature on this classical, but ever young, subject.

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BOURBAKI, N., *Eléments de Mathématique I. Les structures fondamentales de l'analyse, Livre II Algèbre, Chapitre 9. Formes sesquilineaires et formes quadratiques* (Actualités Scientifiques et industrielles 1272, Hermann, Paris, 1959), 218 pp., 38s.

This important instalment of Bourbaki's monumental work concerns bilinear, alternating, quadratic and Hermitian forms and their generalisations. Most of these generalisations have grown naturally from the desire for extreme abstraction with little regard for applications in other branches of pure mathematics, not to mention more menial uses. The reader must accept the harsh discipline of putting the study of the "fundamental structures" of the mathematical skeleton before all else. On the other hand, a generous portion of meat, and indeed many a juicy morsel is to be found in the exercises, which include a wealth of material that would fill a whole volume of an "old-fashioned" mathematical textbook.

The sesquilinear forms mentioned in the title are defined as follows: let  $A$  be a ring with unit element and let  $E$  be a left  $A$ -module. Assume that  $A$  possesses an anti-automorphism  $J$ , that is, a mapping  $a \rightarrow a^J$  satisfying  $(a+b)^J = a^J + b^J$  and  $(ab)^J = b^J a^J$  ( $a, b \in A$ ). Then a mapping  $\Phi$  of  $E \times E$  into  $A$  is called a sesquilinear form on  $E$  if  $\Phi(x+x', y) = \Phi(x, y) + \Phi(x', y)$ ,  $\Phi(x, y+y') = \Phi(x, y) + \Phi(x, y')$ ,  $\Phi(ax, y) = a\Phi(x, y)$ ,  $\Phi(x, by) = \Phi(x, y)b^J$ .

The book is devoted essentially to the algebraical rather than to the arithmetical theory of forms, that is, it is generally assumed that  $A$  is in fact a field. Apart from a discussion of reduction of the various types of forms in a sufficiently abstract setting, the most noteworthy features of the book are an account of Witt's theorem, an interesting chapter on Clifford algebras and a rigorous algebraical treatment of angles.