A School Method for Calculating a Table of Sines to Three Places for Integral Values of the Angle.

The two standard formulae required are used in the forms

$$\cos A = \sqrt{\{\tfrac{1}{2}(1 + \cos 2A)\}} \qquad \ldots\ldots(1)$$

$$\sin \tfrac{1}{2}(A + B) = \tfrac{1}{2}(\sin A + \sin B) / \cos \tfrac{1}{2}(A - B) \qquad \ldots.(2)$$

The values corresponding to 0°, 30°, 45°, 60°, 90° are assumed known.

By (1), using $A = 15°$, compute cos 15° to four figures, and apply the answer in (2) ($A = 0°$, $B = 30°$) to find sin 15°. sin 75° = cos 15°. Tabulate $\sin A$ at intervals of 15°, leaving room for three intermediate entries.

By (1), using $2A = 15°$, compute cos $7\frac{1}{2}°$. Divide the answer into the averages of pairs of sines already tabulated. Complete the table for intervals of $7\frac{1}{2}°$.

Repeat the process for intervals of $3\frac{3}{4}°$. This gives a table of 25 values, which should now be entered to three decimals. Since each difference corresponds to 15 quarters of a degree make a difference table by taking 1/15 of the successive differences. Apply to compute values corresponding to each integral value of the angle by proportional parts. The final table should agree with the printed table within an error of 1 unit in the third decimal. Alternatively a large scale graph may be drawn in sections and the required values read off; this however requires longer time, some skill, and is less reliable.

The above process is well within the capacity of a fifth year class, and the entire calculation can be done in two school periods if the work is suitably distributed, even when no contracted methods are used. Important simplifications however can be introduced, if desired. Thus when x is small it can be shown by squaring that the approximation $\sqrt{(1 - x)} = 1 - \frac{1}{2}x - \frac{1}{8}x^2$, in which the last term should be computed mentally, leaves negligible terms for numbers between cos 15° and 1. Again the factor 1/cos $7\frac{1}{2}° = 1/{\cdot}9914$ may be applied in the form $1 + \frac{1}{100}(\frac{1}{2} + \frac{1}{4} + \frac{1}{10})$ and the factor (used 12 times) 1/cos $3\frac{3}{4}° = 1/{\cdot}9979$ in the form $1{\cdot}002$.

The method can be applied successfully to any degree of accuracy. As the interval is reduced, so is the labour of computing both square root and quotient because the cosine divisor approaches 1.

The interpretation of formula (2) as here applied may be stated in the general form.

In a series of sines of angles which are in arithmetical progression the ratio of the sine of any angle midway between an adjacent pair of angles to the mean of the sines of those angles is constant and is equal to the secant of half the common difference.

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