REMARKS ON A PAPER BY S. TROTT

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(received July 25, 1963)

There are several statements in [3] which require clarification.

Theorem 1 [3, p. 246] states that $U_3 = U_2U_4U_2^{-1}U_4U_2U_4$. In fact this is (essentially) the relation $O = PUPU^{-1}PU$ given in [1, p. 91, (7.35)]. To see this we note that $O = U_3$, $P = U_4$, $U = U_4U_2U_4$ (as explained in [1, p. 88]); since $U_4^2 = E$,

$$U_3 = O = PUPU^{-1}PU = U_4 \cdot U_4U_2U_4 \cdot U_4 \cdot U_4U_2^{-1}U_4 \cdot U_4 \cdot U_4U_2U_4$$

$$= U_2U_4U_2^{-1}U_4U_2U_4 \ .$$

In [3, p. 245] there is the statement: "It has been shown by D. Beldin (Thesis, Reed College, 1957) that M_n is a 2-generator group."

There is the implication that this had not been established earlier. In fact, B. Neumann [2, pp. 375-378] showed that M_n is generated by

$$R = \begin{pmatrix} 0 & -1 & 0 & & & & & & \\ 1 & 0 & 0 & & & & \text{\Large 0} & & \\ 0 & 0 & 1 & & & & & & \\ & & & \cdot & & & & & \\ & & & & \cdot & & & & \\ & & & & & \cdot & & & \\ & & & & & & 1 & 0 & 0 \\ & \text{\Large 0} & & & & & 0 & -1 & 1 \\ & & & & & & 0 & 1 & 0 \end{pmatrix},$$

https://doi.org/10.4153/CMB-1964-006-7 Published online by Cambridge University Press

$$S = \begin{pmatrix} 0 & -1 & 0 & 0 & & & \\ 0 & 0 & -1 & 0 & & \Large 0 & \\ 0 & 0 & 0 & -1 & & & \\ & & \Large 0 & & \ddots & & \\ 0 & 0 & 0 & 0 & \cdots & -1 \\ -1 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

when n is odd, $n > 3$; and M_n is generated by R and $Q = -S$ when n is even, $n > 3$. Trott's generators

$$U_2 = \begin{pmatrix} 1 & 0 & 0 & & \\ 1 & 1 & 0 & \Large 0 & \\ 0 & 0 & 1 & & \\ & \Large 0 & & \ddots & \\ & & & & 1 \end{pmatrix},$$

$$K = \begin{pmatrix} 0 & 1 & 0 & 0 & & \\ 0 & 0 & 1 & 0 & \Large 0 & \\ 0 & 0 & 0 & 1 & & \\ & & \Large 0 & & \ddots & \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ (-1)^n & 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

(Trott uses the symbol U instead of K, but this could be confused with Neumann's U, which is different) are "simpler" than Neumann's; but Neumann gave a set of defining relations using R and S, whereas Trott did not.

In terms of the generators

$R_1 = U_1 = U_4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $U_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, Trott [3, p. 252] obtained, for M_2, the defining relations

$$(1) \qquad R_1^2 = (U_1^{-1} R_1 U_2 R_1)^6 = E, \quad U_2^{-1} R_1 U_2 R_1 U_2^{-1} = R_1 U_2 R_1 U_2^{-1} R_1 U_2 R_1,$$

by showing that (1) and

$$R_2 = R_1 U_2^{-1} R_1 U_2 R_1 , \quad R = U_2 R_2$$

are together equivalent to 7.21 [1, p. 85] (which define M_2) and

$$R_1 = R_1, \qquad U_2 = R_3 R_2 .$$

It should be pointed out that (1) is immediately seen to be equivalent to the relations

$$(2) \qquad (RU_2)^2 = (R^3 U_2^2)^2 = (R^2 U_2^2)^6 = E$$

which define M_2 [1, p. 88]; in fact (1) is obtained from (2) by letting $R_1 = RU_2$.

REFERENCES

1. H. S. M. Coxeter and W. O. J. Moser, Generators and relations for discrete groups. Springer, Berlin, 1957.

2. B. H. Neumann, Automorphismengruppe der freien Gruppen. Math. Ann. 107(1933) pp. 367-386.

3. S. M. Trott, A pair of generators for the unimodular group. Can. Math. Bull. 5(1962) pp. 245-252.

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