## Mathematical Notes.

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A New Form of Boyle's Law Tube.-The following apparatus for verifying Boyle's Law in an elementary manner is of interest from the point of view of pure mathematics as well as that of the laboratory. The method of construction, which I have described in the July (1910) number of the School World, presents a little difficulty in the first attempt, but is easily mastered in the second.


Fig. 1.

The apparatus consists of a tube of 2 mm . bore, closed at both ends, with an air column of $8^{\prime \prime}$ to $10^{\prime \prime}$ at one end, a mercury thread of $20^{\prime \prime}$ to $25^{\prime \prime}$ in the middle, and a good vacuum as long as possible at the other. If the end of the air column is pivoted at a point $O$ (Fig. 1) on a squared blackboard so that the other end of the air column $A_{1}$ lies on a heavy horizontal line, then, on rotating the tube, the point A keeps on this line. The law follows immediately from the similarity of triangles $\mathrm{OA}_{1} \mathrm{~A}_{2}$ and $\mathrm{B}_{2} \mathrm{CA}_{2}$, for

$$
\frac{p}{p_{0}}=\frac{v_{0}}{v} \text { and therefore } p v=p_{0} v_{0}
$$

The point B describes a "Conchoid of Nicomedes," of which the horizontal line is an Asymptote.

If the upper end of the tube be now opened so as to admit air at atmospheric pressure, represented by a barometric column of height $\pi$, the end of the air column will describe an Ellipse, of which the point O is a focus and the horizontal line the directrix. The eccentricity is the ratio of the length of the mercury thread to the barometric height $\left(=p_{0} / \pi\right)$.


Fig. 2.

The tube in this form affords a blackboard method of describing an Ellipse.

The other end of the mercury thread describes an Oval of the fourth degree (Fig. 2).

When the length of the mercury thread is equal to the barometric height, the ellipse becomes a parabola and the oval asymptotic.

For the demonstration of Boyle's Law the lower end of the mercury column may be used as a pivot. In this case the end B describes a circle and the point $O$ a horizontal straight line.

## William Miler

The Reciprocal Polar of a Circle.-The reciprocal polar figure of a circle $s$, with regard to another circle $c$, is a conic, one of whose foci is the centre of the reciprocating circle. The polar reciprocal of $s$ with regard to $c$ is obtained by taking the polars of points on $s$ with regard to $c$, and then constructing the envelope of these lines.


Fig. 1.
Let $p, q$ (Fig. 1) be the polar of P and Q and $p_{2}$ their point of intersection. $\quad p_{2}$ is then the pole of the line PQ , and the polar

