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# Images of hyperbolic reflection groups

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Hyperbolic triangle and tetrahedral groups have been studied for their group-theoretic interest, as well as for their connection with the geometry of discrete groups, 3-manifolds and 3-orbifolds. This thesis presents three aspects of these groups.

Stemming from a classical result about the modular group, Graham Higman showed that all but finitely many of the alternating groups are quotients of the hyperbolic triangle group $\Delta(2, 3, 7)$. Such groups are said to have the *residual alternating property* (RAP). Various authors have extended this result to show that certain infinite families of hyperbolic triangle groups also possess the RAP. We set out to establish, with the help of coset diagrams, a similar result for the family $\Delta(2, 4, r)$, where $r \geqslant 6$.

Regular maps facilitate a greater understanding of the automorphisms of Riemann surfaces. Besides, they are interesting structures in their own right, being in some sense analogues to the Platonic solids, but on surfaces different from the 2-sphere. A classical result of Coxeter shows that among the closed non-orientable surfaces, the Klein bottle and the non-orientable surface of genus 3 do not admit regular maps, although it is a simple matter to find examples on the projective plane. We have attempted to see in general which of the closed non-orientable surfaces do admit regular maps. Specifically, representations of twenty five hyperbolic, and two spherical triangle groups provide infinite families of regular maps on at least 77.5 per-cent of non-orientable genera.

Finally, there is a long history of obtaining hyperbolic and spherical 3-manifolds from the identification of the faces of regular solids. Classical examples include the Poincaré homology sphere (1904), the Weber-Seifert manifold (1933), and the Gieseking manifold (1911). The last of these has attained more topical relevance as the minimal volume cusped hyperbolic 3-manifold. We approach the problem of classifying, up to isometry, all the manifolds obtainable in such a manner by looking for subgroups with special characteristics in hyperbolic tetrahedral groups. We find a total of 39 distinct

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hyperbolic and eight distinct spherical 3-manifolds in this way.

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