Comets

Short-Period Comet Production in Close Encounters with Jupiter

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Abstract: A method for calculating the resultant probability distributions of orbital elements for a small body (a comet, asteroid or meteoroid) after a gravitational encounter with a planet is described. This technique incorporates the frequency of such encounters so that the chance of attaining a certain new orbit per unit time is derived. The use of this technique is illustrated by considering the effect of Jupiter upon the orbits of near-parabolic comets with perihelia near that planet ($q = 5.2$ AU) and in the inner solar system ($q = 1.0$ AU), with prograde ($i = 10^\circ$) and retrograde ($i = 170^\circ$) paths. As indicated by previous authors the prograde comets are more easily captured into short-period ($P < 20$ yr) and intermediate-period ($20 < P < 200$ yr) orbits; however, in contrast to most previous work in agreement with the results of Stagg and Bailey (submitted to Mon. Not. R. Astron. Soc.) it is found that the comets with smaller perihelia, rather than those with perihelia near Jupiter, have higher capture probabilities. This is apparently due to the fact that a small deflection only is needed to sufficiently decelerate a comet onto a smaller orbit if it makes a near-perpendicular crossing of Jupiter's path, whereas a larger deflection (to achieve a large orbital change) is needed if the paths are near-parallel. With comparatively modest amounts of computer time this method may be used to calculate the relative capture probabilities as a function of $i$ and $q$ for all values of interest, and is thus a useful precursor to integrations following orbital evolution, since it indicates the most likely avenues whereby shorter-period comets are derived from the near-parabolic flux.

Introduction

The question of the origin of short-period comets is a problem which has attracted astrophysicists for many years. Tisserand (1889) and Callandreau (1892) investigated the possibility that such orbits are created when long-period comets pass close to Jupiter, with the cometary orbital energy occasionally being reduced sufficiently that the period is much shortened. It seems likely that the idea was not new even at that time; Laplace is usually credited with being the first person to suggest this possibility. Newton (1893) showed that the probability per perihelion passage of a randomly-oriented comet in a parabolic orbit being captured into an orbit of semi-major axis less than some value $a$ is

$$P_{\text{cap},a} = C_a (M_P/M_\odot)^2 (a/a_p)^2$$

(1)

where $M_P$ and $a_p$ are the mass and semi-major axis of the perturbing planet, $M_\odot$ is the solar mass, and $C_a$ is a slowly-varying function of $a$ ($C_a = 4/3$ for $a > 3a_p$). Newton considered only the effect of Jupiter, which is generally the major perturbing planet for comets which pass within 6 AU of the Sun; this work was extended by Russell (1920) who included the other planets. This formula allowed the first estimates to be made of the production of short-period (SP) comets from long-period (LP) orbits. [Throughout this paper we will refer to SP orbits as being those of period $P < 20$ yr, Intermediate Period (IP) of $20 < P < 200$ yr, and LP those of $P > 200$ yr (cf Kresak 1982; Fernández 1984); the term ‘capture’ is used to imply a transformation from a LP orbit to a stipulated smaller orbit (i.e. either SP or IP) with additional conditions also being at times applied to the perihelion distance, $q$]. The results of Newton and Russell were subsequently used by various authors (e.g. van Woerkom 1948; Vsekhsvyatskii 1962, 1967; for reviews of this work see Bailey et al. 1986, 1989; Stagg and Bailey 1989; Weissman 1985) in order to compare different ideas upon the origin of the SP and IP comets; in particular it was possible to show that capture into such orbits in single close encounters with any of the planets is unlikely, with many encounters being necessary in general.

With the advent of modern electronic computers it was possible to expand upon this work so as to investigate how the capture probabilities vary with the orbital parameters, in particular the inclination ($i$) and perihelion distance ($q$), of parabolic comets. In this connexion the work of Everhart has been especially significant in advancing our understanding of the process whereby comets may be captured from LP into IP or SP orbits. Everhart (1968, 1969) investigated the changes in orbital energy (i.e. semi-major axis) caused in parabolic initial orbits by the action of single planets, and showed that Newton’s formula was correct but that his values for the constant were slightly wrong for the smaller orbits ($a < 3a_p$). Everhart (1972, 1973a) discussed the implications of these results for hypotheses of the origin of SP comets, and later expanded his analysis so as to include the effect of the initial orbit being elliptical (Everhart 1977), which allowed him to describe in general terms the general orbital evolution of all types of comet (Everhart 1982).

Similar approaches to that of Everhart have also been used by a number of different authors (e.g. Rickman and Vaghi 1976; Froeschlé and Rickman 1980; Fernández 1981, 1982; see also Carusi and Valsecchi 1987), and it is clear that a number of substantial problems still exist which require solution before we can feel secure that our understanding of the origin of SP/IP comets is broadly correct. For example, there is apparently an excess of retrograde over prograde comets in the discovered LP population (as listed in Marsden 1986), as has been pointed out by several authors (e.g. Fernández 1981, 1982): is this excess real, or is it a statistical fluke? If real, is it due to retrograde comets being anomalously bright (possibly due to more-frequent impacts with interplanetary meteoroids), or is it due to the prograde LP comets being preferentially lost from that population as a result of captures/ejections caused by the planets? What are the rôles of temporary satellite captures and horseshoe orbits in the overall evolutionary sequence (Kazimirchak-Polonskaya and Shapov 1976; Everhart 1973b, 1982; Carusi and Valsecchi 1981)? If it is possible to explain the origin of SP/IP comets from initially near-parabolic orbits, then it is possible to use this knowledge in order to constrain models of the Oort Cloud, and also to more fully understand the impact rate of extraterrestrial objects upon the Earth, as has been discussed by Bailey and Stagg (1988) and Stagg and Bailey (1989)? Does the reservoir of comets thought to exist
beyond the planetary region conform to the initial model proposed by Oort (1950), at heliocentric distances of $10^4 - 10^5$ AU, or does it exist with an inner core at $10^3 - 10^4$ AU, or does it exist with an inner core at $10^3 - 10^4$ AU as proposed by several authors (e.g. Fernández 1985a; Weissman 1985)? Is such an inner core flattened, and is there a belt of comets just beyond the planetary system, at 50-100 AU, the Kuiper Cloud (Kuiper 1951; Whipple 1964, 1972, 1975; Mendis 1973; Fernández 1980; Hills, 1981; Bailey 1983a,b; Olsson-Steel 1988a; Jackson and Killen 1988)? These are a few of the questions which need to be attacked, and are of central importance to our understanding of the origin and evolution of the solar system: for example the existence of a cometary belt just beyond Neptune could be viewed as an important indicator of how the solar system formed, since it is believed that the 'conventional' Oort Cloud derived from comets ejected from the outer solar system as the planets in that region were forming, so that their compositions do not represent the undifferentiated stuff of the solar nebula—whereas the Kuiper Cloud comets would still be orbiting in their formation zone.

A completely rigorous solution to the problem of the origin of the SP/IP comets would require the precise numerical integration of the orbits of a very large number of randomly-distributed initially-parabolic orbits, and in the present epoch this is not possible even using dedicated parallel processors such as that constructed by Applegate et al. (1986) (see also Sussman and Wisdom 1988), due to the huge number of calculations required. One method of overcoming this problem is to arbitrarily increase the planetary mass input to an integration program so that the orbital evolution of the comets considered is accelerated, making numerical integrations feasible. Duncan et al. (1988) have used this approach, incrementing the masses by an order of magnitude, and interpret their results in terms of the majority of SP comets originating in a flattened belt just beyond Neptune (i.e. the Kuiper Cloud); however, Stagg and Bailey (1989) have criticized the method used by Duncan et al., claiming that it underestimates the importance of the rare, but dynamically significant, very close encounters.

Everhart's approach, later followed by other researchers, was to construct a distribution of changes in the orbital energy of $10^4$ random hypothetical comets which he followed through the solar system, accumulating their orbital changes in close planetary encounters. He then followed the changes in other orbits by choosing perturbations at random from this distribution. Assuming that $i$ was constant whilst the change in $q$ could be calculated by assuming conservation of the Tisserand parameter,

$$C = \frac{a_p}{a} + 2 \cos \frac{i}{2} \left(\frac{q - 2}{a_p}\right)^{1/2} = \frac{a_p}{a} + 2 \cos \frac{i}{2} \left(\frac{a (1 - e^2)}{a_p}\right)^{1/2}$$

where $e$ is the orbital eccentricity. This Tisserand parameter is a near-conserved quantity over the duration of an encounter (close or distant) between a small-mass object (such as a comet, asteroid or meteoroid) and a planet, and so is often referred to as being the 'Tisserand Invariant' or 'Tisserand Constant' (e.g. see Carusi and Valsecchi 1985); however, the finite eccentricities of the planetary orbits lead to its non-conservation, especially over the course of many encounters (Opik 1976); when more than one planetary orbit is crossed, the value of $C$ referred to a particular planet will by no means be conserved, and thus a comet may be passed from the control of one planet to another (Everhart 1973a; 1982). Everhart's technique has been discussed in more detail by Stagg and Bailey (1989).

One important conclusion found by Everhart was that the low-inclination comets have much higher probabilities of being captured, and so he considered initial inclinations of $i < 9^\circ$ only in constructing his perturbation distributions: that is, his results would imply that the capture probability is zero for all $i > 9^\circ$.

However, more recent analyses (e.g. Fernández and Ip 1983) have shown that Everhart's capture probabilities could be applicable for inclinations as high as $30^\circ$, in which case the net capture probability could be an order of magnitude higher than Everhart's values, emphasizing that we still know little of the way in which the observed distribution of SP and IP comets have come about. Equally well, Everhart (1968) found that the capture probability is highest if the comet has perihelion close to one of the planets, and in particular near Jupiter (i.e. $4 < q < 6$ AU). This is to be expected, since the frequency of close encounters goes up markedly for such a configuration (e.g. see Olsson-Steel 1987b), and in addition for prograde orbits the encounter velocity is then minimized, which leads to the gravitational interaction causing a larger change in the cometary orbit (e.g. Carusi and Valsecchi 1980); in the same situation a retrograde orbit has its encounter velocity maximized, so that the orbit change is very small (cf. results discussed later in this paper).

It should not be thought that the questions being asked here involve only comets in exclusion: there is now a realization that the overall complex of planet-crossing bodies is derived at least in part from the influx of LP comets which are captured in SP/IP orbits. Until recently it had been believed that SP comets were the major source of interplanetary dust and small meteoroids (Olsson-Steel 1986) but it has now been recognized that LP comets can make a major direct contribution to the population of smaller particles (Fulle 1987, 1988), and also that there are meteoroid streams associated with several Apollo asteroids which may be either collisional debris or remnant tails from when the asteroids were active cometary nuclei (Olsson-Steel 1988b). The physical decay of comets, with concomitant production of meteoroids and dust, has been the subject of much study (e.g. Kresák 1981; Fernández 1985b), as has the orbital evolution of SP/IP comets and their dynamical relationship to planet-crossing asteroids (Carusi et al. 1985; Belyaev et al. 1986; Hahn and Rickman 1985). The physical relationship between Earth-approaching asteroids and comets has also been the subject of many investigations—see Hartmann et al. (1987) or Degewij and Tedesco (1982) for reviews. In turn the meteoroids released maintain the zodiacal dust cloud through catastrophic collisions (Whipple 1967; Leinert et al. 1983; Steel and Elford 1986; Olsson-Steel 1986), whilst it has recently been discovered that there are various other mechanisms and sources which can help replenish this cloud (e.g. Sykes and Greenberg 1986; Gustafson et al. 1987).

Ideally, one would like to be able to solve this longstanding problem in the 'brute force' way, by following a very large number of hypothetical random parabolic comets from their dislodgement from the Oort Cloud (although there is still much debate about the form and extent of that reservoir: see for example Weissman 1985; Bailey and Stagg 1988; Marochnik et al. 1988) through successive passages through the planetary region until such time as a SP/IP orbit is attained or, more likely, the comet suffers a planetary impact or rejection from the solar system. However, even if a statistically-valid number of integrations of orbits with differing $i$ and $q$ were possible, there would still be other associated problems; for example, do the non-gravitational effects continue in the same sense for many orbital revolutions? Equally well, how many SP comets are
produced by the fragmentation of LP comets as they pass through the Roche Radius of a planet? This may be especially prevalent if comets in general are of as low density as is P/Halley (Rickman 1986; Sagdeev et al. 1988).

Thus it is clear that there are many questions still to be answered, or even addressed, regarding the supply of comets into SP/IP orbits. In this paper we initiate a program which is directed towards answering one of the central problems: how does the capture probability vary with initial inclination and perihelion distance?

**Method**

The method used in the present paper has been described in detail elsewhere and will only be briefly discussed here. The first step is to determine, for a particular planet and a certain input hypothetical cometary orbit, the frequency of close encounters. The method for achieving this was developed by Kessler (1981) in order to calculate the probability of a collision between two arbitrary orbiting objects, and the algorithm was also given by Steel and Baggaley (1985). The probability of a collision is

\[
P_c = \sum_j S_{1,j} S_{2,j} V_j \sigma_c \Delta U_j
\]  

(3)

where the summation is over all j volume elements \( \Delta U_j \) which are accessible to both objects, \( S_{1,j} \) and \( S_{2,j} \) are the spatial densities of the objects in that volume element, \( V_j \) is the relative velocity of the two, and \( \sigma_c \) is the collision cross-section, which will be a function of \( V_j \) due to gravitational focusing; its value is

\[
\sigma_c = \pi R_p^2 \left( 1 + \left( V_e / V_j \right)^2 \right)
\]  

(4)

where \( R_p \) is the radius of the planet and \( V_e \) is its surface escape velocity. This technique makes the assumption that the longitude of the node and the argument of perihelion of both objects are random; this is valid in almost all cases (and certainly for comets crossing many planets) but may be violated in the case of stable liberations or resonances, as displayed (for example) by the Trojan asteroids or some Earth-crossers.

The heliocentric space which can be occupied by both objects is split up in terms of the radial distance from the Sun, and also the latitude; the ecliptic latitude may be used (i.e. with the inclination referred to the ecliptic), but it is better to reference the inclination to the planet in question since there is only one latitude bin (latitude \( \beta = 0^\circ \) since the planet is confined to that plane) to be considered, so that the integration is over radial distance only, and the execution time is consequently shortened. Steel and Elford (1986) gave an alternative expression for calculating the spatial density in any position which has some advantages in certain situations, and calculated the collisional lifetime of meteoroids against catastrophic collisions with zodiacal dust particles. Using the same technique Steel and Baggaley (1985) found the impact rate with each of the terrestrial planets for the discovered population of Aten-Apollo-Amor asteroids, and Olsson-Steel (1987b) calculated the collision probabilities with each of the planets for all known comets.

The above expression applies to the case of a collision. In reality there are many close approaches to a planet which do not result in an actual impact, and the frequency of these can be calculated by putting the appropriate cross-section into equation (3). Most often the definition of a close encounter with a planet is taken to be passage within the so-called 'sphere of influence' of that planet, which may be defined in various ways (see Olsson-Steel 1986, 1987a); here the expression

\[
s_d = 1.15 \left( M_p / M_0 \right)^{1/3}
\]  

(5)

(where \( r \) is the Sun-planet distance at any time) is used for the radius of this sphere, so that the encounter cross-section is

\[
\sigma_d = \pi s_d^2
\]  

(6)

The meaning of this is that at some large distance from the planet ('large' meaning sufficiently distant that the comet's path is not yet deviated by the planetary gravitational field), the instantaneous trajectory would have a maximum impact parameter \( s = s_0 \). In reality if an encounter occurs then the distance at closest approach is smaller than this due to the path deviation produced by the planet. Since \( (M_p/M_0)^{1/3} = 0.000954 \) when \( r = a_{Jupiter} = 5.2 \) AU one finds \( s_0 = 0.59 \) AU = 1238 × \( R_{Jupiter} \).

In the past there have been various approaches to the problem of estimating the change in a cometary orbit in a close encounter (e.g. see Everhart 1969; Weidenschilling 1977; Opik 1976) which have varying degrees of merit (in terms of precision and processing time); as is most often used by others, the interaction approximation we have used is simply the gravitational analogue of Rutherford Scattering in atomic physics, whereby

\[
s = \left( GM_p / V^2 \right) \cot(\chi / 2)
\]  

(7)

where \( G \) is the universal constant of gravitational and \( \chi \) is the angular deflection produced by impact parameter \( s \); here equation (7) is used to calculate for given values of \( s \). Note that the use of this formula implies that the planetocentric velocity as the comet exists the sphere of influence is the same as at which it entered, which also means that the Tisserand parameter (with respect to that planet) is unaltered. This model is often termed the 'two, two-body approximation': before entry to (and after exit from) the sphere of influence the cometary orbit is assumed to be an unperturbed heliocentric ellipse, whilst the trajectory within the sphere is taken to be a planetocentric hyperbola. Although it might seem that this interaction model is too simplistic to be of use, in fact recent comparisons between exact integrations and the results gained along these lines have shown that it may be used with confidence in all but the slowest, closest encounters (Greenberg et al. 1988). A similar approach to this has also been described by Torbett (1986).

The geometry of the situation is shown in Figure 1. Line \( PP' \) is in the plane of the planet's orbit, and that body is assumed to be a scattering centre at \( C \) which is momentarily moving on a circular orbit towards \( P' \); \( P-C-Sun \) is thus a right angle. The small solid circle represents the geometrical cross-section of the planet; the small dashed circle encompasses the collisional radius \( s_q \) of the planet (i.e. \( s_q \) is the impact parameter which would produce a grazing impact, and \( s_q = \sqrt{(\mu / \pi)} \)), whilst the large dashed curve is of radius \( s_g \). The line \( XX' \) is parallel to the plane of the comet's orbit, and passes through the centre of the planet, and \( CZ \) is perpendicular to \( XX' \) (i.e. \( CZ \) points to the pole of the cometary orbit); in general the plane containing \( XX' \) is not in the same as that containing \( PP' \). The area between \( s_q \) and \( s_g \) is then divided up into small segments as shown by the arcs and radial lines in the bottom half of Figure 1. Equal angular divisions in \( \theta \) are used (the origin for \( \theta \) is arbitrary but has been shown here as being measured from \( CZ \)), but the divisions in \( s \) are not equal, since this would lead to undersampling of the closer approaches (which result in the largest orbital changes): thus equal divisions in angular deflection \( \chi \) are utilized (cf equation 7). In the diagram point \( A \) represents a particular trajectory at a large distance from the planet (i.e. the impact parameter to be used would be equal to the length \( CA \)), whilst \( B \) is the point at which the comet's actual path cuts the plane containing \( X, X', Z, \) and \( C \).
Figure 1 — The geometry of the close encounter computations. PP' is in the plane of the planet's orbit, and is at right angles to the direction of the Sun: the planet is assumed to be a scattering centre at C which is instantaneously on a circular orbit (i.e. its velocity vector is directed towards P'). XX' is a line parallel to the comet's orbital plane, whilst CZ points to the pole of the cometary orbit. The plane containing X, X' and Z is perpendicular to the comet's velocity vector; in general PP' is not in this plane. The impact parameters s have values ranging from zero to \( s_d \), which is the radius of the sphere of influence of the planet, as derived from equation (5); any \( s < s_d \) results in an impact upon the planet. For a particular encounter the impact parameter might be \( s = s_A \), with the position of \( A \) being described by angle \( \theta \); due to the gravitational attraction of the planet the comet would actually pass it at a closest approach distance \( CB \). The encounter cross-section is split up into many segments of known area, as indicated in the lower part of the diagram: each of these is described by a particular \((s, \theta)\).

In each volume element of heliocentric space occupiable by both planet and comet, all segments of the sphere of influence are considered and for each (such as the blackened segment in Figure 1) the new elements \((a, e, i)\) which would result given the particular values of \( s \) and \( \theta \) are calculated via the algorithm fully described by Olsson-Steel (1987a). Knowing the cross-section for each segment it is possible to substitute this into equation (3) and thus accumulate probability distributions of new orbital elements as a result of these close encounters; it is assumed that in reality the trajectory taken by any comet is random so that all possible positions for \( A \) are equally likely. In most cases the majority of the 'close encounters' are ineffective in changing the orbit, so that the distributions have peaks near the input cometary elements; it is the tails of the distributions, resulting from the closer approaches, which are of most interest.

Some indication of the processing time required for the technique described in this paper is necessary should others be intending to apply it to similar problems. Obviously in the general case the process involves a numerical integration which is four cycles deep; that is, over (i) heliocentric distance, \( r \); (ii) (ecliptic) latitude, \( \beta \); (iii) all impact parameters, \( s \); and (iv) all planetocentric inclination angles, \( \theta \). By referring the cometary orbit to the inclination of the planet's orbit, only \( \beta = 0^\circ \) is possible, which effectively obviates cycle (ii). However, the amount of processing time is still appreciable unless comparatively coarse divisions in the other three parameters are used. Typically we have used \( \Delta r = 0.05 \text{ AU} \), \( \Delta \beta = 4^\circ \), and \( \Delta s = 0.02 \) (impact parameter \( s \) is derived from \( \chi \) via equation 7). It was found that the use of smaller divisions did not materially affect the resultant probability distributions. Typically the results presented here then required of the order of 3 minutes of CPU time on a DEC VAX-785: more efficient programming and/or the use of a lower-level language (Pascal was used here) would undoubtedly reduce this considerably. The processing time necessary is thus very much shorter than the hours or days (or longer) which are needed in numerical integrations of cometary orbits.

This technique was used initially by Olsson-Steel (1986) in order to show that sporadic meteoroids are produced by the disruption of meteoroid streams in close approaches to the planets, in particular Jupiter. Following this the effects of close approaches upon the orbits of 966 Hidalgo and 2060 Chiron, both of which cross two of the giant planets, were investigated (Olsson-Steel 1987a). Next the technique was used to prove that P/Halley has an orbit which is dynamically extremely stable, with minor changes being expected over time-scales of order \( 10^3 \text{ yr} \) or more (Olsson-Steel 1987c), and later to indicate that the most likely origin of that comet is in the Kuiper Cloud (Olsson-Steel 1988a). The results of close encounters between Neptune and Pluto, which may occur if the presently-observed resonance is not stable over the age of the solar system (Sussman and Wisdom 1988), were investigated in another paper (Olsson-Steel 1988c), and it was shown that if Pluto is not 'protected' from close encounters then severe orbital disruption is expected on an astronomically-brief time-scale \((\lesssim 2 \times 10^5 \text{ yr})\). Similarly it has been shown that the handful of discovered (non-Trojan) Jupiter-crossing asteroids have dynamical lifetimes of only \( \lesssim 10^4 - 10^5 \text{ yr} \) (Olsson-Steel 1988d), and thus require constant replenishment: These objects may well be recently-extinguished or dormant comets (Olsson-Steel 1987d, 1988e).

**Application to long-period comet orbits**

In this initial paper our main intention is merely to describe the method which is being used in order to investigate the orbital changes, capture probabilities etc. which occur as a result of close encounters between LP comets and the planets, and then to illustrate the power of the method with a few sample LP orbits. In this section we present the results of our calculations for four hypothetical LP orbits, and in the following section discuss the implications of these results as regards the origin and evolution of SP/IP comets in general.

Our sample orbits are made up of the four possible combinations of the elements \( (q = 1.0, 5.2 \text{ AU}; i = 10^\circ, 170^\circ) \). The value \( q = 5.2 \text{ AU} \) is chosen since this is the value of \( a_{	ext{Jupiter}} \), and as discussed in the introduction this order of perihelion distance is believed to have the highest capture probability; note that the full range of heliocentric distances for the planet is generally used in computing the probability distributions, but in this case the value of \( q \) means that only encounters at heliocentric distances \( 5.2 < q < 5.455 \text{ AU} \) are possible; \( 5.455 \text{ AU} \) is the aphelion distance of Jupiter. The value of \( q = 1.0 \text{ AU} \) is used to contrast with this higher initial perihelion distance.

Following Stagg and Bailey (1989) we have used a cut-off of \( q \leq 1.5 \text{ AU} \) in order to define 'easily observable SP/IP comets', since it is near that point that the SP/IP \( q \)-distribution maximizes, even though the onset of formation of a water-derived coma occurs near 3 AU. [Note that Duncan et al. (1988) preferred to use \( q \leq 2.5 \text{ AU} \) as their limit]. Thus it is possible for the \( q = 1.0 \text{ AU LP comets} \) to become observable SP/IP comets with just a reduction in semi-major axis \( (a) \), whereas for the same to be true for the \( q = 5.2 \text{ AU LP comets} \) a reduction in both \( q \) and \( a \) is needed. The value of \( i = 10^\circ \) is used in line with the expectation that such low-inclination orbits, with small planetocentric encounter velocities, are the most efficiently captured; similarly as above the value \( i = 170^\circ \) is chosen as a contrast.
Table 1: Orbital parameters, collision, encounter and ejection probabilities, and minimum and maximum encounter velocities with Jupiter, for the four near-parabolic comets.

<table>
<thead>
<tr>
<th>Orbit</th>
<th>Perihelion Distance (AU)</th>
<th>Inclination (degrees)</th>
<th>( P_{\text{coll}} ) (per orbit)</th>
<th>( P_{\text{enc}} ) (per orbit)</th>
<th>( P_{\text{eject}} ) (per orbit)</th>
<th>( V_{\text{min}} ) (km/sec)</th>
<th>( V_{\text{max}} ) (km/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5.2</td>
<td>10</td>
<td>( 1.46 \times 10^{-6} )</td>
<td>( 2.76 \times 10^{-2} )</td>
<td>( 1.30 \times 10^{-2} )</td>
<td>6.1</td>
<td>7.0</td>
</tr>
<tr>
<td>B</td>
<td>5.2</td>
<td>170</td>
<td>( 3.90 \times 10^{-7} )</td>
<td>( 1.32 \times 10^{-1} )</td>
<td>( 6.44 \times 10^{-2} )</td>
<td>30.2</td>
<td>31.5</td>
</tr>
<tr>
<td>C</td>
<td>1.0</td>
<td>10</td>
<td>( 2.06 \times 10^{-7} )</td>
<td>( 2.47 \times 10^{-2} )</td>
<td>( 1.23 \times 10^{-2} )</td>
<td>16.7</td>
<td>18.1</td>
</tr>
<tr>
<td>D</td>
<td>1.0</td>
<td>170</td>
<td>( 1.48 \times 10^{-7} )</td>
<td>( 3.80 \times 10^{-2} )</td>
<td>( 1.86 \times 10^{-2} )</td>
<td>25.8</td>
<td>27.9</td>
</tr>
</tbody>
</table>

Figure 2 — Probability distributions for new orbital elements for near-parabolic Orbit A (\( q = 5.2 \) AU, \( i = 10^\circ \)) resulting from close encounters with Jupiter. All four plots give the logarithm to the base 10 of the probability per million orbits (or per \( 10^{12} \) yr since the original period is \( 10^6 \) yr). For the semi-major axis and the perihelion distance the probability is per AU in each of these elements; for the eccentricity it is per step of 0.01 in \( e \); for the inclination it is per degree. The solid line shows the probability distribution for all periods \( P \); the dashed line is for all \( P < 200 \) yr (i.e. \( SP + IP \)); the dotted line is for all \( P < 20 \) yr (i.e. \( SP \) only).

In all four cases the semi-major axis is \( a = 10^4 \) AU, which is used for the sake of simplicity since then the period is \( P = 10^6 \) yr. The eccentricities calculated from \( a \) and \( q \) are then \( e = 0.99990 \) for \( q = 1.0 \) AU, and \( e = 0.99948 \) for \( q = 5.2 \) AU, both of which are sufficiently close to being parabolic as to make no discernable difference to the results gained with the technique used here. The only planet considered in the present computations is Jupiter: the effect of the Earth (or Mars) for the \( q = 1.0 \) AU orbits is very small compared to that of Jupiter, whilst the other giant planets (which have significant but smaller influences upon such orbits as used here, as will be shown in later papers) are also excluded for the sake of brevity.

In Table 1 the four orbits are listed and are denoted A to D for ease of future reference. Also given in that table are the probabilities of a collision or encounter (passage within the sphere of influence) with Jupiter occurring, the probability of an ejection on a hyperbolic heliocentric orbit resulting, and the minimum and maximum possible encounter velocities. Note that the only new orbits which contribute to the probability distributions plotted in Figures 2-5 are those which are elliptical: the ejections are excluded. Since these orbits are very close to being parabolic, only a very small acceleration is needed for an ejection to occur and thus just less than half of all encounters result in an ejection (i.e. \( P_{\text{enc}} \approx 2 \times P_{\text{eject}} \); passage behind the planet results in an acceleration—an increase in the comet's heliocentric orbital energy—whilst passage in front decelerates the comet). Neglecting gravitational focussing the encounter probability scales as the relative velocity (equation 3: note that the spatial densities at all points are identical for orbits with complementary inclinations—e.g. \( i = 10^\circ, 170^\circ \)—if the other elements are the same) so that, for instance, \( P_{\text{enc}} \) (Orbit B) \( = 5 \times P_{\text{enc}} \) (Orbit A) since the encounter velocity of B is about five times that of A. This does not, however, apply to the relative values of \( P_{\text{coll}} \); for impacts upon Jupiter the gravitational focussing is very significant since the escape velocity of that planet (\( V_e = 61 \) km/s) is much higher than the encounter...
velocities (cf equation 4), and in fact the slower encounters (low inclination orbits) generally have higher values for $P_{\text{enc}}$. This is the reverse of the situation for the terrestrial planets (for details see Olsson-Steel 1987b).

In Figure 2 we present the distributions of new elements $a$, $e$, $i$ and $q$ for input orbit A. In each plot the solid line shows the distribution for new orbits of any period (i.e. SP + IP + LP comets), the dashed line shows only those orbits of $P<200$ yr (i.e. SP + IP comets), whilst the dotted line is for $P<20$ yr only (SP comets only). The total area under the solid line (all periods) is equal to the value of $P_{\text{enc}}$ given in Table 1, except that the full range of new semi-major axes ($0<a<\infty$) cannot be plotted.

The distribution for $a$ is quite erratic for $a>20$ AU, and this is largely due to the binning used in accumulating the distribution. The minimum value of $a$ attained is near 3 AU: note that expressions for the minimum possible values of $a$ and $q$ ($q_{\text{min}}$ and $q_{\text{min}}$) under the interaction model used here were given by Opik (1976; see also the discussion by Olsson-Steel 1988c). The total probability of attaining $P<20$ yr is $5.8 \times 10^{-5}$ per orbit, whilst that for $P<200$ yr is $8.4 \times 10^{-3}$. The value of $P_{\text{cap.a}}$ found from equation (1) — applicable to a randomly-oriented comet, hence of any inclination — for the IP orbit (i.e. $a>3a_p$, as is required such that $C_a \approx 4/3$) is $\sim 5.2 \times 10^{-5}$; the capture probability is thus about 16 times higher for entry to a $P<200$ yr orbit for $i=10^\circ$ comet than for the average over all inclinations, testifying to the higher efficiency of captures from low-$i$ orbits.

As expected, the SP orbits produced have reduced eccentricities, in the range $0.0<e<0.78$; within these limits the values are quite evenly spread. The IP orbits range from $0.28<e<0.86$, with the majority being towards the upper limit.

Of most interest are the plots for $i$ and $q$. As discussed above, most of these 'close' encounters are in fact rather distant and leave the orbit largely unaltered from the original; thus the orbits

<table>
<thead>
<tr>
<th>$P&lt;20$ yr</th>
<th>$P&lt;200$ yr</th>
<th>Any period</th>
</tr>
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<tbody>
<tr>
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<td>$1.9 \times 10^{-6}$</td>
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</tr>
<tr>
<td>$q&lt;2.0$ AU</td>
<td>$1.0 \times 10^{-5}$</td>
<td>$1.0 \times 10^{-5}$</td>
</tr>
<tr>
<td>$q&lt;2.5$ AU</td>
<td>$1.5 \times 10^{-5}$</td>
<td>$1.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>$q&lt;3.0$ AU</td>
<td>$2.1 \times 10^{-5}$</td>
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<td>$q&lt;4.0$ AU</td>
<td>$4.0 \times 10^{-5}$</td>
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<td>$5.3 \times 10^{-5}$</td>
<td>$4.8 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

As expected, the SP orbits produced have reduced eccentricities, in the range $0.0<e<0.78$; within these limits the values are quite evenly spread. The IP orbits range from $0.28<e<0.86$, with the majority being towards the upper limit.

Of most interest are the plots for $i$ and $q$. As discussed above, most of these 'close' encounters are in fact rather distant and leave the orbit largely unaltered from the original; thus the orbits

Figure 3 — As for Figure 2 except for the source orbit being Orbit B ($q=5.2$ AU, $i=170^\circ$).
which remain LP mostly still have $i = 10^\circ$. However, the majority of the IP and SP orbits produced have increased inclinations, the peaks occurring at $-15^\circ$ and $-26^\circ$ respectively; in fact all orbits of $P < 200$ yr have $i \geq 16^\circ$ and all those of $P < 20$ yr have $i \geq 27^\circ$. This is easily understood with reference to the Tisserand parameter (equation 2): a decrease in $a$ and $e$ generally leads to an increase in $i$.

The plot for the perihelion distance shows a peak at the original value of $q = 5.2$ AU. This is as expected since tangential encounters will most often leave the trajectory unaltered but result in a small acceleration/deceleration such that the size of the orbit is changed. However, there is a significant fraction of orbits which have $q$ much-decreased, and those with $q < 3.5$ AU are all SP, whilst those of $3.5 < q < 4.7$ AU are all IP. This is consistent with the results of Everhart (1972) who found that large values of $\Delta q$ corresponded with large changes in the orbital energy, $\Delta E/\Delta$. The probabilities per perihelion passage of attaining perihelion distances below limits of interest are shown in Table 2; the figures given there indicate that about one in 175,000 LP comets on orbits like A are expected to be diverted in single close encounters into observable SP orbits ($q < 1.5$ AU).

Similarly Figure 3 represents the distributions of new orbital elements for orbit B. Note that the distribution in $a$ is given from 0 to 500 AU in Figure 3, whereas in the other Figures the range is from 0 to 50 AU only. Additionally some smoothing has been applied to this plot: although it appears that values of $a$ down to $-60$ AU are possible, in reality there are none with $a < 80$ AU. Thus no SP or IP comets are producible from this retrograde orbit through single close encounters with Jupiter: only about one in 500,000 perihelion passages results in $a$ being reduced to below 100 AU. The eccentricities of the new orbits are almost all still $e > 0.98$, with the minimum possible being $e = 0.93$. The inclinations, ranging from $164^\circ < i < 180^\circ$, are largely unaltered, and most significantly from the point of view of the production of observable SP/IP comets there are no new orbits of $q < 5.0$ AU. The reason for the inefficiency of such an orbit as B in producing SP/IP comets in interactions with Jupiter is that the encounters are close to being anti-parallel, with the relative velocity consequently being very high (Table 1).

In Figure 4 we show the probability distributions for new orbital elements being derived from Orbit C, which has $q = 1.0$ AU and $i = 10^\circ$. Again the plot for $a$ is rather erratic due to the binning used, but it does show that the probability of attaining a SP orbit ($P < 20$ yr) is $-5.5 \times 10^{-6}$ per orbit, and $-1.5 \times 10^{-4}$ for $P < 200$ yr. The majority of orbits of $e < 0.8$ produced are SP, those of $0.8 < e < 0.98$ are IP, and of course those of $e > 0.98$ are all LP.

The inclination plot is of some interest since it indicates that an appreciable fraction of the SP comets produced should be retrograde, whereas there are no observed retrograde SP comets. The peak of the SP curve occurs at $i = 80^\circ$, and $-13\%$ of these SP orbits have $0^\circ < i < 20^\circ$, $-78\%$ have $0^\circ < i < 90^\circ$, and $-22\%$ are retrograde. For the IP comets $-80\%$ have $0^\circ < i < 20^\circ$ and none are retrograde; the peak for these is at $i = 11^\circ$. The comets which remain in LP orbits all have $i \geq 50^\circ$, and the vast majority have $i = 10^\circ$.

For Orbit C almost all SP comets produced have $q$ reduced from the original value of 1.0 AU; the peak to this distribution is at $q = 0.1-0.2$ AU. The IP comets produced similarly have reduced perihelia, with the peak at $q = 0.7-0.8$ AU, whereas the remnant LP comets have $q$ almost unchanged. Thus the SP/IP orbits generally have $q$ reduced but $i$ increased. As before we are particularly interested in the production of ‘observable’ SP/IP comets, for which the new values of $q$ are critical; thus we give in Table 3 the probabilities of attaining $q$ below certain values.

In Figure 5 we show the distributions for Orbit D ($q = 1.0$ AU, $i = 170^\circ$). Now the probability of attaining a SP orbit ($P < 20$ yr) is $-4.5 \times 10^{-7}$ per orbit, and $-3.5 \times 10^{-5}$ for $P < 200$ yr. These
Table 3: Probabilities (per perihelion passage/orbit) of attaining new perihelion distances from Orbit C ($q = 1.0$ AU, $i = 10^\circ$), as plotted in Figure 4.

<table>
<thead>
<tr>
<th>$P &lt; 20$ yr</th>
<th>$P &lt; 200$ yr</th>
<th>Any period</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q &lt; 0.5$ AU</td>
<td>$4.0 \times 10^{-6}$</td>
<td>$1.6 \times 10^{-5}$</td>
</tr>
<tr>
<td>$q &lt; 1.0$ AU</td>
<td>$4.8 \times 10^{-6}$</td>
<td>$1.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>$q &lt; 1.5$ AU</td>
<td>$5.1 \times 10^{-6}$</td>
<td>$1.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>$q &lt; 2.0$ AU</td>
<td>$5.2 \times 10^{-6}$</td>
<td>$1.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>$q &lt; 2.5$ AU</td>
<td>$5.3 \times 10^{-6}$</td>
<td>$1.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>$q &lt; 3.0$ AU</td>
<td>$5.4 \times 10^{-6}$</td>
<td>$1.5 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 4: Probabilities (per perihelion passage/orbit) of attaining new perihelion distances from Orbit D ($q = 1.0$ AU, $i = 170^\circ$), as plotted in Figure 5.

<table>
<thead>
<tr>
<th>$P &lt; 20$ yr</th>
<th>$P &lt; 200$ yr</th>
<th>Any period</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q &lt; 1.0$ AU</td>
<td>$6.6 \times 10^{-9}$</td>
<td>$3.4 \times 10^{-5}$</td>
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<tr>
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<tr>
<td>$q &lt; 2.0$ AU</td>
<td>$0$</td>
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</tr>
<tr>
<td>$q &lt; 2.5$ AU</td>
<td>$0$</td>
<td>$2.1 \times 10^{-5}$</td>
</tr>
<tr>
<td>$q &lt; 3.0$ AU</td>
<td>$1.0 \times 10^{-7}$</td>
<td>$3.5 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Figure 5—As for Figure 2 except for the source orbit being Orbit D ($q = 1.0$ AU, $i = 170^\circ$).

The majority of orbits of $e < 0.6$ produced from D are SP, and those of $0.6 < e < 0.96$ are all IP. All eccentricities are $e > 0.09$, and of course the near-circular orbits produced would all have perihelia close to Jupiter ($q > 5$ AU): in fact all SP orbits
which result have $q$ increased to become $2.8 < q < 5.2$ AU, and similarly the IP orbits produced have $1.2 < q < 5.2$ AU. Numerical values for the $q$-distribution are given in Table 4.

Although the prograde Orbit C can produce retrograde SP/IP comets (Figure 4), the converse is not true for retrograde Orbit D (Figure 5). The SP orbits all have $146^\circ < i < 180^\circ$, whilst the IP orbits have $120^\circ < i < 180^\circ$. Although $i < 120^\circ$ is possible for the LP orbits, the majority of the resultant orbits have inclinations close to the original value of $170^\circ$.

Discussion

One of the main conclusions to result from the early work of the capture of LP comets into smaller orbits which was outlined in the introduction was that single close encounters causing gross changes of the orbits do not occur often enough in order to explain the number of SP/IP comets which are actually observed, although this is of course heavily dependent upon the flux of LP comets in the region of the giant planets, an area in which we are comparatively ignorant. In addition the single close encounters apparently would give rise to a distribution of SP/IP comets, especially in terms of the prograde-retrograde ratio, which is at variance with the observed distribution (Everhart 1982). Thus the consensus has been that many more distant encounters are required in order for the capture of a comet from a LP into a SP/IP orbit to occur: this is often called the ‘diffusive picture’, since there is a gradual diffusion in orbital energy ($1/a$) space until a sufficiently small period is attained. The recent numerical experiments of Stagg and Bailey (1989: hereafter SB) have confirmed that this is generally the avenue through which SP/IP comets come about, for initial orbits of $q < 9$ AU.

The other scenario whereby SP/IP comets are captured in single close encounters may be termed the ‘stochastic picture’, and SB have indicted that this process is of especial importance for comets with initial LP orbits having perihelia in the outer solar system, whereas the experiments of Duncan et al. (1988) effectively were confined to the diffusive regime, with the infrequent but dynamically important large perturbations in close encounters being undersampled. Conversely, SB assumed that the inclination of each comet was preserved throughout the capture process, such an assumption having been shown to be invalid by Duncan et al. There are therefore reasons to be cautious in one’s interpretation of the results of both Duncan et al. (1988) and Stagg and Bailey (1989).

In view of the above, of what use are the present results, and how might the technique described here be utilized in order to aid our understanding of LP comet capture? Firstly, a direct comparison with the capture of LP comets via the diffusive process (but with the infrequent large orbital changes accommodated) would be of interest, and we will consider the results of SB here. These authors investigated the capture of initially parabolic comets of inclinations $i = 3^\circ$, $9^\circ$ and $27^\circ$ with perihelion distances in 1 AU bins running from $q = 2$ to 34 AU. For each $(q, i)$ several thousand comets were followed through their orbital evolution, with a capture into an observable SP/IP orbit being taken to have occurred if $P < 200$ yr and $q < 1.5$ AU was achieved. In this way it was possible for the first time to build up a picture of the dependence of capture probability upon $q$ and $i$. For $i = 9^\circ$ SB found that the capture probabilities ($P_{\text{cap}}$) are as follows:

<table>
<thead>
<tr>
<th>$q$ (AU)</th>
<th>$P_{\text{cap}}$ (Any $Q$)</th>
<th>$P_{\text{cap}} (Q &gt; 10AU)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-3</td>
<td>$5.6 \times 10^{-3}$</td>
<td>$5.5 \times 10^{-3}$</td>
</tr>
<tr>
<td>4-5</td>
<td>$4.1 \times 10^{-3}$</td>
<td>$0.2 \times 10^{-3}$</td>
</tr>
<tr>
<td>5-6</td>
<td>$4.0 \times 10^{-3}$</td>
<td>$0.1 \times 10^{-3}$</td>
</tr>
<tr>
<td>6-7</td>
<td>$2.0 \times 10^{-3}$</td>
<td>$0.1 \times 10^{-3}$</td>
</tr>
<tr>
<td>7-8</td>
<td>$0.6 \times 10^{-3}$</td>
<td>0.0</td>
</tr>
</tbody>
</table>

For larger $q_{\text{init}}$ their values of $P_{\text{cap}}$ are much reduced below these figures, accentuating the belief that Jupiter is the major agent causing the capture of LP comets with $q_{\text{init}} < 6$ AU. A comparison with the results gained here for Orbit A ($q = 5.2$ AU, $i = 10^\circ$), as shown in Table 2 and plotted in Figure 2, shows that our $P_{\text{cap}} = 5.7 \times 10^{-4}$ is much smaller than the value of $4.0 \times 10^{-3}$ derived by SB, confirming that the vast majority of captures for LP comets with such elements are indeed through multiple small perturbations rather than single close encounters. Stagg and Bailey find that most of the captured comets from $q = 5-6$ AU orbits end up with aphelion distance $Q < 10$ AU (which we may view as being the SP component); similarly our results indicate that all comets directed into such orbits by single close encounters are SP rather than IP. Contrary to the results of Everhart, SB find that in fact it is the smaller values of $q_{\text{init}}$ which have higher capture probabilities, at least for the inclinations ($i = 3^\circ$, $9^\circ$ and $27^\circ$) which they considered. This is backed up by the present results. Although SB did not consider any $q_{\text{init}} < 2$ AU, it is clear from the values given above that $P_{\text{cap}}$ rises sharply with decreasing $q_{\text{init}}$. Although the chances of being captured into the ($P < 200$ yr, $q < 1.5$ AU) orbits are comparable from Orbits A and C, our values from Tables 2 and 3 indicate that the chance of being captured into a ($P < 200$ yr, $q < 1.5$ AU) orbit from Orbit A ($q = 5.2$ AU) in a single close encounter is $5.7 \times 10^{-4}$ per orbit, about 26 times less than the value of $\sim 1.5 \times 10^{-3}$ from Orbit C ($q = 1.0$ AU). This is apparently in contradiction to the results of Everhart. From Figure 2 it is clear that SP/IP orbits of small $q$ can only be attained from Orbit A with a severe reduction in the eccentricity, whereas Figure 4 shows that such orbits may be attained from Orbit C with a modest decrease in the eccentricity: the former requires a large deflection as the planet and the comet pass on near-parallel paths, the latter a smaller deflection as they pass on near-perpendicular paths. Clearly it would be of interest to find how $P_{\text{cap}}$ varies with $q$ from 0 to 6 AU, to study the general trend and compare it to SB’s results.

Secondly, although it seems clear that the overall capture mechanism involves multiple encounters, it does seem necessary in many cases that the LP comet has at least one closer encounter resulting in a large deceleration, such that the eccentricity is reduced to become rather less that unity (i.e. $e \leq 0.98$) so that the comet is then largely protected from ejections. This is discussed in detail by SB; see also Olsson-Steel (1988a). One facet of this problem which could be solved using the present technique is how the ejection probability varies with $e$, $q$ and $i$ when $e$ is close to one, since this will critically affect the flux of parabolic comets which is required in order to maintain the SP/IP population. For example the values of $P_{\text{enc}}$ and $P_{\text{ject}}$ shown in Table 1 indicate that 47% of all encounters between an orbit like A and Jupiter will result in an ejection; thus if the orbital elements are largely unchanged in these encounters then after five encounters only $\sim (0.53)^5 \approx 0.42$ will remain. However, if generally the eccentricity drops to become $e < 0.98$ for the non-ejected orbits, as is indicated by Figure 2, then a rather larger proportion of the comets will remain after five or more encounters. Equally well, the experiments of SB have
shown that although the capture of LP comets with \( q < 9 \) AU is basically diffusive in nature, in fact for comets with perihelia in the outer solar system the captures mainly occur as the result of the stochastic situation: that is, the infrequent close encounters are the major contributors to the changes in the orbital energies of LP comets with \( q > 9 \) AU. The technique described in this paper may be directly applied to such orbits.

Thirdly, in order to save on the amount of computation time which would be necessary in order to investigate the capture probabilities for the full range of possible values of \( q \) and \( i \) of parabolic comets, it is necessary to be able to make an educated guess at the most likely parameters, whilst not ignoring other possibilities. Thus although it is known that low-inclination prograde LP comets supply the bulk of the SP/IP population, so that SB used only \( i = 3^\circ, 9^\circ \) and \( 27^\circ \) in their numerical experiments, in fact the results presented here show that retrograde LP comets of small-\( q \) may also be captured into SP/IP orbits, which remain retrograde (Figure 5), whilst prograde LP comets of small-\( q \) may produce retrograde SP orbits (Figure 4). Clearly important clues as to the production of the observed cometary population, and to the avenues of attack in future numerical integrations, can be gained from applying this technique to the full gamut of possible initial orbits, with comparatively little computer time being necessary.

Fourthly, as the case in most numerical experiments of this type a basic assumption is that the Tisserand parameter is conserved with respect to the planet of interest, at least until such time as the comet is passed on to the control of another planet, generally the next planet sunward (e.g. see the discussion by SB). If this is true then the effect of many distant encounters may be estimated from the plots presented here, since the change in the element in a distant encounter will not be sufficient to markedly affect the next set of probability distributions. Thus a series of more-distant encounters, although not individually effective in altering the orbit, may in total produce the same net effect as one closer encounter, and the plots are of use in investigating the evolution of an orbit which has perturbations dominated by one planet only. For cometary orbits with \( q \leq 6 \) AU, for example, Jupiter is the only planet of significance, whilst for \( q > 25 \) AU Neptune dominates.

Fifthly, although the various papers reviewed by SB and mentioned in the Introduction follow the orbital evolution of input parabolic comets in order to find the probabilities of capture into observable SP/IP orbits, often the distributions of the elements of the resultant comets are not accumulated or published, which makes any comparison with the observed population difficult. For example, the prograde-retrograde ratio produced is of considerable interest (e.g. see Everhart 1982).

Finally, this technique is of considerable use of investigating the possible origins of comets in peculiar orbits, and indicating the most likely avenues by which those orbits have come about. For instance Olsson-Steel (1988a) has discussed the four retrograde, intermediate-period comets (P/Halley, P/Swift-Tuttle, P/Tempel-Tuttle and P/Pons-Gambart) and suggested that these might have been flipped into their present orbits in close encounters with Neptune or Uranus from trajectories which were originally prograde. Equally well, although most SP comets have perihelia in the inner solar system, aphelia near Jupiter and low-inclination, prograde orbits, there are a few oddballs which are difficult to fit into our overall picture (see the orbits listed in Marsden 1986). One such orbit is P/Hartley-IRAS, which has \( q = 1.282 \) AU, \( e = 0.834 \), and \( i = 95^\circ \); from the evidence of Figure 4 it is tempting to suggest that this comet was captured by Jupiter from a near-parabolic orbit of \( q = 1.5 \) AU and \( i = 10^\circ \).

Conclusions

The main aim of this paper has been to describe the technique which has been developed at The University of Adelaide for investigations of the frequency and effects of close encounters between small objects (comets, asteroids and meteoroids) and the planets, and then to illustrate its usage with reference to the capture by Jupiter of comets from near-parabolic paths into short- and intermediate-period orbits.

The four hypothetical cometary orbits which we have considered (\( q = 5.2 \) AU, \( i = 10^\circ \); \( q = 5.2 \) AU, \( i = 170^\circ \); \( q = 1.0 \) AU, \( i = 10^\circ \); and \( q = 1.0 \) AU, \( i = 170^\circ \)) were chosen so as to show how the capture probabilities are critically dependent upon the input orbit. It has been shown that, in line with previous results, prograde comets are much more likely to be captured into short- and intermediate-period orbits. However, contrary to previous analyses, but in agreement with the results of Stagg and Bailey (1989), it does not appear that parabolic comets with perihelia near Jupiter have larger capture probabilities; rather, the input orbits with smaller perihelion distances are more likely to be diverted into smaller orbits. Although the technique used here considers only the effect of single close encounters between a comet and a planet, rather than the longer-term orbital evolution with many more encounters (both near and distant), it has been demonstrated that this is a method of considerable utility as regards identifying the most likely entry corridors from which the short- and intermediate-period comets are derived. Future papers will be concerned with detailing the dependence of the capture probability upon the initial inclination and perihelion distance.

Acknowledgements

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