

# Mathematical Notes.

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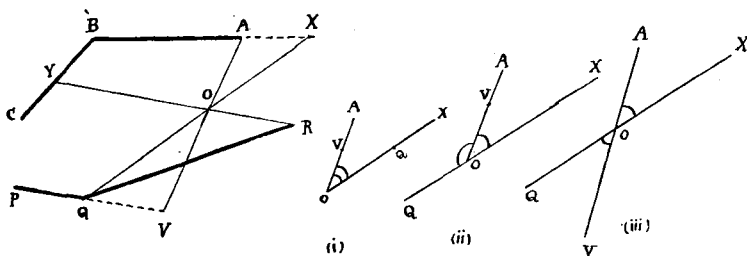
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## Two Extensions of Ceva's Theorem.

1. If  $ABCDE \dots$  is a polygon of an odd number of sides,  $O$  any point in its plane, and if  $AO, BO, CO, \dots$  cut the sides opposite  $A, B, C, \dots$  respectively,  $AB$  in  $X, BC$  in  $Y, CD$  in  $Z, \dots$  then

$$\frac{AX}{XB} \cdot \frac{BY}{YC} \cdot \frac{CZ}{ZD} \dots = +1.$$



Let  $PQ$  be the side opposite  $A$ . Then  $AB$  will be the side opposite  $Q$ .

Let  $AO$  cut  $PQ$  in  $V, QO$  cut  $AB$  in  $X, RO$  cut  $BC$  in  $Y$ . Then according as  $V$  and  $A, X$  and  $Q$  are on the same or different sides of  $O, \widehat{AOX}$  will be identical with, supplementary, or vertically opposite to  $\widehat{VOQ}$ . In any case,  $\sin AOX = \sin VOQ$ .

$$\text{Now } \frac{AX}{XB} = \frac{\triangle AOX}{\triangle XOB} = \frac{OA \cdot \sin AOX}{OB \cdot \sin XOB},$$

$$\frac{BY}{YC} = \frac{OB \cdot \sin BOY}{OC \sin YOC},$$

$$\dots = \dots, \dots,$$

$$\frac{PV}{VQ} = \frac{OP \cdot \sin POV}{OQ \cdot \sin VOQ},$$

$$\dots = \dots, \text{ etc.}$$

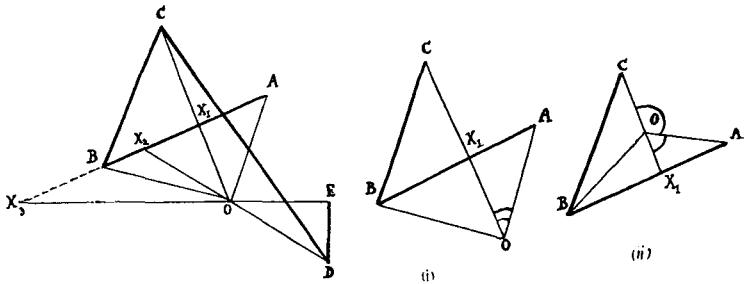
(1)

In the continued product  $OB$  and  $\sin AOX$  in the numerator cancel  $OB$  and  $\sin VOQ$  in the denominator, and so on, so that

$$\frac{AX}{XB} \cdot \frac{BY}{YC} \cdot \frac{CZ}{ZD} \dots = +1.$$

2 If  $ABCDE \dots HK$  be a polygon of  $n$  sides,  $O$  any point in its plane, and if  $CO, DO, EO, \dots KO$  cut  $AB$  in the  $n-2$  points  $X_1, X_2, X_3, \dots X_{n-2}$ , and  $DO, EO, \dots AO$  cut  $BC$  in the  $n-2$  points  $Y_1, Y_2, \dots Y_{n-2}$ , ..... and lastly if  $BO, CO, \dots HO$  cut  $KA$  in  $V_1, V_2, \dots V_{n-2}$ , then

$$\left(\frac{AX_1}{X_1B} \cdot \frac{AX_2}{X_2B} \cdot \frac{AX_3}{X_3B} \dots \frac{AX_{n-2}}{X_{n-2}B}\right) \left(\frac{BY_1}{Y_1C} \cdot \frac{BY_2}{Y_2C} \dots \frac{BY_{n-2}}{Y_{n-2}C}\right) \left(\dots\right) \left(\frac{KV_1}{V_1A} \dots \frac{KV_{n-2}}{V_{n-2}A}\right) = +1.$$



For  $\widehat{AOC}$  is identical with or supplementary to  $\widehat{AOX_1}$ .  
 $\therefore \sin AOX_1 = \sin AOC$ .

$$\text{Now } \frac{AX_1}{X_1B} = \frac{OA \cdot \sin AOX_1}{OB \cdot \sin X_1OB} = \frac{OA \cdot \sin AOC}{OB \cdot \sin COB}.$$

So for all the other ratios.

$$\therefore \text{the continued product} = \left[\frac{OA}{OB} \cdot \frac{OB}{OC} \dots \frac{OK}{OA}\right]^{n-2} \times \left[\left(\frac{\sin AOC}{\sin COB} \cdot \frac{\sin AOD}{\sin DOB} \dots \frac{\sin AOK}{\sin KOB}\right) \left(\frac{\sin BOD}{\sin DOC} \cdot \frac{\sin BOE}{\sin EOC} \dots\right) \left(\dots\right) \times \left(\frac{\sin KOB}{\sin BOA} \dots \frac{\sin KOH}{\sin HOA}\right)\right] = +1.$$

When  $n=3$ , each of the above theorems reduces to Ceva's Theorem.

A. C. AITKEN.