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Is Mandatory Project Evaluation Always Appropriate? Dynamic Inconsistencies of Irreversible and Reversible Projects

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Is Mandatory Project Evaluation Always Appropriate? Dynamic Inconsistencies of Irreversible and Reversible Projects

Tatsuhito Kono and Hiromichi Notoya

Abstract

Benefit-cost analysis (BCA) is used to optimize investment in public projects. Indeed, in many countries, BCA is mandatory for provision of most public services. However, once BCA is mandated, residents can strategically alter their behaviors based on a dynamic inconsistency that the BCA-based optimal service level depends on residents' behaviors. This paper discusses this dynamic inconsistency problem, taking transportation services as an example. We show that the problem may decrease both social welfare and the utility of residents as compared with the first-best case, and that the occurrence of second-best outcomes depends on the reversibility of the project and the general-equilibrium interdependency with another project.

KEYWORDS: project evaluation, dynamic inconsistency, transport service, migration

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1. Introduction

The efficient implementation of public policies is important for economic development. Currently, in many countries throughout the world, benefit-cost analysis (BCA) is used for determining efficient expenditure and infrastructure levels of public policies. For example, specific BCA procedures are prepared by ministries for types of public infrastructure such as various modes of transportation in many countries and by many institutions: Germany, Japan, the United Kingdom, the United States, the European Union and the World Bank.[1] Furthermore, BCA is mandatory for provision of most public services such as transportation (Hayashi and Morisugi (2000)).

The fundamental principle of BCA is that the net gain for society should be a criterion to determine whether or not a service should be provided. The evaluation of social net gain would be simple if preferences could be observed directly. However, in reality, preferences are not directly observable. Consequently, BCA attempts to estimate net gain using other observable economic variables such as a change in consumer surplus based on demand and prices.[2]

This estimation method of BCA, however, implies that the BCA-based optimal level of service depends on residents' selection of choice variables, such as demand and residential location. The inconsistency between optimal policies that differ before and after an agent's behavior (e.g., the resident's selection behavior in this specific BCA situation) is known as "dynamic (or, time) inconsistency," described originally by Kydland and Prescott (1977). The definition of dynamic inconsistency, based on Kydland and Prescott (1977), is as follows. *Let* $\pi = (\pi_1, \pi_2)$ *be a sequence of policies for periods 1 and 2 and* $x = (x_1, x_2)$ *be the corresponding sequence for economic agents' decisions. An agreed-upon social objective function* $B = (x_1, x_2, \pi_1, \pi_2)$ *is assumed to exist. Agents' decisions in each period depend on all*

[1] Over the past 25 years, there has been a growing interest in evaluating the efficiency of public regulatory programs (Sieg, et. al; 2004). One example is Section 624 of the Regulatory Right-to-Know Act of 2001 in the United States. This legislation requires the Office of Management and Budget to submit an annual report on the costs and benefits of federal regulations, considering them in the aggregate.

[2] A change in the consumer surplus by a public project can be observed *ex post,* but must be estimated *ex ante*. Benefit-cost analysis is an essential step to take before implementing a public project. Hence, the demand function for calculating the change in the consumer surpluses has to be estimated. This estimation is difficult and often involves an error. However, *ex post* observability may reduce the level of error in the estimation.

policies[3] *and past decisions, as denoted by* $x_1 = X_1(\pi_1, \pi_2)$ *and* $x_2 = X_2(x_1, \pi_1, \pi_2)$. *In this situation, if the optimal policy* π_2 *for period 1 is the same as the optimal policy* π_2 *for period 2* (that is, given the first period behavior set (x_1, π_1))*, then the policy is dynamically consistent.* Otherwise, it is inconsistent. Inconsistency arises because the optimal policy for period 2 depends on the economic agents' behavior in period 1. Inconsistency is a problem if the economic agents take advantage of such inconsistencies in the policy.

Past studies have addressed various dynamic inconsistency problems. In particular, numerous studies have examined problems of monetary policy by modeling behaviors of government and people in a Stackelberg game (e.g., Barro and Gordon (1983), Calvo (1978a), Calvo (1978b), and Kydland and Prescott (1977)). The inclusion of monetary policy in those models is well-known to engender a problem of inefficiently high inflation because the optimal policymaker behavior has a dynamic inconsistency that people tacitly infer. Dynamic inconsistency arises not only in monetary policy but also in other various fields.[4] However, past studies have not analyzed the dynamic inconsistency of BCA-based public policies.

In an important discussion on a public policy's dynamic inconsistency, Kydland and Prescott (1977, p.477) consider the problem of building houses in a floodplain as one such example[5]: *Assume the socially desirable outcome is not to have houses in a particular flood plain but, given that they are there, to take certain costly flood-control measures.・・・ Rational agents know that, if he and others build houses there, the government will take the necessary flood-control measures. Consequently, in the absence of a law prohibiting the construction of houses in the floodplain, houses are built there, and the army corps of engineers subsequently builds the dams and levees.*

As described above, Kydland and Prescott (1977) underscore the dynamic inconsistency problem as it might be applied to allocation of flood-control measures: residents strategically migrate to elicit the government's reaction to their migration. But Kydland and Prescott (1977) do not specify why and to what extent costly flood-control measures such as dams and levees are taken by the government

[3] This implies that the agents expect the future policies.
[4] Examples of past studies in other fields are Boadway et al. (1996) on education, Glazer (2000) on transportation congestion tolls, and Richer (1995) on urban development.
[5] Kydland and Prescott (1977) explore a general analysis of a dynamic inconsistency problem in a model and then put forth many examples of such problems. The above example of flood-control measures is one example among them.

if houses are built in the flood plain. At present, many countries use BCA to determine the optimal level of such flood-control facilities. In this situation, the dependency of the optimal level of public facilities on the population distribution is the source of dynamic inconsistency. In the real world, similar dependencies are observed in many other public facilities, such as transportation, local environmental projects such as large parks, and other public flow services (e.g., pension services), because the level of public facilities and services affects migration, for example, “voting with their feet” as discussed by Tiebout (1956).

This paper, using public transport services as an illustrative example, is intended to present a concrete discussion of the dynamic inconsistency problem of BCA-based public policies, extending the “flood-control measure” discussion of Kydland and Prescott (1977).[6] The reason for choosing transport services as our illustrative example is that the benefit of increasing the service is easily represented in terms of consumer surpluses[7] and so the dynamic inconsistency problem is illustrated simply.

We first demonstrate that the BCA-based optimal level of transport services is dependent on the migration of residents because the number of residents affects traffic volume, which is one BCA component. Hence, the residents can migrate strategically (or developers may strategically induce numbers of residents to migrate), taking advantage of the BCA to increase their respective utility levels (or developers’ rent). As a result, a greater than socially optimal level of transport service might be supplied. This dynamic inconsistency structure with strategic migration is essentially identical to that of building houses in a floodplain as described by Kydland and Prescott (1977). That is, “transport services” and “strategic migration” in the current paper correspond respectively to “dams and levees” and “strategic migration to a floodplain” in Kydland and Prescott (1977).

As distinct from Kydland and Prescott (1977), we take account of two types of public services: 1) a fixed-capital-stock service that is irreversible, and 2) a flow service with a fully variable service level. The level of the first type of service, e.g. dams, levees, and roads, can increase but not decrease; in contrast, the level of the second type, e.g., bus service, can increase and decrease freely according to demand. Such differences between irreversible and reversible projects can impose

[6] See footnote <5>.

[7] For example, in the case of flood-control facilities, we have to use the expected utility and the corresponding welfare measure that would be more complex than consumer surpluses.

different effects on the results.[8] We demonstrate that, in fact, a flow service with a fully variable service level can decrease not only social welfare but also the residents' utility, as compared with the first-best situation; whereas a fixed-capital-stock service can decrease only social welfare.

Section 2 presents the model and derives the BCA used to assess the transport projects, while Section 3 explores how dynamic inconsistency affects the residents' equilibrium utility level and social welfare depending on the transport service situation. Section 4 concludes the paper.

2. Model

The model has one employment zone and two residential zones ($i = 1, 2$). The two residential zones are hereafter labeled "zone 1" and "zone 2." The number of zones is the minimum necessary for analyzing migration, although real cities have more than two zones. Each zone has one transport node which is linked to the employment zone. Examples of transport nodes are a railway station, a bus terminal, or a main road. The transport route extending from the node of zone 1 to the employment zone is labeled *route 1*, and the other route, extending from the node of zone 2 to the employment zone, is labeled *route 2*. The city structure is illustrated in Fig. 1.

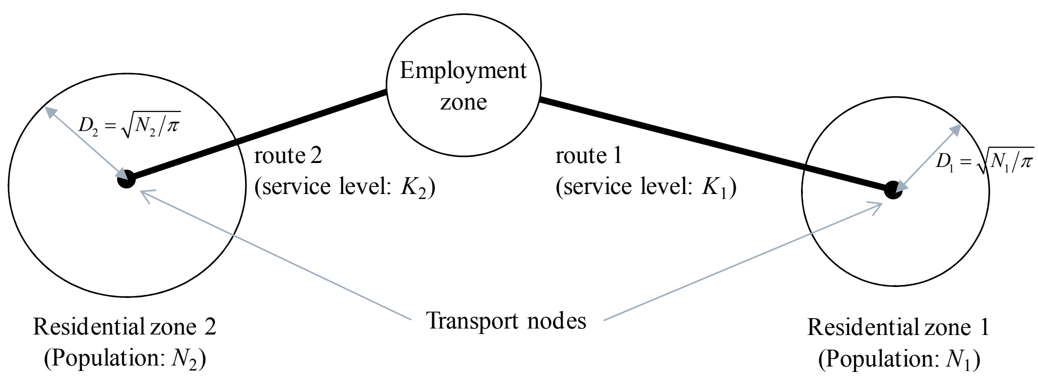

Figure 1. Urban structure

[8] Monetary policy as explored in the vast past literature as a cause of dynamic inconsistency does not use these two type characteristics. Turnovsky, S.J., and W.A. Brock (1980) analyze another set of two policies, monetary instruments and fiscal instruments, to clarify a difference between the two.

Transportation costs. The commuting cost per resident is composed of the "access" transport cost and the "line-haul" transport cost. The access transport cost is the cost from the residence to the transport node in each zone. Assuming one unit cost per unit distance for the access transport cost, the cost is equal to the distance d_i. On the other hand, the line-haul transport cost c_i is expressed as follows. Transportation costs c_i of route *i* are expressed by the unit of distance cost function, $c(K_i)$, where K_i is the transport service level per unit of length of route *i*, multiplied by length s_i of route *i* ($i = 1, 2$):

$$c_i = c(K_i)s_i, \tag{1}$$

where $\partial c / \partial K_i < 0$, and $\partial^2 c / \partial K_i^2 > 0$.[9] The supply of transport service K_i is assumed, for simplicity, to require amount K_i of the composite good.

The agents of the model are the local government, $\bar{N}$ homogeneous residents, absentee landowners, and a perfectly competitive composite-good producer. The $\bar{N}$ residents of the zones live in either zone 1 or zone 2. Fundamentally, household members are assumed in this simplified analysis to migrate without any cost, so the utility levels must be equal for zones 1 and 2. The migration cost can be negligible if the cost is spread over a lifetime. However, migration might impose some costs in some real situations. For that reason, we also discuss cases of such costly migration as the analysis demands.

To focus on the dynamic inconsistency problem, we assume the following. The land in zones 1 and 2 is owned by many absentee landowners. A perfectly competitive composite-good producer operates in the employment zone. The producer does not need land and, for simplicity, the area of the employment zone is zero. Consequently, the model has no price distortions and externalities such as congestion.[10]

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[9] In reality, as opposed to our setting, $\partial^2 c / \partial K_i^2$ might be negative for very small K_i, reflecting the scale economies. For large K_i, however, such scale economies disappear. For example, suppose there is an increase in the number of road lanes. When increasing the number from 1 to 2, the travel time might decrease exponentially (i.e., $\partial^2 c / \partial K_i^2 < 0$), but when increasing the number from 3 to 4, the change in the travel time becomes less extreme (i.e., $\partial^2 c / \partial K_i^2 > 0$). In this paper, we assume the interval where $\partial^2 c / \partial K_i^2 > 0$, because the second-order condition of the optimality of the service level should hold.

[10] The combination of the dynamic inconsistency problem and other externalities (e.g., congestion and price distortions) is an important topic for future discussion.

Households. Households are homogeneous and independent of their respective locations. Because each household is assumed to consume one unit of land, the utility level depends on the net consumption (that is, the wage net of transport cost and rent). A household at a point distance d_i from the transport node in zone i commutes to the employment zone with the line-haul transportation cost c_i, earns wage w and pays rent $r_i(d_i)$. The net wage for a worker residing at distance d_i from the transport node in zone i is w-$r_i(d_i)$-$\bar{z}$ (d_i+c_i), where $\bar{z}$ is the constant number of commuting trips per household. The net wage is spent on the composite good x_i. The utility[11] of a household at distance d_i from the transport node in zone i, $V_i(d_i)$, is expressed as

$$V_i(d_i) = w - r_i(d_i) - \bar{z}\{d_i + c_i\}. \tag{2}$$

In equilibrium, the utility of residents within zone i is uniform. Hence, the utility per resident in zone i is represented by that of the residents at the edge of the zone, $V_i(D_i) = w - r_i(D_i) - \bar{z}\{D_i + c_i\}$, where we express the distance from the transport node to the zone edge as D_i. We normalize the land rent at both zone edges at zero (that is, $r_i(D_i) = 0$), so the zone i equilibrium utility, V_i, is

$$V_i = w - \bar{z}\{D_i + c_i\}. \tag{3}$$

Regarding $r_i(d_i)$, because $V_i(d_i) = V_i(D_i)$, we have $r_i(d_i) = \bar{z}\{D_i - d_i\}$. Because unit distance transport cost is constant in every direction, each zone has a circular shape of radius D_i. If the population size of zone i is N_i, then $D_i = \sqrt{N_i/\pi}$. The residential areas of zones 1 and 2 change depending on the population size N_i. Correspondingly, the utility V_i depends on N_i.

Producer. The production function is assumed to be a constant-returns-to-scale function of labor, as

$$X^f = \omega l^f, \tag{4}$$

where X^f is the quantity of composite goods produced, ω is a constant parameter, and l^f is demand for labor. Under perfect competition, wage w is equal to ω:

[11] The utility, of course, depends on the housing lot. However, the lot size is fixed, so it is unnecessary to consider lot size.

$$w = \omega . \tag{5}$$

Absentee landowners and absentee taxpayers. Assuming no opportunity cost for the supply of land, land will be supplied as long as the price is positive. Consequently, the total revenue of landowners who own land in zone i, Π_i^A, is expressed as

$$\Pi_i^A = 2\pi \int_0^{D_i} d_i r_i(d_i) dd_i = 2\pi \int_0^{D_i} d_i \bar{z} \{D_i - d_i\} dd_i = \pi \bar{z} \frac{D_i^3}{3}, \tag{6}$$

Absentee landowners must pay $(s_1 K_1 + s_2 K_2)$ for transport services.

As described above, the model assumes that only the absentee landowners pay the transport service costs[12] through a land rent tax, which is an unrealistic assumption made just to simplify the calculation. In reality, transport services are paid for also by income taxes and other revenue sources. However, that assumption is not essential for our purpose. Essentially the same results can be derived if residents were also to pay the transport service costs, but the derivation would be extremely complex. The assumption necessary to derive the same results is that not only zone 1 and 2 residents, but also others pay the transport service costs. In the real world, transport services are financed by various taxes such as those on income and property. In addition, financing is often derived not only from the service area, but also from other areas. Therefore, a necessary assumption to derive the same results pertains in the real world (see Remark 2 for a related discussion).

Market clearing conditions. First, for simplicity, assuming that all residents are employed, the total population in zones 1 and 2 is the total labor,

$$N_1 + N_2 = l . \tag{7}$$

Next, total population is fixed,

$$N_1 + N_2 = \bar{N} , \tag{8}$$

where N_i represents the population in zone i, and $\bar{N}$ is the total population (fixed).

[12] Absentee taxpayers can be separated from absentee landowners. That separation does not affect the result.

Next, households migrate without any cost, so the utility levels must be equal in zones 1 and 2 unless the number of residents is zero in either zone:

$$V_1 = V_2 \ \text{ when } N_1 > 0 \text{ and } N_2 > 0 . \tag{9}$$

Next, the total demand for a composite good for residential use and the transport service equals the supply from the producer,

$$\sum_{i\in\{1,2\}} x^i N^i + \sum_{i\in\{1,2\}} s^i K^i = X^f . \tag{10}$$

***Benefit-cost analysis* (BCA).** The BCA method can be derived from the above model, that is, eqs. (1)–(10). Social welfare, B, is defined as the sum of the residents' utilities measured in terms of a composite good plus the absentee landowners' profits from the land[13],

$$B = \sum_{i\in\{1,2\}} N_i V + \sum_{i\in\{1,2\}} \Pi_i^A - \sum_{i\in\{1,2\}} s_i K_i . \tag{11}$$

Using eqs. (1)–(10), the derivative of social welfare, dB $\left(= \sum_{i=\{1,2\}} N_i dV + \sum_{i=\{1,2\}} d\Pi_i^A - \sum_{i\in\{1,2\}} s_i dK_i\right)$ is

$$dB = \sum_{i\in\{1,2\}} [\underbrace{-N_i \bar{z} \frac{\partial c}{\partial K_i}(K_i)}_{\text{marginal benefit}} - \underbrace{1}_{\text{marginal cost}}] s_i dK_i . \tag{12}$$

See *Appendix A* for the derivation of eq. (12).

Equation (12) expresses the benefit-cost analysis[14] of eqs. (1)–(10). The term $-N_i\bar{z}(\partial c/\partial K_i)s_i dK_i$ $(i = 1,2)$ in eq. (12), where $N_i\bar{z}$ is the total trip demand and $(\partial c/\partial K_i)s_i dK_i$ is the reduction in transport costs per trip, expresses the benefit arising from improved transport service of route i ($i \in \{1,2\}$).[15] The term $s_i dK_i$ in eq. (12) represents the additional transport service level on the route. The supply of

[13] Note that the firms' profits are zero because of homogeneous production technology.
[14] Indeed, the BCAs designed by ministries in most developed countries primarily adopt methods that mirror those expressed in eq. (12) (see Hayashi and Morisugi (2000)).
[15] The total trip demand and the reduction in transport costs per trip should be estimated before implementing the policy. See note 2 above.

the transport service level $s_i dK_i$ requires the amount $s_i dK_i$ of the composite good by assumption.

The optimal transport service level based on the BCA is met when the marginal net benefit is zero[16], i.e. $dB/dK_i = 0$. Applying this condition to eq. (12) yields the following lemma.

Lemma 1 (Dependency of BCA on residents' migration). *The optimal level of the transport service* $(i \in \{1,2\})$*, as judged by the BCA*[17] *holds that*

$$\frac{dB}{dK_i} = -N_i \bar{z} \frac{\partial c}{\partial K_i}(K_i) - 1 = 0 \quad (i \in \{1,2\}) . \tag{13}$$

The first term, $-N_i \bar{z}(\partial c/\partial K_i)$, in (13) expresses the marginal benefit; the second term, 1, denotes the marginal cost of the transport service. Equation (13) shows that the optimal level of the transport service K_i $(i = 1,2)$, as judged by the BCA, depends on zone *i* population N_i. In other words, the optimal K_i is a function of N_i, as shown by $K_i(N_i)$. As the zone *i* population N_i increases, the optimal K_i increases.

Lemma 1 states that the optimal service level judged by the BCA depends on the residents' migration (possibly induced by developers). This dependency, which is very natural, is the dynamic inconsistency of the BCA and the source of residents' strategic behavior. The BCA assumes that residents migrate passively in response to a change in transport service but not that they migrate strategically.

16 The marginal net benefit should be zero when the social welfare *B* is maximized with regard to K_i. In other words, the first-order condition should hold. In practical situations, the net benefit (or the ratio of the benefit to the cost) is used as a criterion. To maximize social welfare, public projects should be implemented as far as their net benefits are greater than zero (or the ratio of the benefit to the cost is greater than one).

17 Increasing the level of transport service, K_i, will have a (general equilibrium) feedback on migration and thus on N_i. (This migration is later referred to as "passive migration in response to policy" in Remark 1.) However, based on the envelope theorem, equation (13) is identical regardless of whether or not such feedback is taken into account. Indeed, equation (13) is derived from the general equilibrium approach (see Appendix A for the detailed derivation). Equation (13) is based on a small change in K_i. Therefore, when the benefit of a large investment is calculated, equation (13) should be integrated from the initial level of K_i to the final level. In this process, N_i increases as K_i increases. Accordingly, the benefits when taking account of migration can differ from those without taking account of it, and the difference can be large as demonstrated in Sieg et al. (2004) and Walsh (2007).

However, in reality, residents can migrate strategically. Or, rent-seeking developers might support such strategic migration. In this case, the developers gain the increased profit by obtaining some part of the increase in the residents' utility levels. The difference between 1) passive migration in response to policy and 2) strategic migration is important for our analysis. We define them in Remark 1.

Remark 1. Two types of migration of resident exist:

1) *Passive migration in response to policy*: Residents migrate passively in response to changes in the transport service level. This migration is determined based on the conventional general equilibrium.
2) *Strategic migration taking advantage of policy*: Residents migrate strategically to take advantage of the mandatory BCA method to increase their utilities.

As we have already discussed, residents usually do not bear the full costs of public projects, including transportation projects. In other words, public projects often face "free rider" problems. If residents bear the full cost, "strategic migration" will not arise because residents can not be free riders. This point is summarized in the following Remark.

Remark 2. Strategic migration can take advantage of policy when residents do not bear the full costs of the policy.

In our model, and as with most public projects, residents do not bear the full costs of such projects.[18]

The following section explores situations in which residents migrate strategically, contrasting "passive migration in response to policy" with "strategic migration taking advantage of policy".

3. Dynamic Inconsistencies of BCA-based Transport Policies

The local government can improve transport services of the two routes between the employment zone and each of the two residential zones (see Fig. 1). Two route

[18] In the sense that residents do not pay the full costs of the policy, a kind of externality arises in the policy.

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improvement scenarios are possible. In the first scenario, the government can improve only one of the two routes. In the second, the government can improve both routes. The fixed level in the first scenario might occur when no investment can improve the service level because of an existing state-of the art facility, or when geographic or municipal difficulties prohibit investment. The second scenario can illustrate the effect of dependency between the two routes on the results.

This section presents an exploration of how the dynamic inconsistency of BCA affects the residents' equilibrium utility level and social welfare under four transport-policy situations, arising from combinations of transport services of two types (fixed capital service or variable flow service) and route-improvement scenarios of two types (one of the two routes or both routes). The following are the four transport-policy situations: ***Situations* I–IV**.

***Situation* I:** The government can improve the transport service level of route 2 only. Hence, the level of transport service of route 1 is fixed. Furthermore, the transport service is the fixed capital service such as a road.

***Situation* II:** The government can improve the transport service level of route 2 only. But, in this situation, the transport service level is variable at any time. In other words, the transport service is a flow service, such as bus service.

***Situation* III:** The government can improve transport services of both routes 1 and 2. The transport service is a fixed capital service.

***Situation* IV:** The government can improve transport services of both routes 1 and 2. The transport service is a flow service, which is variable at any time.

Situation* I: *Fixed capital transport service of route 2 only

Assuming ***Situation* I**, we analyze how strategic behavior of residents affects the equilibrium and the social welfare using Fig. 2, which is drawn from the model. Government and residents are the players in this game. In the game framework, the relation between strategies and the payoff plays a vital role. Figure 2 can show the relation between any amount of migration (i.e. strategies) and the residents' utility level (i.e., payoff) concisely as in "payoff matrix". Therefore, it can readily present the mechanism of the dynamic inconsistencies of a BCA-based policy.[19]

The upper graph in Fig. 2 shows the relationship between population allocation and the level of transport services. The *x*-axis denotes the allocation of the population between zones 1 and 2. The population in zone 1, N_1, is measured according to the distance from the left side. The population in zone 2, N_2, is measured from the right side. The total population, $\bar{N}$, is fixed. The *y*-axis expresses the level of the transport service of route 2: K_2.

The upper graph in Fig. 2 has two lines, $N_2(K_2)$ and $K_2(N_2)$:

$N_2(K_2)$ marks the equilibrium population allocation , N_2, given the transport service level K_2. In other words, this line represents "passive migration in response to policy" as defined in Remark 1. For this population distribution, the levels of utility for the two zones are equal unless the total population resides in one zone (that is, $V_1 = V_2$ when $N_1 > 0$ and $N_2 > 0$ as shown in Eq. (9)).

$K_2(N_2)$ marks the optimal level of transport service, K_2, from the viewpoint of BCA at a given population allocation, N_2. It is the level of transport service satisfying eq. (13): $dB/dK_2 = 0$ (As **Lemma 1** indicates, K_2 depends on N_2).

The area with slanting lines (that is, the area to the left of $K_2(N_2)$) in the upper graph of Fig. 2 has positive net benefit for a marginal increase in the transport service, i.e., $dB/dK_2 > 0$, because the population in zone 2, N_2, a positive factor within the marginal benefit in eq. (13), is greater than line $K_2(N_2)$ at a given level of K_2. From the viewpoint of BCA, the level of transport service K_2 should be increased in this area.

[19] For our analysis, local information (e.g., differential) is not sufficient, but global information is needed. So, figures are useful. Actually, simple algebra can replace the discussion using figures. But, such algebra does not show the mechanism in an intuitive and simple way as a figure does. Although we use figures, the slopes of the related lines, which denote the combination of migration (i.e. strategies) and the utility level to residents (i.e., payoff), in the figures are rigorously derived not to lose the rigor of the discussion.

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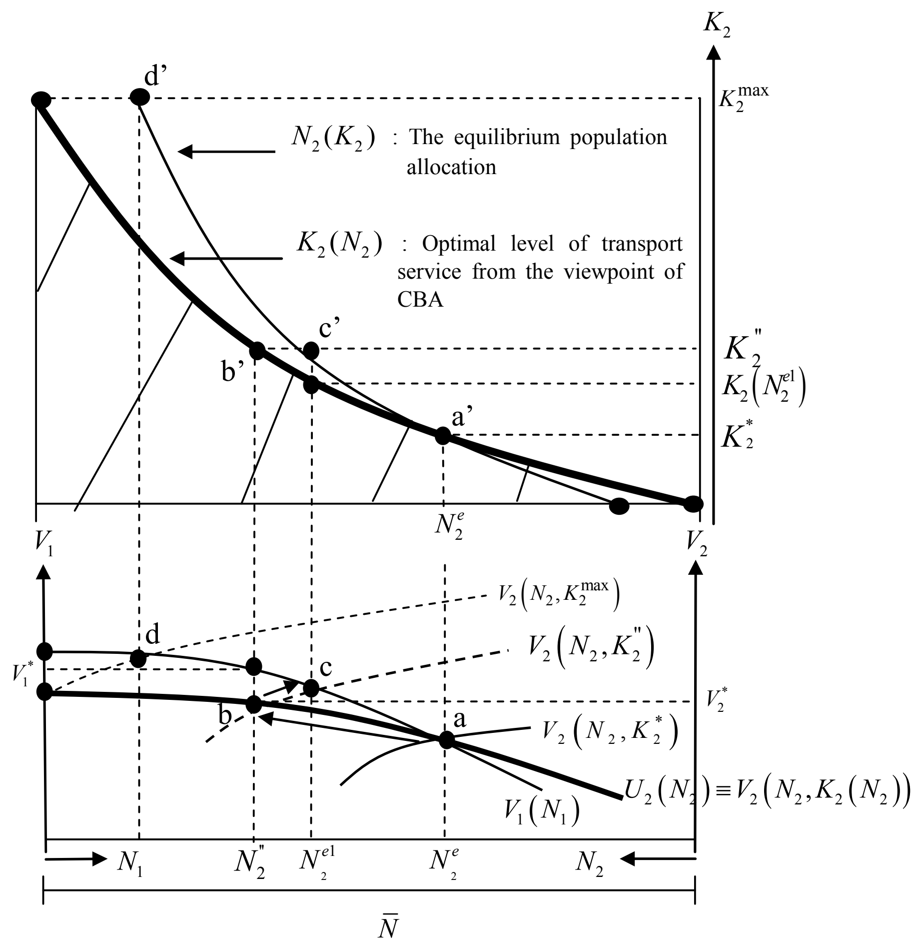

Figure 2. Population allocation, utility and the transport service level (*Situations* I, II)

The lower graph in Fig. 2 depicts the relationship between the population allocation and utility levels in the two residential zones. The *x*-axis denotes the populations of zones 1 and 2: N_1 and N_2. The *y*-axis shows the utility levels in the residential zones. The lower graph presents three solid curved lines, $V_1(N_1)$, $V_2(N_2, K_2^*)$, and $U_2(N_2)$:

$V_1(N_1)$ expresses the utility level of zone 1 residents, V_1, when the number of zone 1 residents, N_1 is given exogenously. (Note: Because K_1 is fixed in ***Situation* I**, we suppress K_1 in this function.)

$V_2(N_2, K_2)$ is the utility level of zone 2 residents, V_2, when both the number of zone 2 residents, N_2, and the level of the transport service, K_2 are given exogenously.

$U_2(N_2)$ expresses the utility level in zone 2 when the number of zone 2 residents, N_2, is given exogenously and the optimal transport service level, K_2, from the perspective of the BCA is determined based on N_2. Therefore, $U_2(N_2) \equiv V_2(N_2, K_2(N_2))$.

The intersection of $V_1(N_1)$ and $V_2(N_2, K_2)$ in the lower graph indicates, $N_2(K_2)$, the equilibrium population allocation between zones 1 and 2, given K_2.

The slope properties of all lines in the upper and lower graphs in Fig. 2 are derived from the model: eqs. (1)–(10) plus (13) (see *Appendices B* and *C*). These slope properties are important for deriving the propositions because they express the relation between migration (i.e. strategies) and the residents' utility level (i.e., payoff). The lower graph has five properties:

Property 1L: *$\partial V_1(N_1)/\partial N_1 < 0$ and $\partial V_2(N_2, K_2)/\partial N_2 < 0$ because of the expansion of the zone area (or, the increase in rent at each point with the increase in the travel cost at the city edge) with the increased respective zone population.*[20]

Property 2L: *$\partial U_2(N_2)/\partial N_2 (\equiv \partial V_2/\partial N_2 + \partial V_2/\partial K_2 \cdot \partial K_2/\partial N_2) > \partial V_2(N_2, K_2)/\partial N_2$ because the level of "transport service K_2 based on the BCA", $K_2(N_2)$, is increased with respect to the increased N_2 (i.e., $\partial V_2/\partial K_2 \cdot \partial K_2/\partial N_2 > 0$)*

Property 3L: *$\partial U_2(N_2)/\partial N_2$ is not necessarily negative, whereas $\partial V_2(N_2, K_2)/\partial N_2 < 0$. $\partial U_2(N_2)/\partial N_2$ can be positive if the level of "transport service K_2 based on the BCA", $K_2(N_2)$ in $U_2(N_2)$ $(\equiv V_2(N_2, K_2(N_2)))$, greatly increases the utility.*

Property 4L: *At point* a*, corresponding to point* a*', the three lines V_1, V_2 and U_2 are all equal because the equilibrium population allocation $V_1(N_1) = V_2(N_2, K_2)$ and the optimal level of the transport service $K_2 = K_2(N_2)$ imply that $V_1(N_1) = V_2(N_2, K_2) = V_2(N_2, K_2(N_2)) \equiv U_2(N_2)$.*

Property 5L: *To the left of point* a*, line $U_2(N_2)$ lies below line $V_1(N_1)$; to the right of point* a*, line $U_2(N_2)$ lies above line $V_1(N_1)$.*

[20] The property $\partial V_1(N_1)/\partial N_1 < 0$ *and* $\partial V_2(N_2, K_2)/\partial N_2 < 0$ holds in other settings, too. For example, as the population increases, the residential utility decreases because of the decrease in lot size and the congestion in public facilities. These factors are not considered in our model to simplify the discussion.

Most ***properties***, in particular ***1L*** and ***2L,*** are intuitive. They can also be derived strictly. See eq. (G) in *Appendix B* for ***Property 1L***. See eq. (H) and the discussion in *Appendix B* for derivations of ***Properties 2L*** and ***3L***. ***Property 5L*** relates directly to the second property of the upper graph. *Appendix C* demonstrates how ***Property 5L*** is derived from the second property of the upper graph.

The upper graph depicts two important properties: ***Properties 1U*** and ***2U.***

Property 1U: *Point* a*', which is the intersection of* $N_2(K_2)$ *and* $K_2(N_2)$*, corresponds to point* a *in the lower graph because the combination of* $V_1(N_1) = V_2(N_2, K_2)$ *and* $K_2 = K_2(N_2)$*, which both hold at point* a*, directly imply that* $N_2(K_2)$ *and* $K_2(N_2)$ *intersect. These points* a *in the lower graph and* a*' in the upper graph are the most efficient (first best) points.*[21] *If residents take no strategic actions, these points* a *and* a*' are equilibrium points.*

Property 2U: *To the right of point* a*', line* $N_2(K_2)$ *lies below line* $K_2(N_2)$ *and to the left of point* a*', line* $N_2(K_2)$ *lies above line* $K_2(N_2)$.

Property 2U implies that line $N_2(K_2)$, which denotes the equilibrium population allocation, lies in the area with slanting lines $dB/dK_2 > 0$ when K_2 is too small; line $N_2(K_2)$ lies in the area $dB/dK_2 < 0$ when K_2 is too large. ***Property 2U*** is realistic, and we therefore assume that it holds in the model.[22]

If residents do not take strategic advantage of the BCA method by migrating, public investment based on BCA yields the stable equilibrium expressed by point a', which is the intersection of $N_2(K_2)$ and $K_2(N_2)$ in the upper graph, and point a, which is the intersection of $V_1(N_1)$ and $V_2(N_2, K_2^*)$ in the lower graph. These points are the first-best social welfare points (see ***Property 1U***).

Can the first-best social welfare point a (or point a') be attained even if residents strategically take advantage of the BCA method? We will examine whether or not the first-best point is stable (or proof) against residents' strategic migration.

[21] As shown in Section 2, the model has no externalities such as congestion and fiscal externalities. Hence, the conventional competitive market equilibrium (or non-strategic equilibrium) is the most efficient (i.e., first-best) point.

[22] In other words, property *2U* assumes that optimal K_2 is an interior solution.

Definition 1 (Stability of the first-best point against strategic migration). The first best point (or non-strategic equilibrium) is *stable* (or *proof*) against strategic migration if no resident can increase his utility level by his migration at the first best point.

People migrate if the utility increases, i.e. $dN_1 = \phi(V_1 - V_2)$ where ϕ is a positive parameter. Therefore, such migration takes place if a strategic migration induces an increase in utility, which implies that the equilibrium is unstable. This "unstable" situation can be explained using Fig. 2. First, presume that the economy is at the first-best point, point a, in the lower graph or a' in the upper graph. From these points, if the number of residents in zone 2 increases from N_2^e to $N_2^{''}$, the BCA method indicates that $K_2(N_2)$ is the optimal level of transport service; consequently, the level of transport service is improved to $K_2^{''}$, so that the residential utility in zone 2 increases from $V_2\left(N_2, K_2^*\right)$ to $V_2\left(N_2, K_2^{''}\right)$. It is assumed that the level of transport service in ***Situation* I** does not decrease. Therefore, the level of transport service, $K_2^{''}$ remains. To reach the eventual equilibrium of utility between the two residential zones, the residents migrate again until the utilities are equal between the two residential zones. The final equilibrium is at point N_2^{e1}.

In summary, when residents take advantage of the BCA, they can strategically migrate to point b, where the population in zone 2 is $N_2^{''}$. Then residents migrate to point c, where the zone 2 population is N_2^{e1}.[23] Residents increase their utility by this behavior. In contrast, the final equilibrium at point c or c' indicates an excess supply in the level of transport service, that is $dB/dK_2 < 0$, decreasing social welfare from the first-best situation (point a'). Social welfare decreases with the increased taxes for "too much" transport service.

The discussion above assumes a certain amount of migration of $\left(N_2^{''} - N_2^e\right)$. Even if the migration is smaller or larger, the residents can, by the same logic, increase their utility. Points d and d' express the points that are attainable by the initial migration of all residents to zone 2. In addition, even if only one resident

[23] This two-stage migration, which might involve short-term migration, is not so unrealistic as it may appear. In real situation, when the developer induces the strategic migration, population $N_2^{''}$, which can be achieved by the first stage migration, might be announced as a "planed population" by the developer. Correspondingly, the government takes this planned population $N_2^{''}$ as N_2 in the BCA formula (13) because the government has no sufficient information on migration (or, residents' preference).

migrates from the first-best situation, the final equilibrium differs from the first-best social welfare point. The result is summarized as Proposition 1.

> **Proposition 1.** *If residents take account of the mandatory BCA method to migrate, then* ***Situation*** **I** *always involves a decrease in social welfare from the first-best situation, with an increase in the residents' utility beyond that of the first-best situation and with an excess supply in the level of transport services.*

Residents' strategic behavior implies that they are Stackelberg leaders and that the government conducting the BCA is the follower. BCA inherently supposes that residents are followers in the sense that the residents react to the BCA-based policy change. Then, the residents' strategic behavior prevents the achievement of the first-best equilibrium.

How much the final equilibrium differs from the first-best situation depends on the number of residents who migrate strategically to increase their utility. In reality, the migration cost cannot be neglected, and strategic migration might not occur when the migration cost is greater than the increased utility. However, if numerous people migrate, the increase in utility tends to be large. In the real world, rent-seeking developers can support such a large migration. The above dynamic inconsistency problem might arise in such a case.

Situation II: Variable transport service of only route 2

As in ***Situation*** **I**, in ***Situation*** **II** the government can only improve the transport service level of route 2, but, as distinct from ***Situation*** **I,** transport services are flow services, which are variable at any time. Figure 2 is useful to explore ***Situation*** **II** also. Variable services means that the transport service level is always adjustable so that $dB/dK_2 = 0$ (i.e., optimal from BCA) if the service is provided based on BCA, which implies that the utility attainable in zone 2 is expressed as line $U_2(N_2)$ $(\equiv V_2(N_2, K_2(N_2)))$. Therefore, to explore equilibrium in ***Situation*** **II**, it is sufficient to check only the relationship between the utility level in zone 1, $V_1(N_1)$ and the utility attainable in zone 2, $U_2(N_2)$, as indicated in Fig. 2.

The configuration of $V_1(N_1)$ and $U_2(N_2)$ is described by ***Properties 2L, 3L, 4L*** and ***5L.*** The combination of these four properties directly implies that **Lemma 2** holds.

Lemma 2 (The configuration of $U_2(N_2)$, $V_1(N_1)$ and $V_2(N_2)$.) To *the left of the first-best point* (*point* a *in* Fig. 2)*, line* $U_2(N_2)$ *lies below line* $V_1(N_1)$ *but above line* $V_2(N_2)$ *; to the right of the first-best point, line* $U_2(N_2)$ *lies below line* $V_2(N_2)$ *but above line* $V_1(N_1)$*, where* $V_1(N_1)$ *and* $V_2(N_2)$ *are abbreviations for* $V_1(N_1, K_1)$ *and* $V_2(N_2, K_2)$ *passing through the first-best point. In addition, the slope of* $U_2(N_2)$ *can be positive with respect to the increase in* N_2*.*

The relationship between $V_1(N_1)$ and $U_2(N_2)$ in Fig. 2 satisfies **Lemma 2**. As in our treatment of ***Situation*** **I**, we will see whether or not the first-best point (or non-strategic equilibrium) is stable (or proof) against residents' strategic migration. First, assume that the population allocation is at point a, the first-best social welfare point. Next suppose there is a strategic migration by which the residents in zone 2 increase from the first-best point, say "from N_2^e to $N_2^{"}$". In this case, the levels of utility are V_1^* in zone 1 and V_2^* in zone 2 (see Fig. 2 for V_1^* and V_2^*). Because $V_1^* > V_2^*$ at $N_2^{"}$, based on **Lemma 2**, the zone 2 population will decrease until the utility levels in the zones are equal. Thus, the final equilibrium point is N_2^e: the first-best point! Similarly, if the residents in zone 1 increase, we can show the stability of point a. Hence, the first-best point a is asymptotically stable.[24] That result is summarized as Proposition 2.

Proposition 2. ***Situation*** **II** *always yields the first-best equilibrium even if the residents take account of the BCA method to migrate.*

Hence, in Situation II, neither residents nor developers behave strategically. The reason for this is because even if residents are Stackelberg leaders, the instant adjustment in the level of the policy service with the strategic behavior cancels the benefit of the strategic behavior.

Situation **III**: ***Fixed capital transport services of both route 1 and route 2***

In ***Situation*** **III**, the government improves the fixed capital transport services of both route 1 and route 2. The fixed capital service incurs no depreciation. Figure 3

[24] That implies that as time goes to infinite, then the solution will be the first best.

shows the graphs in this situation. The upper right graph is added to Fig. 2 to reflect the change in route 1. In addition, the thick dotted line $U_1(N_1)$ is added to the lower graph.

In this situation, the first-best point is represented by points f, f', and f'' in Fig. 3. As in ***Situation*** **I**, assume that the number of zone 1 residents increases from the first-best point N_1^e to N_1'. That increase improves the level of transport service to K_1' according to the BCA. Because of this improvement, utility in zone 1, $V_1(N_1, K_1^*)$, shifts upward to $V_1(N_1, K_1')$. As discussed in ***Situation*** **I**, the equilibrium will be at points g, g', and g''. This final equilibrium shows that the transport service level implies $dB/dK_1 < 0$ and $dB/dK_2 < 0$ in the upper graph and that the point is not the first best. If all residents migrate strategically to zone 1 first, the final equilibrium point will be point h, which is also not the first-best situation.

The discussion presented above incorporates only the migration of residents to zone 1. In the case of an increase in zone 2 residents, a similar discussion is possible. If all residents were to migrate to zone 2, the final equilibrium would be point i, which is also not the first-best situation. The result is summarized as Proposition 3.

> **Proposition 3.** *If residents take account of the BCA method to migrate, then* ***Situation*** **III** *always involves a decrease in social welfare from the first-best situation, with an increase in the residents' utility beyond that of the first-best situation.*

As in Proposition 1, residents' strategic behavior implies that they are Stackelberg leaders and that the government conducting the BCA is the follower. BCA inherently supposes that residents are followers in the sense that the residents react to the policy change. Then, the residents' strategic behavior prevents the achievement of the first-best equilibrium. Social welfare decreases with increased taxes for transport services. Absentee landowners suffer a tax increase for "too much" transport service. The amount that the final equilibrium differs from the first-best situation increases with the volume of strategic migration as in ***Situation*** **I**. Similarly to ***Situation*** **I**, rent-seeking developers can support a large migration. The above dynamic inconsistency problem might arise in such a case.

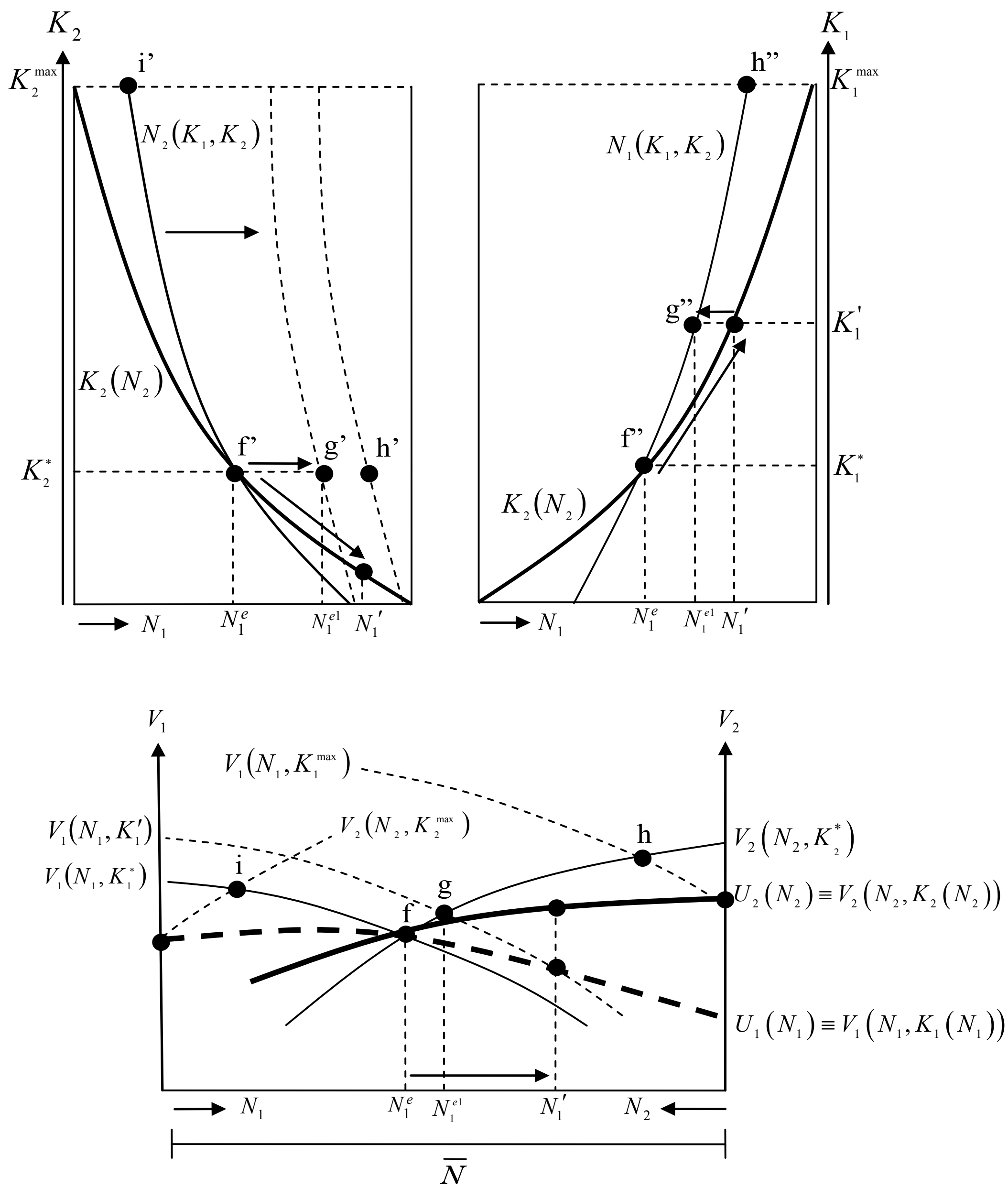

Figure 3. Population allocation, utility and the transport service level (*Situations* III, IV)

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Situation IV: *Variable transport services of both route 1 and route 2*

As in ***Situation*** **III**, in ***Situation*** **IV** the government can change the transport service levels of both route 1 and route 2. However, as distinct from ***Situation*** **III**, the transport service level is a flow service, which is variable at any time. Figure 3 is useful for ***Situation*** **IV**. The variable level means that the transport service level always holds as $dB/dK_i = 0 \ \ (i = 1,2)$ if the level is determined by BCA. Therefore, to explore the equilibrium, it is sufficient to check only the relationship of the utility attainable in zone 1, $U_1(N_1)$, and the utility attainable in zone 2, $U_2(N_2)$, neglecting $V_1(N_1, K_1)$ and $V_2(N_2, K_2)$.

Situation **IV** yields two greatly different results: the first-best point f may be stable or unstable. The stable case can be illustrated as in Fig. 3; the unstable cases are shown in Figs. 4 and 5.

We first discuss the stable case of point f. The lower graph in Fig. 3 shows lines $U_1(N_1)$ and $U_2(N_2)$. The level of $U_2(N_2)$ is higher than the level of $U_1(N_1)$ if the number of the residents in zone 1 increases from point f. This relationship of $U_1(N_1)$ and $U_2(N_2)$ is compatible with all the properties: ***Properties 1L–5L.*** As discussed in ***Situation*** **II**, the combination of ***Properties 2L, 3L, 4L,*** and ***5L*** yield **Lemma 2**, which describes the configuration of lines $U_2(N_2)$, $V_1(N_1)$ and $V_2(N_2)$. A similar relationship (**Lemma 3)** holds regarding lines $U_1(N_1)$, $V_1(N_1)$ and $V_2(N_2)$ if the four properties are reinterpreted by replacing the $U_2(N_2)(\equiv V_2(N_2, K_2(N_2)))$ by $U_1(N_1)$ $(\equiv V_1(N_1, K_1(N_1)))$.

> **Lemma 3** (The configuration of $U_1(N_1)$, $V_1(N_1)$ and $V_2(N_2)$.) To *the left of the first-best point* (*point* f *in* Figs. 3, 4, *and* 5)*, line* $U_1(N_1)$ *lies below line* $V_1(N_1)$ *but above line* $V_2(N_2)$*; and to the right of the first-best point, line* $U_1(N_1)$ *lies below line* $V_2(N_2)$ *but above line* $V_1(N_1)$*, where* $V_1(N_1)$ *and* $V_2(N_2)$ *are abbreviations for* $V_1(N_1, K_1)$ *and* $V_2(N_2, K_2)$ *passing through the first-best point. In addition, the slope of* $U_1(N_1)$ *can be positive with respect to the increase in* N_1.

Here, if the number of residents in zone 1 increases to N_1', the transport service level in zone 1 increases to $K_1'(= K_1(N_1'))$, and the transport service in zone 2 decreases to $K_2(N_1')$. At this new point, the utility level is higher in zone 2 than in zone 1: $U_2(N_1') > U_2(\bar{N} - N_1')$. Hence, the residents in zone 1 migrate to zone 2 until the utility levels are equal between the zones. This migration from zone 1 to 2 continues until point f. Conversely, if the number of residents in residential zone 1 decreases from point f, the level of $U_1(N_1)$ is greater than the level of $U_2(N_2)$. Consequently, through a similar argument, the first-best points f, f', and f'' are asymptotically stable if based on Fig. 3.

We now describe the unstable case of point f. That case is first depicted in Fig. 4. The differences between Figs. 3 and 4 show configurations between lines $U_1(N_1)$ and $U_2(N_2)$. Figure 4 shows that if the number of residents in zone 1 increases from point f, the level of $U_1(N_1)$ is greater than the level of $U_2(N_2)$, in contrast to Fig. 3. This configuration of lines $U_1(N_1)$ and $U_2(N_2)$ in Fig. 4 can be compatible with ***Properties 1L–5L***.

In this configuration, point f is unstable. For example, if the number of zone 1 residents increases to N_1', the transport service level based on the BCA in zone 1 increases to K_1', and decreases to K_2' in zone 2. In this configuration, the utility in zone 1 is higher than in zone 2. This indicates the instability of point f. This increase in the utility that exists in zone 1 resulting from the increase in the number of zone 1 residents further increases the number of zone 1 residents until all residents live in zone 1. A similar argument is applicable to the case of increased zone 2 residents from point f. In summary, in Fig. 4, the final equilibria are at points l and k. The equilibrium implies that all people live in only one zone, either 1 or 2. At point l or k, the utility level of residents is higher than at point f. However, the landowners' profit decreases, which decreases the social welfare.

Figure 5 depicts another unstable case of point f. It differs from Fig. 4 in that the utility level of the residents at the final equilibrium l or k is lower than at point f. This lower utility at the final equilibrium l or k occurs when an expansion of the zone area produces a large decrease in utility. In Fig. 5, the mandatory BCA decreases both the residents' utility and social welfare relative to the first-best situation.

The above results are summarized as Proposition 4.

Proposition 4. *If residents take advantage of the BCA method to migrate, then* ***Situation*** **IV** *has two fundamentally different results: a stable case and an unstable case with respect to the first-best social welfare point. In the stable case, the final equilibrium is the first-best situation. In the unstable case, the social welfare decreases compared with the first-best situation, and the residents' utility can also decrease.*

Whether the first-best social welfare point is stable or not depends on the slopes of $U_1(N_1)$ and $U_2(N_2)$ at the first-best point (see eq. (H) in *Appendix B* for the mathematical expressions of the slopes). In Situation IV, as in Situation II, even if residents are Stackelberg leaders, the instant adjustment in the level of the policy service with the strategic behavior may cancel the benefit of the strategic behavior. However, the adjustment in public services in the two zones can make the first-best solution unstable because the two slopes of the utility lines are simultaneously changed[25] by the adjustment, unlike in Situation II and Proposition 2.

The instability of the first-best point results from the incorporation of the BCA (eq. (13)) into the normal economic system (eqs. (1)–(10)). That is, although the first-best point is stable and in equilibrium based on eqs. (1)–(10) without eq. (13), the introduction of eq. (13) renders the first-best point unstable.[26] The problem is that BCA is intended to assess the efficiency of the economic system from outside the economy, but the mandated BCA requirement incorporates the BCA into the economic system.

[25] In Situation II, unlike in Situation IV, only one slope of the zone 2 utility line is changed by the instant adjustment in the public service level. In this situation, instability does not occur.

[26] Boadway and Flatters (Fig. 2, p. 618; 1982) also show, using a similar diagram of an unstable equilibrium of utilities between regions, that the utility level at a stable corner solution can be lower than the unstable equilibrium of utilities between regions. However, their results are different from ours in our Figs. 4 and 5. Two utility curves in Boadway and Flatters (1982) show the normal utility levels in the two regions (i.e. their curves are corresponding to our V_1 and V_2), whereas our curves incorporate the BCA into the utilities (i.e. $U_1(N_1)$ and $U_2(N_2)$ in our notation). One important point is that our unstable equilibrium of utilities (i.e., point f) is the first best (stable) solution if the BCA is not incorporated into the consumers' behavior.

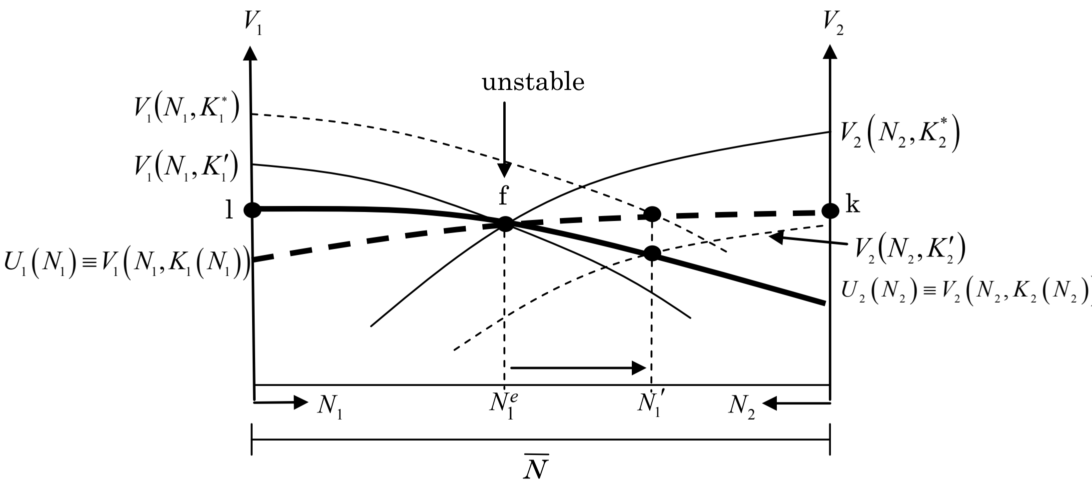
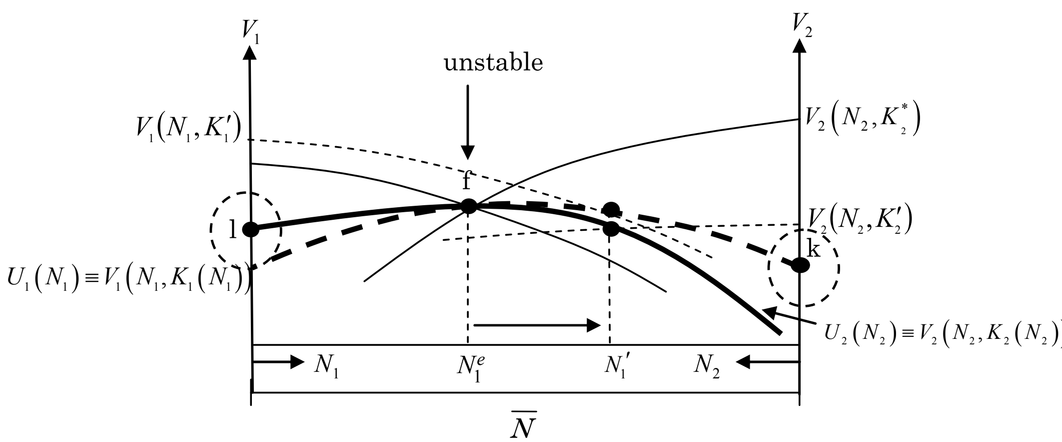


Figure 4. Population allocation and the utility level (*Situation* IV: unstable case 1)



Note: Only the location of points l and k in the two dotted circles is different from those in Fig. 4.

Figure 5. Population allocation and the utility level (*Situation* IV: unstable case 2)

4. Conclusion

This paper discusses dynamic inconsistency problems of BCA-based transport policy, which can decrease social welfare. We examine four situations, arising from

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all combinations of two types of transport service (fixed capital service or variable flow service) and two types of route-improvement scenario (one of the two routes or both routes).

We obtained different results for these four situations, as shown in **Propositions** 1-4. We conclude that the occurrence of second-best outcomes caused by strategic migration depends on the reversibility of the project and the interdependency of the project with another project. First, the difference in the reversibility of the project directly affects the strategic behavior of residents because, as shown in the analysis, the residents' utility levels that are attainable by strategic migration differ for irreversible (or fixed) capital service and reversible (or variable) flow service.[27] These differences cause the differences in outcome between ***Situation*s I** and **II** and between ***Situation*s III** and **IV**. Second, the number of routes that the government can improve is also important, because the existence of more than two routes implies the existence of some dependency between the routes through a general equilibrium link. Such a dependency leads to the difference in outcome between ***Situation*s II** and **IV**. The dependency exists in ***Situation* IV**, in which two routes are improved by the government, whereas no such dependency exists in ***Situation* II**, where only one route is improved. As a result, ***Situation* IV** may yield an equilibrium that is not first-best, whereas ***Situation* II** gives only a first-best equilibrium.

Transportation improvements are considered in the present paper, but general public services including flood control measures like those described in Kydland and Prescott (1977, p.477) would have fundamentally identical structures. Flood control measures such as dams and banks correspond to ***Situation* I** in our analysis. As in Kydland and Prescott (1977), ***Situation* I** has a dynamic inconsistency problem, resulting in too much investment. The flood control measure as well as our "transportation" case has the dependency of the optimal level of public facility on migration, which is the source of dynamic inconsistency. The same dependency can be observed in local environmental projects such as parks and other public flow services (for example, pension services).

To prevent or mitigate the dynamic inconsistency problem of BCA, further study is necessary. The scenario in which all transportation improvement costs are

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[27] Specifically, the residents' respective utilities which are attainable through their strategic migration are expressed by V_i in the case of fixed capital service and by U_i in the case of variable flow service.

paid by "the affected residents" would obviate the dynamic inconsistency problem because residents would not be allowed to be free riders.[28] That scenario is not always feasible, however, because identifying the affected residents is sometimes difficult. For example, it is difficult to identify residents who do not migrate, but increase their utilities because of others' strategic migration. Furthermore, if residents are heterogeneous, payments should be dependent on that heterogeneity so that no resident strategically increases their own utility at the expense of others. Designing this type of payment system would be difficult.

Appendix A: Derivation of the benefit-cost analysis.

The derivative of eq. (11) with eq. (9) and $dN_2 = -dN_1$ yields

$$dB = \sum_{i\in\{1,2\}} N_i dV_i + \sum_{i\in\{1,2\}} d\Pi_i^A - \sum_{i\in\{1,2\}} s_i dK_i \qquad \text{(A)}$$

Combining eqs. (1), (2) and (3) shows

$$V_i = w - \bar{z}\{D_i + c_i\}\,. \qquad \text{(B)}$$

Total differentiation of eq. (B) yields

$$dV_i = -\bar{z}\{dD_i + {\partial c_i}/{\partial K_i}\, s_i\}\,. \qquad \text{(C)}$$

Totally differentiating eq. (6) gives

$$d\Pi_i^A = \pi \bar{z} D_i^2 dD_i\,. \qquad \text{(D)}$$

Then, substituting eqs. (C) and (D) into eq. (A) yields

$$dB = \sum_{i\in\{1,2\}} -N_i \bar{z}\{dD_i + {\partial c_i}/{\partial K_i}\, s_i\} + \sum_{i\in\{1,2\}} \pi \bar{z} D_i^2 dD_i - \sum_{i\in\{1,2\}} s_i dK_i\,. \text{(E)}$$

Considering that $N_i = \pi D_i^2$, eq. (E) is transformed to eq. (12), which expresses the benefit-cost analysis.

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[28] In Britain and Japan, for some types of public infrastructure such as small parks and nearby roads, the infrastructure implementations required by the residents are paid by developers. If such payment perfectly excludes the dynamic inconsistency, it will not generate free riders. However, if developers sold housing only, they would not maintain the level of such infrastructure because of no increase in their profit. In this situation, dynamic inconsistency arises between the developers and residents. Hence, such a payment system also can not perfectly preclude the dynamic inconsistency of the current paper.

Appendix B: Derivation of Properties 1L, 2L, and 3L.

Property 1L is proved as follows. Substituting eq. (1) and $D_i = \sqrt{N_i/\pi}$ into eq. (B) yields

$$V_i = w - \bar{z}\{\sqrt{N_i/\pi} + c(K_i)s_i\}. \tag{F}$$

Equation (F) implies that the utility V_i is expressed as a function of N_i and K_i, that is, $V_i = V_i(N_i, K_i)$ because the other variables w, s_i, and $\bar{z}_i$ are fixed.

Differentiating (F) with respect to the zone population gives

$$\frac{dV_i}{dN_i} = -\frac{1}{2(\pi N_i)^{1/2}}\bar{z}. \tag{G}$$

Equation (G) implies ***Property 1L:*** the derivative of V_1 and V_2, with respect to the respective zone populations, is negative because of the expansion of the zone area (= the increase in rent at each point).

Properties 2L and ***3L*** are derived as follows. U_2 is the achievable utility level in residential zone 2 when the transportation facility level is based on the BCA, $U_2(N_2) \equiv V_2(N_2, K_2(N_2))$, where $K_2(N_2)$ expresses the BCA-based level of transport service, K_2, satisfying eq. (13). The change in utility U_2 with respect to the change in N_2 is

$$\frac{\partial U_2}{\partial N_2} = \underset{(-)}{\frac{\partial V_2}{\partial N_2}} + \underset{(+)}{\frac{\partial V_2}{\partial K_2}} \underset{(+)}{\frac{\partial K_2(N_2)}{\partial N_2}}. \tag{H}$$

In that equation, $\partial K_2/\partial N_2$ expresses the change in the optimal level of transport service with respect to the increased population. The sign of this term, $\partial K_2/\partial N_2$, is easily shown to be positive by totally differentiating eq. (13) with respect to K_2 and N_2 and rearranging slightly. The sign of $\partial V_2/\partial N_2$ is negative from eq. (G). Consequently, from eq. (H), $\partial U_2/\partial N_2 > \partial V_2/\partial N_2$ because $\partial V_2/\partial K_2 \cdot \partial K_2(N_2)/\partial N_2 > 0$. That implies ***Property 2L:*** the derivative of V_2 with respect to N_2 is less than the derivative of U_2 with respect to N_2 because the level of the transport service, K_2, based on the BCA, i.e. $K_2(N_2)$ in $V_2(N_2, K_2(N_2))$, is increasing with respect to the increase in zone 2 population. In addition, the derivative of $U_2(N_2)$ is not necessarily negative. That derivative can be positive if, in eq. (H), $|\partial V_2/\partial K_2 \cdot \partial K_2(N_2)/\partial N_2| > |\partial V_2/\partial N_2|$. This is ***Property 3L***.

Appendix C: Derivation of Property 5L from Property 2U.

The derivation can be illustrated simply using Fig. 2. First, take one population allocation as an example using Fig. 2. Selecting population allocation N_2^{e1} as a population allocation to the left of point a or a', the transport service given that population allocation at equilibrium, $K_2^{"}$ is greater than the optimal transport service level based on BCA, $K_2(N_2^{e1})$; that is, $K_2^{"} > K_2(N_2^{e1})$, as shown in the upper graph. At that population allocation, the equilibrium condition $V_1(\overline{N} - N_2^{e1}) = V_2(N_2^{e1}, K_2^{"})$ and definition $U_2(N_2^{e1}) \equiv V_2(N_2^{e1}, K_2(N_2^{e1}))$ hold. Because $K_2^{"} > K_2(N_2^{e1})$, $V_1(\overline{N} - N_2^{e1}) = V_2(N_2^{e1}, K_2^{"}) > V_2(N_2^{e1}, K_2(N_2^{e1})) \equiv U_2(N_2^{e1})$. Therefore, $V_1(N_1)$ is greater than $U_2(N_2)$.

At any level of transport service to the right of point a or a', the same logic is applicable, hence, to the right of point a, line $V_1(N_1)$ lies below line $U_2(N_2) \equiv V_2(N_2, K_2(N_2))$ in the lower graph. Consequently, ***Property 5L*** follows from ***Property 2U***.

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