

# Multiwavelength Light-Curve Analysis of Cepheid Variables

A. Bhardwaj<sup>1</sup>, S. M. Kanbur<sup>2</sup>, M. Marconi<sup>3</sup>, H. P. Singh<sup>1</sup>,  
M. Rejkuba<sup>4</sup> and C-C. Ngeow<sup>5</sup>

<sup>1</sup>Department of Physics & Astrophysics, University of Delhi, Delhi, India  
email: [anupam.bhardwaj@gmail.com](mailto:anupam.bhardwaj@gmail.com)

<sup>2</sup>State University of New York, Oswego, NY, USA

<sup>3</sup>INAF-Osservatorio astronomico di Capodimonte, Napoli, Italy

<sup>4</sup>European Southern Observatory, Garching, Germany

<sup>5</sup>National Central University, Jhongli, Taiwan

**Abstract.** This poster presented results from a detailed analysis of observed and theoretical light-curves of classical Cepheid variables in the Galaxy and the Magellanic Clouds. The theoretical light-curves were based on non-linear convective hydrodynamical pulsation models; the observational data were taken from ongoing wide-field variability surveys. The variation which we found in theoretical and observed light-curve parameters as a function of period, wavelength and metallicity was used to constrain the input physics to the pulsation models, such as the mass–luminosity relations obeyed by Cepheid variables. We also accounted for the variation in the convective efficiency as entered into the stellar pulsation models and its impact on the theoretical amplitudes and Period-Luminosity relations for Cepheid variables.

**Keywords.** (Stars: variables:) Cepheids - stars: evolution - stars: pulsations - (galaxies:) Magellanic Clouds

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## 1. Introduction

Classical Cepheid variables are well-known standard candles and fundamental tracers of young stellar populations in their host galaxies. Cepheid variables exhibit a very strong Period–Luminosity relation (P–L), [Leavitt & Pickering \(1912\)](#), that has been used extensively for extragalactic distance determination and for estimating an accurate and precise value of the Hubble constant ([Freedman \*et al.\* 2001](#), [Riess \*et al.\* 2016](#)). These radially pulsating stars are also very sensitive probes for the theory of stellar evolution and pulsation ([Cox 1980](#)).

Theoretical studies based on non-linear, convective hydrodynamical pulsation models by [Bono \*et al.\* \(1999, 2000\)](#), [Marconi, Musella & Fiorentino \(2005\)](#), [Marconi \*et al.\* \(2013\)](#), and references therein, have been able to predict the observed pulsation properties and the morphology of the light-curves of Cepheid variables. In terms of mean light properties, theoretical (P–L) relations for Cepheid variables were found by [Caputo, Marconi & Musella \(2000\)](#), [Fiorentino \*et al.\* \(2007\)](#) and [Bono \*et al.\* \(2010\)](#) to be consistent with observations at multiple wavelengths. More recently, [Marconi \*et al.\* \(2017\)](#) used pulsation models to match the observed light and radial-velocity variations of fundamental and first-overtone mode Cepheids in the Small Magellanic Cloud at multiple wavelengths. Similar efforts to reproduce Cepheid properties have used stellar evolution models; for example, [Anderson \*et al.\* \(2016\)](#) investigated the effects of rotation, and showed that Cepheid luminosity increases between its crossings of the instability strip. But despite

this recent progress, there are several challenges for stellar pulsation modelling, such as reproducing light variations close to the red-edge of the instability strip, and disentangling the effects of helium and metallicity dependence on Cepheid properties; a detailed review has been given by Marconi (2017).

Over the past few years, a huge amount of variable-star data has become available at multiple wavelengths from time-resolved wide-field variability surveys. Bhardwaj *et al.* (2015) exploited time-series data from these large surveys to analyse light-curves of Cepheid variables in the Galaxy and the Large Magellanic Cloud (LMC) at optical, near-infrared and mid-infrared wavelengths. Bhardwaj *et al.* (2017) extended that work to carry out a comparative study of theoretical and observed light-curve parameters of Cepheid variables at multiple wavelengths in order to explore constraints for stellar pulsation models. This poster summarises the main results from those analyses.

## 2. Theoretical and Observational Framework

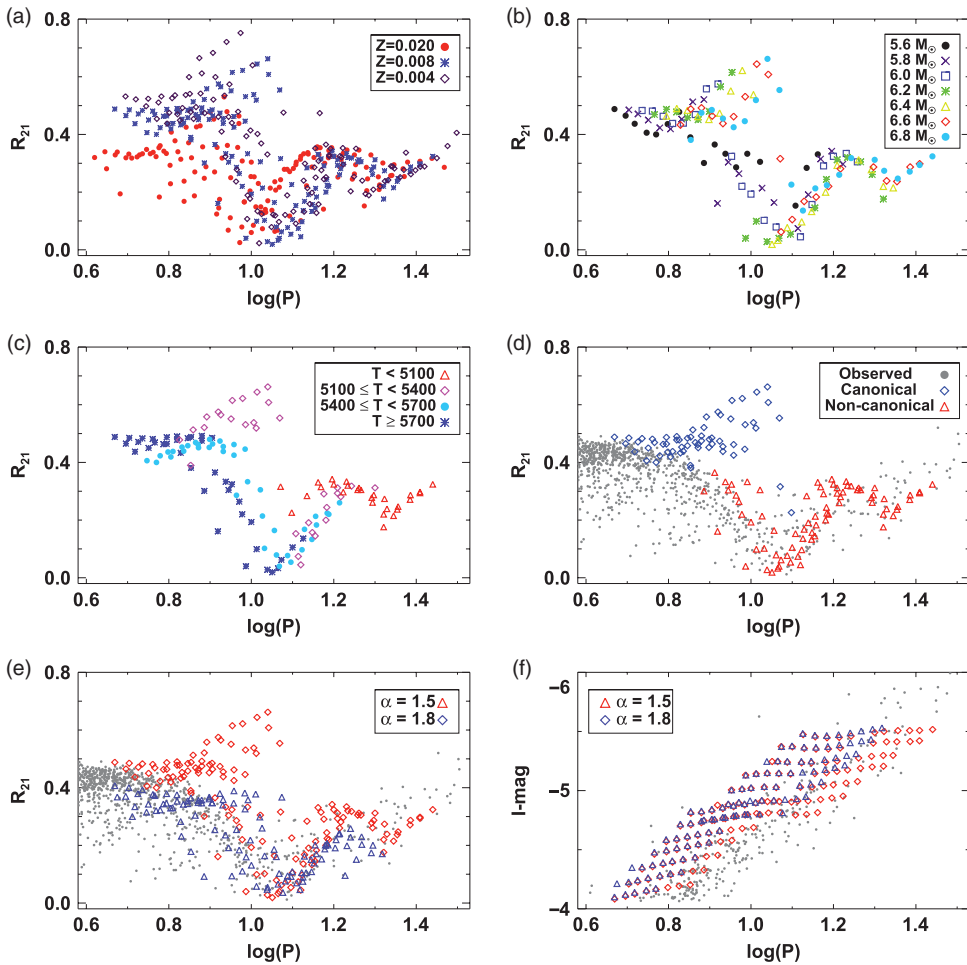
We used full amplitude, non-linear, convective hydrodynamical models to generate Cepheid light-curves as discussed in Marconi *et al.* (2013). In brief, for a fixed composition representative of Cepheid variables in the Galaxy and the Magellanic Clouds, we adopted mass–luminosity (M–L) relations predicted from stellar evolutionary calculations (canonical relations). For a given mass, we also adopted a luminosity level brighter by 0.25 dex to allow for possible mass-loss and overshooting (non-canonical relations). We explored a wide range of temperatures for each combination of X,Y,Z and (M–L) to produce bolometric light variations. Those bolometric light-curves were transformed into visual and near-infrared filters. The corresponding observational data set consisted of the observed light-curves of Cepheid variables at multiple wavelengths, compiled from the literature by Bhardwaj *et al.* (2015). The Cepheid light-curves could be analysed using the Fourier decomposition method as suggested by Simon & Lee (1981).

We fitted a Fourier sine series to the Cepheid light-curves in the following form:  $m = m_0 + \sum_{k=1}^N A_k \sin(2\pi kx + \phi_k)$ , where  $m$  represents the observed magnitude and  $x$  is the pulsation phase. The optimum order-of-fit ( $N$ ) was determined from the size of the least-square residuals. Fourier amplitude ratios and phase differences are defined as:  $R_{k1} = \frac{A_k}{A_1}$ ;  $\phi_{k1} = \phi_k - k\phi_1$ , for  $k > 1$ . Simon & Lee (1981) suggested that the lower-order Fourier parameters are sufficient to reproduce most characteristic features of Cepheid light-curves.

## 3. Comparison of Cepheid Light-Curve Parameters

Comparisons of theoretical and observed Cepheid light-curve parameters at multiple wavelengths, along with detailed discussions of the variations of Fourier amplitude and phase parameters with period, wavelength and metallicity, can be found in Bhardwaj *et al.* (2015) and Bhardwaj *et al.* (2017). Figure 1 shows the variation of the  $I$ -band Fourier amplitude ratio ( $R_{21}$ ) with period as a function of the input parameters for different pulsation models. In Figure 1(a), the variation in  $R_{21}$  is shown as a function of metallicity. We find that  $R_{21}$  values increase with decreasing metal-abundance for models of short-period Cepheids ( $\log(P) < 1$ ). We also note that the central minimum of  $\sim 10$  days shifts to longer periods for lower metal-abundance. Bhardwaj *et al.* (2015) also found that the central period of the Hertzsprung progression (Hertzsprung 1926) shifts to longer periods with decreasing metallicity and also for longer wavelengths.

In Figure 1 the variation in  $R_{21}$  is displayed as a function of stellar mass in panel (b) and as a function of temperature in panel (c). Figure 1(d) shows the variation in theoretical  $R_{21}$  values for canonical and non-canonical (M–L) relations, and compares them with  $R_{21}$  values obtained from the observed Cepheid light-curves in the LMC from the OGLE-IV



**Figure 1.** Variation of  $I$ -band amplitude ratio ( $R_{21}$ ) with period for Cepheids in the LMC, as a function of (a) metal-abundance, (b) stellar mass, (c) temperature, (d) luminosity (canonical models are fainter by 0.25 dex than non-canonical models), and (e) mixing-length ( $\alpha$ ). Models in panels (b)–(f) correspond to the composition  $Y=0.25$ ,  $Z=0.008$ , which is representative of Cepheids in the LMC. Panels (d) and (e) compare the results from theoretical models with  $R_{21}$  values from the observed light-curves of Cepheids in the LMC. In panel (f) the  $I$ -band ( $P-L$ ) is shown for different values of  $\alpha$  and compared with observed ( $P-L$ ) relations for classical Cepheids in the LMC.

catalogue (Soszyński *et al.* 2015). A comparison of panels (b)–(d) suggests that canonical models are discrepant with respect to observations in the period range  $0.8 < \log(P) < 1.1$ . Those canonical models have masses greater than  $6M_{\odot}$  and relatively lower temperatures ( $5100 \leq T < 5400$  K), suggesting a location closer to the red-edge of the instability strip. Fiorentino *et al.* (2007) have shown that an increase in the convective efficiency narrows the width of the instability strip as the red-edge becomes hotter. Panel (e) also shows that the discrepancy in  $R_{21}$  values can be remedied by increasing the mixing-length parameter. However, that results in an offset of the bolometric mean-magnitudes and affects the slope and zero-point of the theoretical calibrator ( $P-L$ ) relations, as shown in panel (f). The primary and secondary minima in  $R_{21}$  values around 10 and 20 days, respectively (panel c) can also be correlated with the observed non-linearities in Cepheid ( $P-L$ ) relations in the LMC at similar periods, as found in Bhardwaj *et al.* (2016).

#### 4. Conclusions

This analysis has explored constraints for stellar pulsation models. The variation in light-curve parameters as a function of period, wavelength and metallicity shows that canonical and non-canonical models can be differentiated on the Fourier plane. At optical wavelengths, the amplitude parameters display a greater offset with respect to observations, but that can be resolved by increasing the convective efficiency in the pulsation models.

A more quantitative and comparative light-curve analysis could provide deeper insights into the theory of stellar evolution, since the physics in the pulsation models is dependent on stellar evolutionary calculations. Light-curve analyses also enable us to study Cepheid properties as a function of pulsation phase, and used to probe the interaction between stellar photosphere and hydrogen ionisation front; for example, see [Simon, Kanbur & Mihalas \(1993\)](#) and [Bhardwaj \*et al.\* \(2014\)](#). Light-curve analysis will also be essential for constructing templates for identifying and classifying variable stars in the era of upcoming time-domain surveys.

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