ACCURATE PARALLAXES AND STELLAR AGES DETERMINATIONS

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Abstract. Uncertainties on stellar ages due to the physical description of the stellar material entering the models and to measurements of global parameters and chemical composition are estimated in the case of A/F stars.

With combined efforts on atmosphere modeling (to improve global parameters), on asteroseismology (to improve the physical description of the stellar interior) and on distance determination at the level of the sub-milliarcsecond (to improve luminosity), an accuracy on age determination of about 10% is foreseeable.

1. Introduction

Precise stellar ages determinations are now required for a variety of topics, as galactic evolution or cosmological time scale. The "primary" determination of ages relies on comparisons of models of internal structure with the best available data on individual stars, or stellar groups.

Though the principle of the method is general, its application to different types of stars requires specific developements. We consider the determination of the ages of individual stars and we focus on late A or F spectral type stars on the main sequence, which age range (several 10^6 to a few 10^8 yr) is appropriate for galactic evolution studies.

Stellar age is currently derived from global observable parameters (luminosity and effective temperature), matching the position of a real star and of models in the HR diagram. Uncertainties on models are often ignored. We will show here that, in order to take advantage of the forseeable accuracy on the determination of the global parameters, this simple (first degree) method is inappropriate. One needs to constrain seriously other parameters, with the help of other variables, i.e. asteroseismologic quantities.

2. The "Model-Observation" Confrontation

Stellar interior models are built by integration of the hydrodynamical equations with initial and boundary conditions, assuming spherical symmetry of the star and neglecting rotation and magnetic field. Mass M and chemical composition, i.e. helium content Y and metallicity Z, have to be specified as initial conditions. A physical description of the material is needed, it is assumed here to be globally correct but for some adjustment

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parameters (P_i). The solution also depends on time, hereafter called "age" A. Consequently any output global characteristic Q_j of the model, such as observable quantities like effective temperature or luminosity, is obtained as a function Ψ of these inputs, i.e. $Q_j = \Psi(A, M, Y, Z, P_i)$.

In order to obtain an age one has to find the model which reproduces the observed quantities. It requires in principle to inverse Ψ, which is generally non linear, and sometimes not bijective. We consider here A/F stars on the main sequence, in a region of the HR diagram where the Vogt-Russell theorem applies and where the star position is very sensitive to age. The correct model and its age are obtained by iteration, starting from an approximate model and linearizing about this solution to get the necessary derivatives. A "differential" formalism writes:

$$dQ_j = D\Psi(dA, dM, dY, dZ, dP_k) \quad (1)$$

where $D\Psi$ is the linear operator built with the partial derivatives of the observable quantities with respect to the variables V_i (i.e. initial conditions, "physical" description and age). If the number of accessible observable quantities is larger than the number of parameters a χ^2 minimization technique can be used to estimate the parameters and their uncertainties. Brown *et al.* (1994) have proposed to solve this problem in the case of solar type stars through a singular value decomposition of $D\Psi$.

However, in the "classical" problem, luminosity, effective temperature and metallicity Z are the only observable quantities and the age is estimated by comparison in the HR diagram for a "given" chemical composition. All the variables cannot be determined directly which means that age determinations will rely on the choice of the additional parameters entering physical description.

3. Evaluation of $D\Psi$

Numerical computations of a "reference" sequence and of a set of sequences where one variable is modified with respect to the reference variable V_{j0} are performed to calculate:

$$\partial Q_k/\partial V_i \ \text{ at } \ V_j = V_{j0} \ \text{ for all } \ j \neq i \quad (2)$$

3.1. THE "REFERENCE" MODEL $A_0, M_0, Y_0, Z_0, P_{K0}$

The models are calculated with the CESAM code (Morel, 1993, 1994). The reference model is built with the most standard updated physics and a solar chemical composition. We took the nuclear reaction rates from Caughlan and Fowler (1988) and used the third generation of OPAL opacities (Iglesias *et al.*, 1992). Convection is treated with the mixing-length theory. We considered that an overshooting process extends the size of the convective core over a distance $d_{Ov} = O_v H_p$ where H_p is the pressure scale height. The reference value $O_v = 0.2$ comes from Schaller *et al.* (1992) who derived it by comparison of the observed main sequence width of clusters with that given by theoretical isochrones. The solar mixture and metal content ($Z_\odot = 0.019$) are from Grevesse (1991). The constraint that the solar model must yield at solar age the observed luminosity and radius gives an initial helium content $Y_\odot = 0.287$ and a mixing-length parameter $\alpha_0 = 1.67 H_p$.

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3.2. THE UNCERTAINTIES

3.2.1. *The Physical Terms P_k*

Many aspects of the physical description of stellar structure remain unknown. We focus here on the current uncertainties in the range of mass considered.

* The $^{14}N(p,\gamma)^{15}O$ reaction is the slowest reaction in the main CN cycle and controls the energy generation in the CNO cycle. The uncertainty on the cross-section S_0 is of about 17% as discussed by Parker and Rolfs (1991).

* Recent OPAL opacities have been validated by different facts, as the coherent modeling of double mode cepheids. To estimate the effects of the remaining uncertainties we use a linear parameter O_p. The O_p-value is 1 if the OPAL tables are used and 0 for the Los Alamos tables (Huebner *et al.*, 1977).

* The question of the universality of the mixing-length parameter α is still open (see i.e. Neuforge and Fernandes, 1994). We assume here an uncertainty of 20% around the solar value corresponding to our standard physical ingredients.

* There has been an already long debate on whether overshooting should be taken into account. Some authors do not consider this process while Napiwotski *et al.* (1993) give $d_{Ov} = 0.15\ H_p$ and Schaller *et al.* (1992) propose $d_{Ov} = 0.20\ H_p$.

3.2.2. *The Chemical Composition Terms*

They have to be determined for each object.

* The helium content is difficult to observe in A/F stars, and also to constrain. One has then to rely on "prejudices" to estimate Y and its uncertainty. We consider (Fernandes *et al.*, 1994) that the enrichment law, as constrained by the width of the main sequence, is valid here, i.e. $(Y - Y_p)/Z \subset [2,5]$, where Y_p is the primordial value taken at 0.228.

* The metallicity can be determined by observations. For simplicity, we assume that all objects have a solar mixture and that only Z varies.

3.3. THE DIFFERENTIAL OPERATOR $D\Psi$

Derivatives in (2) are estimated as finite differences. The most appropriate estimate is obtained using increments in the variables of the order of the probable uncertainties. Since derivatives strongly depend on mass and on the stage of evolution, we have chosen two "characteristic" models , both slightly evolved ($X_c \sim 0.25$): $M1$ is a $1.4 M_\odot$ star at an age of 2.3 10^9 yr and $M2$ is a $2 M_\odot$ star at an age of 0.81 10^9 yr. The positions in the HR diagram are shown on Fig. 1. and derivatives are given in Table 1 with the increments used for this evaluation.

4. The Leading Factors in Age Determinations

4.1. THE CLASSICAL CASE

Once $D\Psi$ is known, the accuracy on the model parameters are in principle obtained by solving the linear equation $\Delta Q_i = D\Psi \Delta V_j$, where ΔQ_i represent the relative errors on the observable quantities. This inversion "mixes" the role of the various parameters, as also discussed by Brown *et al.* (1994). In the "classical case" we are dealing with (comparison in the HR diagram), there are less observables than unknown quantities and we treat physical terms as parameters varying in a"reasonable" interval and the "chemical

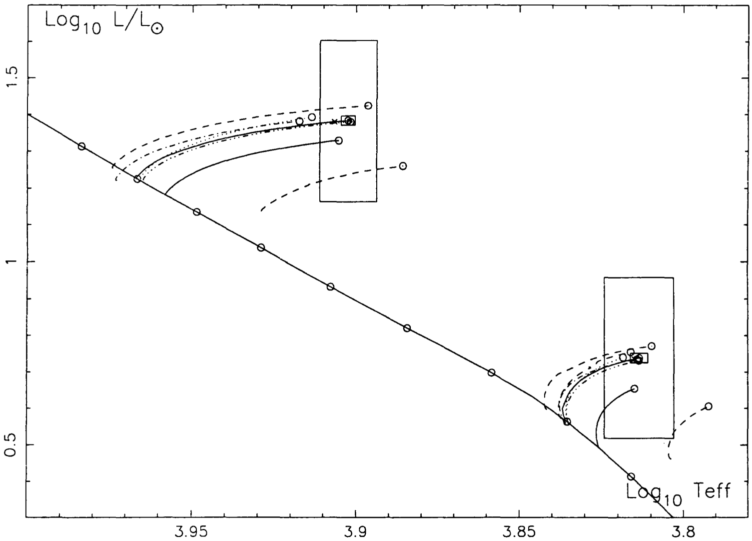

Figure 1. HR diagram showing the evolutionary sequences of the two models $M1$ and $M2$, as computed with the different parameters. The error boxes are given in both cases: $Si1$ (the large one) and $Si2$ (the small one).

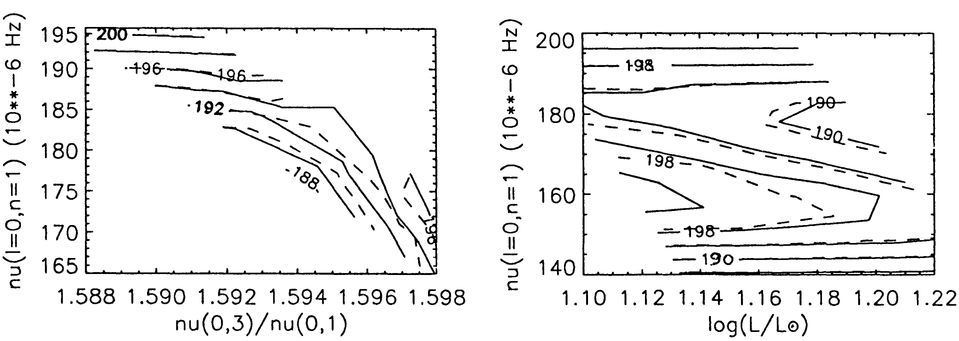

Figure 2. Isocontours of ν_{G1} for models between 1.75 and 1.85 $M_{\odot}$ along the main sequence, (a) in the plane ($\nu_{0,3}/\nu_{0,1}$, $\nu_{0,1}$), (b) in the plane ($logL/L_{\odot}$, $\nu_{0,1}$). Dashed lines refer to models with O_v = 0.20, and solid lines to O_v = 0.19, other physical and chemical parameters being the same.

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TABLE 1.

	model	∂LnA	∂LnM	∂LnS	$\partial LnOp$	$\partial Ln\alpha$	$\partial LnOv$	∂LnY	∂LnZ
increment		0.015	0.05	0.17	1	0.33	0.1	0.0013	0.011
∂LnL	M1	0.25	5.3	-.098	-0.0077	0.046	0.012	1.63	-0.74
∂LnL	M2	0.56	4.9	-0.052	-.0049	0.0048	0.051	2.02	-0.62
∂LnT_{eff}	M1	-0.16	-0.086	0.0014	0.0016	0.12	0.028	-0.19	-0.075
∂LnT_{eff}	M2	-0.21	-0.26	-0.011	0.034	0.0025	0.063	-0.31	-.080

composition terms" as determined through observations. We extract ΔA and ΔM from the uncertainties ΔL and ΔT on $logL$ and $logT_{eff}$, and ΔP_i, ΔZ, ΔY on the parameters.

For simplicity we write $dQ_{Vi} = |\partial Q_k/\partial V_i|$ at $V_j = V_{j0}$ for all $j \neq i$.

$$\Delta A = (dT_M\ \Delta L + dL_M\ \Delta T + \Sigma_i(dL_{Pi}\ dT_M + dL_M\ dT_{Pi})\Delta P_i)/|dL_A\ dT_M - dT_A\ dL_M| \quad (3)$$

where the summation in (3) has to be understood as a r.m.s.

4.2. MAGNITUDE OF THE UNCERTAINTIES

At the primary level discussed here, luminosity determinations rely on distance and bolometric correction BC. We assume that the targets are studied with the most accurate methods and that the temperature indicator is directly the effective temperature, i.e. that no preliminary calibration is needed. Bolometric corrections are computed from an adjustment of a model atmosphere, based on extended measurements of the flux at a wide range of wavelength. Metallicity is given by observations and is supposed to be derived from detailed analysis. We examine a present situation, situation 1 ($Si1$), that includes the HIPPARCOS enormous progress on distance measurements and a future one, situation 2 ($Si2$) that includes a sub-milliarcsecond mission and the foreseeable progresses in stellar atmosphere modeling.

Presently, around 10 000K uncertainties of 200K on effective temperature and 0.1 magnitude on bolometric correction are reachable. With future progress we can expect to get an accuracy of 40K for T_{eff} and 0.02 magnitude for BC, at least for a number of reference objects.

For chemical composition, the present uncertainty on Z is of the order of 0.006 (Edvardsson *et al.*, 1993) and one can hope to reduce it by a factor 3 to 5. The uncertainty on Y depends on the Z-uncertainty as well as on the value of the enrichment factor for the particular star considered. The present uncertainty is estimated at 0.04, and a reduction by a factor 3 is assumed for situation 2.

For luminosity, since these age indicators are quite far, we choose a typical distance $r = 200pc$, necessary to handle a sufficient sample of targets. In the present situation, distances cannot be obtained directly, even with HIPPARCOS, and one has to rely on photometric calibrations. A direct determination will be possible with the next astrometric mission, if it reaches an accuracy 20 times higher than HIPPARCOS. Table 2 summarizes these estimates.

For the "physical " terms, we have chosen: $S = 0.17\ S_0\ =\ 0.54$ keVb, $\Delta O_p = 0.2$, $\Delta\alpha = 0.3$, $\Delta O_v = 0.15$.

TABLE 2.

	$\Delta logT_{eff}$	ΔBC	$\Delta r/r$	$\Delta logL$	ΔZ	ΔY
Si1	0.01	0.1		0.08	0.006	0.04
Si2	0.002	0.02	0.02	0.025	0.002	0.015

Table 3 gives the values of the different contributions for the two models $M1$ and $M2$, in the two situations $Si1$ and $Si2$, and a typical object at 200 pc. We note, for instance, ΔA_T the contribution of the term ΔT in equation (3).

Presently, age determinations between $1.4M_\odot$ and $2M_\odot$ (i.e. ages from 0.7 to 3 10^9 yr) cannot reach an accuracy better than 30 to 40%.

For stars around $1.4M_\odot(M1)$, the situation is quite complex. Age determinations are sensitive to both the observational terms of the atmospheric analysis and the physical terms from the hydrodynamical processes, as outer convection and overshooting. For the $2M_\odot(M2)$ star, the relative importance of the different terms changes from $Si1$ to $Si2$. In situation 1, distances determinations contribute significantly to the uncertainty on age. For situation 2, the overshooting term clearly becomes the leading term. To get a maximum advantage of an increased accuracy on atmosphere modeling and distance determinations, one is bound to fix it more precisely.

TABLE 3. (expressed in %)

		ΔA_T	ΔA_L	ΔA_{S0}	ΔA_{Op}	ΔA_α	ΔA_{Ov}	ΔA_Y	ΔA_Z	ΔA *total*
Si1	M1	12	2.5	0.02	0.2	14	12	21	16	35
Si1	M2	11	7	1.2	4	0.3	24	22	19	40
Si2	M1	3	0.8	0.02	0.2	14	12	10	5	21
Si2	M2	3	2.5	1.2	4	0.3	24	8	5	25
$Si2^*$	M1	3	0.8	0.02	0.2	5	3	5	5	10
$Si2^*$	M2	3	2.5	1.2	4	0.1	6	4	5	10

* including the improvements due to seismology on O_v, α, Y, see $ 5.

5. A Need for Seismology

The advent of helio- and asteroseismology has brought new observable quantities, the eigenfrequencies, which can be used to built a model for a given object. Table 1 can then be complemented by several lines, associated with seismological quantities.

Oscillations are presently observed in several δ Scuti stars (A/F stars) in the low frequency domain which contains low order modes (Michel *et al.*, 1992). Some of these modes (Goupil *et al.*, 1992;Dziembovsky and Pamyatnykh, 1991) are very sensitive to the structure of the inner regions, in particular to the μ gradient layer produced by the recession of the convective core.

As an example, Table 4 gives the derivatives of the frequencies of some low order modes, for a model of $1.8M_{\odot}$, slightly evolved ($X_c = 0.44$), at an age of 767 10^6 yr. This demonstrates the sensitivity of the "mixed mode", G_1-mode ($l = 1, n = 1$, usual nomenclature), to the overshooting parameter O_v, as compared for instance to a radial mode $(0, 1)$. Fig. 2a shows how three well chosen modes can determine O_v. It illustrates nicely the nonlinearity of these indicators, confirming that a preliminary guess of the model is necessary. With three frequencies $\nu_{0,1}$, ν_{G1} and $\nu_{0,3}$, three variables can be determined: M, A and O_v, assuming that the other parameters are precisely known.

The accuracy on frequencies measurements can be very high: in self excited modes, it is only limited by the total duration of observations. If we choose an uncertainty of $0.1\mu Hz$ corresponding to an observing run of 40 days, we can expect to determine O_v at $\sim$ 20 %, i.e. $\sim 0.03H_p$. This reduces its contribution to the error on age to 3 % for $M1$ and 6% for $M2$ and brings it to a level comparable to the other terms. The remaining parameters have then to be adjusted using other good quality observables. Fig. 2b shows that, at the level of accuracy proposed for the sub-milliarcsecond mission, luminosity will be sufficiently precise ($\sim$ 2 %) to become as constraining as the frequencies for stellar modelling.

TABLE 4.

mode	$\nu(\mu Hz)$	$\partial Ln\nu/\partial LnA$	$\partial Ln\nu/\partial M$	$\partial Ln\nu/\partial d_{ov}$	$\partial Ln\nu/\partial Y$
$\nu_{0,1}$	172.78	-.985	-.682	.0889	-2.9
$\nu_{0,2}$	223.66	-.990	-.685	.0899	
$\nu_{0,3}$	275.85	-.970	-.661	.0875	
ν_{G1}	192.06	.342	.259	-.856	.7
$\nu_{0,3}/\nu_{0,1}$	1.595	-.0153	-.0210	.0013	-.06

These desired eigenmodes will probably not be excited and measurable in all objects for which we would like to have a primary determination of age. However, it is reasonable to expect that in the region of the HR diagram that we have selected (Fig. 1), which includes the instability strip, enough objects will oscillate in these modes. These objects will serve to assess the value of the overshooting parameter and tell whether it can be used as an "universal" value, applicable to non oscillating stars. Moreover if a larger number of modes were observed, a parallel treatment, not discussed here, could be done to reduce ∂A_{α} and ∂A_Y.

6. The Sub-milliarcsecond Mission Contribution

Once the overshooting parameter is fixed at $\sim$ 20 % and the errors due to α and Y are reduced, the contribution of the physical terms remains around 6%, i.e. slightly below the foreseen level for the "chemical composition terms". With the present accuracy on distance, luminosity has to be determined through absolute magnitude calibrations and remains one of the major source of error on age. In situation 2, the luminosity term is similar to the others, as given on the two last lines of Table 3. The hypothesis we made on future progress in atmosphere modelling lead to comparable error terms on luminosity due to errors on distances and to errors on bolometric correction. If the sub-milliarcsecond mission was to reach a level of accuracy higher than 20 times the HIPPARCOS one, the contribution of the distance terms would fall below the "atmospheric ones".

The other "physical terms" will probably be improved by the study of a significant sample of objects, as physical effects are "universal". On the contrary, the atmosphere analysis has to be performed on individual objects, and in this primary phase of scaling, it will be difficult to accept general trends, except for groups. The strong dependence of age on Y remains a difficulty which has to be solved by seismology.

We have shown here that a sub-milliarcsecond ~~mission~~ will significantly participate to the reduction of the uncertainties in age determinations below $3\,10^9$ yr, accompanying the efforts in other fields, to reach an age accuracy of $\sim 10\%$.

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