

# Why *prāṇa* is the most excellent among the vital functions, or: the Shapley value in the *Upaniṣads*

Harald Wiese<sup>1</sup> 

Universität Leipzig, Leipzig, Germany

[wiese@wifa.uni-leipzig.de](mailto:wiese@wifa.uni-leipzig.de)

## Abstract

This paper deals with the contest of the “vital functions” for superiority in the *Bṛhadāraṇyaka Upaniṣad* and the *Chāndogya Upaniṣad*, in the *Aitareya Āraṇyaka* and others. The paper concentrates on two particular approaches to deciding the superiority question, namely (i) singly leaving or entering (breath, sight, etc. leave one after another or enter one after another), and (ii) alternating withdrawal (breath leaves and returns, sight leaves and returns). The paper defends two claims. First, it can be shown that the commentators were aware of the generalizable nature of these two approaches. Second, these two approaches are closely related to the so-called Shapley value developed in cooperative game theory.

**Keywords:** Shapley value, Cooperative game theory, Balancing operation, Vital functions, Superiority

## 1. Introduction

Comparisons of the natural body with a political one have been common in many cultures: Egyptian, European, Greek, Indian and Roman.<sup>2</sup> I will focus on a specific aspect of organic theories, namely disputes about rank order. From the *Western* point of view, one of Aesop’s fables is most relevant, which deals with the quarrel between the belly and the feet about their relative importance:<sup>3</sup>

The belly and the feet were arguing about their importance, and when the feet kept saying that they were so much stronger that they even carried the

- 1 The author is grateful for many helpful hints by Ranabir Chakravarti, Sadananda Das, Katharina Lotzen, Karin Preisendanz, Maria Schetelich, Alexander Singer, Walter Slaje, Thomas Voss and Jan Warzok.
- 2 A brief introduction is presented by Ilsley Hicks (1963). Harvey (2007) is a book-length introduction that deals with the body politic from ancient times to the present. Nederman (2004) covers the use of organic metaphors in the *European Late Middle Ages*. Shogimen (2008) shows how the healing of human bodies provides metaphors for dealing with political problems, juxtaposing *Late Medieval Europe* and *Japan*.
- 3 The dating of this fable is very difficult. It may have originated sometime between 1000 BCE and 100 CE, see Harvey 2007: 4–5. The fragmentary Egyptian version (see Erman 1927: 173–4) may be older and date from the first half of the first century BCE (see Ilsley Hicks 1963: 29).

stomach around, the stomach replied, “But, my good friends, if I didn’t take in food, you wouldn’t be able to carry anything.”<sup>4</sup>

In the Indian context, one finds the contest of the “vital functions” breath, speech and the like for superiority. This contest is presented in different versions in the Bṛhadāraṇyaka Upaniṣad, the Chāndogya Upaniṣad (both from seventh to sixth centuries BCE),<sup>5</sup> in the Aitareya Āraṇyaka (sixth to fifth centuries BCE)<sup>6</sup> and others.<sup>7</sup> Olivelle (1998) translates *prāṇa* or *karman* as “vital function”.<sup>8</sup> In contrast, breath as one particular member among the other vital forces is called “breath” or “central breath” (*prāṇa* or *madhyamaḥ prāṇaḥ*). I follow Olivelle in this respect.<sup>9</sup>

Indologists have, of course, noted the “Rangstreitfabel” (Ruben 1947) and the importance of breath (Frauwallner 1997: 41–5). A detailed discussion of the respiratory term *prāṇa*, in particular in contrast to *apāna* (where the former means exhalation and the latter inhalation), is provided by Bodewitz (1986). Zysk (1993) analyses the five bodily winds from *prāṇa* down to *vyāna* and the different understanding adopted in *Āyurveda* on the one hand and in *Yoga* on the other hand. Under the heading of “Soul, body and person in Ancient India”, Preisendanz (2005) presents the differing and changing, but related, conceptions of *prāṇa*, *asu* and *ātman*.

As a specific Indian example of the contest, death succeeds in capturing the vital functions speech, sight and some unspecified others, but not breath in BĀU 1.5.21. This fact shows the superiority of breath (see Section 2.1). The contest is explicitly framed as a competition between the vital functions. Without a competition *expressis verbis*, the superiority of breath also clearly emerges elsewhere.<sup>10</sup> One can conceive of Aesop’s fable and these Indian tales as presenting *idiosyncratic* solutions to the problem of superiority inasmuch as it is not obvious at all how they might be generalized to apply to other problems of superiority, say, concerning the relationship between people working together in a common joint venture or between the countries of the European Union (EU).

In my mind, the above versions clearly differ from other Indian ones where the vital functions avail themselves of some non-idiosyncratic method to assess their superiority. In this paper, I will concentrate on these *generalizable* approaches. I do not want to offer a definition of “generalizability” in general. However, and with a view to the texts covered in this paper, one may argue that generalizability may refer to some mode or manner that is

- (a) a test for something (cf. *parīkṣaṇa*);
- (b) teachable (cf. *prakāropadeśa*);

4 Daly 1961: 148.

5 This chronology follows Olivelle (1998: 12), who cautiously adds “give or take a century or so”. Bronkhorst (2007: 173–262) disputes this chronology and argues that the present form of these Upaniṣads was reached only a few centuries later.

6 See Olivelle 1998: 8, 12–13. The Aitareya Āraṇyaka contains the Aitareya Upaniṣad, which does not address the contest of the vital functions.

7 For example, Śāṅkhāyana Āraṇyaka (ŚĀ 9.1–7) and Kauṣītaki Upaniṣad (KauU 2.14).

8 This translational choice seems sensible also in view of Preisendanz (2005: 125).

9 Generally, translations are mine unless indicated otherwise.

10 For example, ChU 4.3.3 characterizes breath as the “gatherer” (Olivelle 1998: 217) into whom the other vital functions pass when a man sleeps.

- (c) applicable beyond the actual application (cf. *yathā loke* and *cetanāvanta iva puruṣāḥ*); and  
 (d) serves to ward off struggle or competition (cf. *spardhānivāraṇārtham*).<sup>11</sup>

In the ancient Indian texts, there are two generalizable approaches to the problem of superiority. Both involve the difference a vital function makes. The texts employ (i) singly leaving or entering and (ii) alternating withdrawal. I want to turn to the “singly leaving or entering” approach first. In AĀ 2.1.4, the superiority of breath is established in two different ways. The vital functions first leave the body one after another, and then they re-enter, again serially. Breath is the last to leave and the last to re-enter and makes the decisive difference. Turning to the above example of the EU, one may, at least in principle, consider how the remaining countries of the EU would fare if Great Britain and then France, etc. would leave the EU.

The second generalizable approach could be labelled the approach “involving alternating withdrawal” or the “where would you be without me” approach. This approach is seen in BĀU 6.1, ChU 5.1 and ŚĀ 9.1–7.<sup>12</sup> Speech leaves the body and re-enters after a while. The remaining functions<sup>13</sup> are then asked how they fared. Then, the same procedure is followed by other vital functions. It turns out that the departure of breath could not be endured and that, hence, breath is superior. In the example above, a country, like Poland or Portugal, may confront the others with the prospect of leaving the EU. Perhaps the others would fare worse after Poland’s exit than after the exit of Portugal.<sup>14</sup>

The aim of this paper is twofold. Firstly, it can be shown that the commentators were aware of the generalizable nature of the “singly leaving or entering” approach and the approach “involving alternating withdrawal”. Secondly, these two approaches are closely related to the so-called Shapley (1953) value developed in cooperative game theory. I will first present the relevant stories of the contest of the vital functions in the next section. In Section 3, I will then present the Shapley value and discuss the relation between this concept and the literature on the contest of the vital functions. Section 4 concludes the paper.

## 2. The contest among the vital functions

### 2.1. Idiosyncratic approaches

The following story of a contest from the Bṛhadāraṇyaka Upaniṣad is an example for an idiosyncratic approach, in the sense of not presenting a procedure that may be applicable as a solution to a wide range of problems concerning superiority. BĀU 1.5.21 says:

- 11 All these quotations are contextualized and discussed in Section 2.2 and Section 2.3.  
 12 Translation by Keith (1908: 57) and Bodewitz (2002: 73–7).  
 13 The order is speech, sight, hearing, mind, semen, and breath in BĀU 6.1. In ChU 5.1 and ŚĀ 9.1–7, the order is the same, but without semen.  
 14 Yet another method of determining superiority is described in PU 2 (Olivelle 1998: 461). There breath shows its power by “setting off” (*utkram*) and then “setting down” (*pratiṣṭhā*) again. Since the other vital forces – speech, mind, sight, hearing – have to follow suit in the case of both movements, they thereby acknowledge the superiority of breath. This approach is not covered in the current paper.

*prajāpatir ha karmāṇi saṣṭje |*  
*tāni sṛṣṭāny anyo 'nyenāspardhanta |*  
*vadiṣyāmy evāham iti vāg dadhre |*  
*drakṣyāmy aham iti cakṣuḥ*

...

*tāni mṛtyuḥ śramo bhūtvopayeme |*  
*tāny āpnot | tāny āptvā mṛtyur avārundha |*

...

*athemam eva nāpnod yo 'yaṃ madhyamaḥ prāṇaḥ |*  
*tāni jñātuṃ dadhrir | ayaṃ vai naḥ śreṣṭho*

Prajāpati created the vital functions.

Once they were created, they began to compete with each other.

Speech threw out the challenge: “I am going to speak!”

Sight shot back: “I am going to see!”

...

Taking the form of weariness, death took hold of them; it captured and shackled them.

...

The central breath alone, however, death could not capture.

So they sought to know him, thinking: “He is clearly the best among us . . .”<sup>15</sup>

Likewise, testing how the vital functions respond to being “riddled with evil” can be counted among the idiosyncratic solutions to superiority challenges. Using Olivelle’s (1998: 171–3) translation of ChU 1.2.1–7, the gods, who were fighting the demons, venerate the High Chant successively as breath within the nostrils, speech, sight, hearing, mind, and breath within the mouth. The demons “riddle with evil” (*pāpmanā vividhuḥ*) these functions from breath within the nostrils all the way to mind, but they fail to do the same with breath within the mouth.<sup>16</sup>

Commenting on a part of ChU 1.2.8, Śaṅkara (ChU\_Ś: 20, ll. 3–6), who lived perhaps sometime between 650 CE and 800 CE,<sup>17</sup> explains the difference of breath within the nostrils and breath within the mouth:

*nanu nāsikyo 'pi prāṇo vāyvātmā yathā mukhyas, tatra nāsikhyaḥ prāṇaḥ*  
*pāpmanā viddhaḥ prāṇa eva san na mukhyaḥ katham |*  
*naiṣa doṣaḥ | nāsikyas tu sthānakaraṇavaigunyaḥ viddho vāyvātmā 'pi*  
*san, mukhyaḥ sthānadevatābalīyastvān na viddha iti yuktam |*

Objection: Breath within the nostrils is also of the nature of wind, like breath within the mouth. How can it be in that regard that breath within the nostrils which is just breath is riddled with evil, but not breath within the mouth?

15 Olivelle 1998: 57.

16 A very similar sequence is described in BĀU 1.3.1–7. There, using Olivelle’s (1998: 39–41) words, the vital functions speech, breath, sight, hearing, mind, and breath within the mouth have to “sing the High Chant”. The demons “riddle with evil” all these functions except breath within the mouth.

17 See Isayeva 1993: 83–7.

Answer: This fault does not apply. Due to the bad quality regarding its location and sense organ, breath within the nostrils is pierced [riddled with evil] although it is of the nature of wind. [In contrast,] breath within the mouth is not pierced due to its strength of location and [presiding] deity. This is reasonable.

## 2.2. Singly leaving or entering

Singly leaving (and singly entering) the body is the first generalizable approach. It is described in a story within the first chapter of the second book of the Aitareya Āraṇyaka, addressed by AĀ 2.1. In that chapter, the second part deals with the hymn (*uktha*) as in AĀ 2.1.2.1:<sup>18</sup>

*uktham uktham iti vai prajā vadanti  
tad idam evoktham iyam eva pṛthivīto hīdaṃ sarvam uttiṣṭhati yad idam  
kiñ ca |*

People say, “Hymn, hymn.”

The hymn is just this earth. For from it all that exists springs.<sup>19</sup>

Indeed, the hymn is the sky (*antarikṣa*), yonder heaven (*dyau*), man (*puruṣa*) etc.<sup>20</sup> Jumping to the fourth part of AĀ 2.1, Brahman (*brahman*) enters into man (*puruṣa*), first into his feet and finally, having worked his way upwards, into the head of man.<sup>21</sup> The entering of the head by Brahman seems to bring to life the head with its vital functions. Then, the head’s vital functions compete for being the hymn (*uktha*) (AĀ 2.1.4.7–11):

*tā etāḥ śīrṣaṅ chriyaḥ śritāś cakṣuḥ śrotraṃ mano vāk prāṇaḥ | (7)*

*śrayante ’smiñ chriyo ya evam etac chirasaḥ śirastvaṃ veda | (8)*

*tā ahiṃsantāham uktham asmy aham uktham asmīti | (9)*

*tā abruvan hantāsmāc charīrād utkrāmāma*

*tad yasmin na utkrānta idam śarīraṃ patsyati tad uktham bhaviṣyatīti | (10)*

*vāg udakrāmād avadann aśnan pibann āstaiva (11)*

These delights settled in the head, sight, hearing, mind, speech, breath. (7)

Delights settle on him who knows thus why the head is the head. (8)

They strove together, saying, “I am the hymn, I am the hymn.” (9)

They said, “Come, let us leave this body,

then that one of us at whose departure the body falls, will be the hymn.” (10)<sup>22</sup>

18 The last digit is added by the current author in line with the *daṇḍas* used by Keith (1909).

19 Keith 1909: 201.

20 AĀ 2.1.2 with English words from Keith 1909: 201–2.

21 AĀ 2.1.4 with English words from Keith 1909: 204.

22 Keith 1909: 205. The formula in AĀ 2.1.4.8 *ya evam ... veda* is quite common in the Brāhmanas. Olivelle (see, for the specific case of *arka*, BĀU 1.2.1 and Olivelle 1998: 37) would translate it as “who knows the name and nature of head in this way”. It is conceivable to rearrange *tā abruvan* in AĀ 2.1.4.10 so that it immediately follows *uktham asmīti* (AĀ 2.1.4.9). I like to stick to Keith’s translation which is in line with the commentary quoted below.

Speech went out, yet [the body, while] speechless, indeed remained [still] eating and drinking. (11)<sup>23</sup>

The sequence of leaving is the following: speech, sight, hearing, and mind. Finally, breath leaves the body (AĀ 2.1.4.15):

*prāṇa udakrāmat tatprāṇa utkrānte 'padyata*  
Breath went out. When that breath went out, [the body] fell.<sup>24</sup>

The commentary ascribed to Sāyaṇa dating, perhaps, to the fourteenth century CE<sup>25</sup> uses the “eating and drinking” from AĀ 2.1.4.11 to argue quite specifically why breath is the winner (AĀ\_Sā: 111, ll. 11–15):

*vākcakṣuḥśrotramanasām ekaikasminn utkrānte sati tattadindriyasādhyā  
vyāpāramātraṃ lupyate, na tu śarīraṃ patati, kiṃtv annapāne svīkurvan  
yathāpūrvam abhavad eva ... tac charīraṃ prāṇa utkrānte sati patitam  
abhūd, na tv aśnāti nāpi pibati*

When speech, sight, hearing, and mind depart individually, only the effective operation of the respective organs is taken away, but the body does not fall. But taking in food and drink, it indeed remained as before. ... When breath departs, this body fell and did neither eat nor drink.

While the victory of breath must have been obvious to the vital functions, they reaffirm the result by resolving to enter the body rather than leaving it. The sequence of entering is the same as the sequence of leaving. The result is as expected and, this time, the conclusion is accepted as is described in AĀ 2.1.4.20, 24:

*vāk prāviśat aśayad eva | (20)*  
*prāṇaḥ prāviśat, tatprāṇe prapanna udatiṣṭhat tad uktham abhavat | (24)*  
Speech entered, [the body] lay still. (20)<sup>26</sup>

23 While there is nothing wrong with the translation by Keith (1909: 205) (“Speech went forth, yet (the body) remained, speechless, eating, and drinking”), I chose to bring out the apparent meaning more clearly by making some minor changes.

24 My translation is a bit more literal than the one by Keith (1909: 205). Apparently, there is an absolute locative here (*prāṇa(e) utkrānte*). If *tat* and *prāṇa* are two separate words, *tat* might be understood in the sense of *tataḥ* yielding the translation “Then, when breath went out”. If a *tatpuruṣa* compound is meant, one can translate *tatprāṇa utkrānte* as (i) “when the breath of that [head, body, or man] went out” (genitive *tatpuruṣa*) or (ii) “when that breath went out” (nominative *tatpuruṣa* or *karmadhāraya*), i.e. *tasmin prāṇa utkrānte*. I have opted for this latter variant. These differences are unimportant.

25 See Burnell 1873: vi–xv. Slaje (2010) deals with the very doubtful authorship of Sāyaṇa with respect to commentaries on the four Veda-Saṃhitās, but he does not offer an opinion on whether Sāyaṇa might be the author of the Aitareya Āraṇyaka (Slaje 2010: 395, fn. 48).

26 Keith 1909: 205. Current author replaced round brackets with square brackets.

Breath entered. When that breath entered, it [the body] arose and it [breath] became that hymn. (24)<sup>27</sup>

In KauU 2.14,<sup>28</sup> the vital functions enter the body (which is supine) one after another. Only after breath has entered, is the body able to get up.<sup>29</sup> No leaving sequence is described in that Upaniṣad. The procedure of singly entering together with raising the body is of particular relevance in relation to the “etymology” given in BĀU 5.13.1:

*uktham | prāṇo vā uktham | prāṇo hīdaṁ sarvam utthāpayati . . .*  
*Uktha.* The *uktha* (“Ṛgvedic hymn”), clearly, is breath, for breath raises up (*utthā*) this whole world.<sup>30</sup>

For the purpose of this paper, AĀ 2.1.4.9–10 above in this section is central. Sāyaṇa points to the purpose of the discussion, namely, to state the superiority of breath (*prāṇasya śraīṣṭhyam*).<sup>31</sup> He then explains (AĀ\_Sā):<sup>32</sup>

*tāḥ pūrvoktāḥ śriyaś cakṣurādirūpās tadabhimāninyo devatā ahimsanta*  
*parasparaspardhārūpāḥ hiṁsām akurvan |*  
*spardhāviśayo viśpaṣṭam ucyate – aham uktham asmi |*  
*ukthe cakṣuḥsvarūpasya mamaiva dṛṣṭiḥ kartavyety evaṁ cakṣurdevatā*  
*vakti | . . .*  
*tāḥ spardhamānā devatāḥ spardhānivāraṇārthaṁ samayaviśeṣaṁ para-*  
*sparam abruvan*

The previously mentioned delights whose handsome form is sight and so on [referring to *śriyaḥ . . . cakṣuḥ* etc. in the *mūla* text], who are proud [presiding] deities with regard to that [*tad*, namely sight and so on], strove together, i.e. they committed violence in the form of mutual competition. The object of competition is clearly expressed [when they say]: “I am the hymn.”

With respect to the hymn the deity of sight says in this manner: “The faculty of seeing can be performed by me alone who has the specific nature of sight.”

...

In order to ward off [this kind of] competition, these competing deities mutually told<sup>33</sup> a particular agreement [or: agreed on a particular treaty].

27 See fn. 24 for *tatprāṇe*. Concerning *tad uktham abhavat*, the Sāyaṇa commentary offers a little twist: “*tat prāṇasvarūpam utthānahetutvād uktham abhūt* (AĀ\_Sā: 112, ll. 12–13): “*Tat* means the specific nature of breath. By arising, it became the hymn.”

28 *athāto niḥśreyasādānam | etā ha vai devatā ahaṁśreyase vivadamānā asmāc charīrād uccakramuḥ | tad dārubhūtaṁ śiśye | athainad vāk praviveṣa [sic] | tad vācā vadac chiśya eva | . . . athainat prāṇaḥ praviveṣa | tat tata eva samuttasthau | . . .*

29 Translations by Olivelle (1998: 344) and Bodewitz (2002: 40–2).

30 Olivelle 1998: 137.

31 AĀ\_Sā: 110, just before quoting AĀ 2.1.4.9–10, l. 8 from the bottom.

32 p. 110, l. 3 from the bottom up to p. 111, l. 2.

33 The inclusion of *tā abruvan* in the commentary lends support to the translation of AĀ 2.1.4.10 above.

According to the last sentence, Sāyaṇa acknowledges that the agreement (i.e. singly leaving or singly entering) is done for the purpose of warding off competition (*spardhānivāraṇārtham*). Thus, the competition that consists in simply insisting on one's superiority (*aham uktham asmi*) is warded off in favour of a competition by way of a controlled experiment. To the commentator's mind, this experiment amounts to a generalizable manner of deciding the superiority question. This is aspect (d) of generalizability mentioned in the introduction.

### 2.3. Alternating withdrawal

BĀU 6.1.7–8 most clearly brings out the approach involving alternating withdrawal:

*te heme prāṇā ahaṁśreyase vivadamānā brahma jagmuḥ |*  
*tad dhocuh ko no vasiṣṭha iti |*  
*tad dhovāca yasmin va utkrānta idaṁ śarīraṃ pāpīyo manyate sa vo*  
*vasiṣṭha iti || (7)*  
*vāg ghoccakrāma | sā saṃvatsaram proṣyāgatyo vāca katham aśakata*  
*madrte jīvitum iti |*  
*te hocuh yathā kalā avadanto vācā prāṇantaḥ prāṇena paśyantaś cakṣuṣā*  
*śṛṅvantāḥ śrotreṇa vidvāṁso manasā prajāyamānā retasaivam ajīviṣmeti |*  
*praviveśa ca vāk || (8)*

Once these vital functions were arguing about who among them was the greatest. So they went to *brahman* and asked: “Who is the most excellent of us?” He replied: “The one, after whose departure you consider the body to be the worst off, is the most excellent among you.” (7)

So speech departed. After spending a year away, it came back and asked: “How did you manage to live without me?” They replied: “We lived as the dumb would, without speaking with speech, but breathing with the breath, seeing with the eye, hearing with the ear, thinking with the mind, and fathering with semen.” So speech re-entered. (8)<sup>34</sup>

After speech has left and re-entered, the very same procedure is followed by sight, hearing, mind, and semen. When breath is about to leave, the other vital functions realize the serious consequences (BĀU 6.1.13–14):

*atha ha prāṇa utkramiṣyan yathā mahāsuḥayaḥ saindhavaḥ paḍvīśa*  
*śaikūn saṃvṛhed evaṁ haivemān prāṇān saṃvavarha |*  
*te hocur mā bhagava utkramiḥ |*  
*na vai śakṣyāmas tvadrte jīvitum iti |*

34 Olivelle 1998: 143. The compound *ahaṁśreyase* in BĀU 6.1.7 could be in the dative (consonantal stem *ahaṁśreyas*) or in the locative (thematic stem *ahaṁśreyasa*). According to PW, *vivad* is usually employed with the locative of the disputed subject matter. Dative is understood by Śaṅkara who glosses *ahaṁśreyase* with *aham śreyān ity etasmāi prayojanāya* (BĀU\_Ś: 416, l. 13). He uses the similar expression *ahaṁśreṣṭhatāyāi vivadantaḥ* in the commentary on the Chāndogya Upaniṣad (ChU\_Ś: 265, l. 16).

*tasyo me baliṃ kuruteti |  
tatheti ||* (13)

*sā ha vāg uvāca yad vā ahaṃ vasiṣṭhāsmi tvam tad vasiṣṭho 'sīti ...* (14)

Then, as the breath was about to depart, it strongly pulled on those vital functions, as a mighty Indus horse would strongly pull on the stakes to which it is tethered.<sup>35</sup> They implored: “Lord, please do not depart! We will not be able to live without you.” He told them: “If that’s so, offer a tribute to me.” “We will,” they replied. (13)

So speech declared: “As I am the most excellent, so you will be the most excellent.” ... (14)<sup>36</sup>

Apparently, breath’s threat of withdrawal is more damaging to speech than the corresponding threat of speech is to breath. This very fact is the basis for breath’s demand for a tribute.

Commenting on a part of BĀU 6.1.13, Śaṅkara (BĀU\_Ś: 417, ll. 17–20) explains:

*te vāgādayo hocur he bhagavo bhagavan motkramīr  
yasmān na vai śakṣyāmas tvadṛte tvāṃ vinā jīvitum iti  
yady evaṃ mama śreṣṭhatā vijñātā bhavadbhir aham atra śreṣṭhas  
tasya u me mama baliṃ karaṃ kuruta karaṃ prayaccheti  
ayaṃ ca prāṇasamvādaḥ [sic] kalpito viduṣaḥ  
śreṣṭhaparīkṣaṇaparakāropadeśaḥ |  
anena hi prakāreṇa vidvān ko nu khalv atra śreṣṭha iti parīkṣaṇaṃ  
karoti |*

They, i.e. speech and the others, implored: “Oh Lord (using an alternative form of the vocative), please do not depart! For we will not be able to live without you (glossing *tvadṛte* with *tvāṃ vinā*.)” [Breath replies:] “If my superiority is recognized by you in this manner, I am the best here. If that is indeed so, offer a tribute (*bali* glossed with *kara* (tax)) to me (*me* glossed with *mama*)”, i.e. pay a tax.

And this agreement of the vital functions is imagined on the part of a learned person as a teaching of a mode of testing superiority.

For in this manner a learned person performs the test of who, indeed, is the best here.

This version of the story in the BĀU is very close to one found in ChU 5.1. While breath does not explicitly demand a tribute, the other vital functions

35 This first sentence is taken from Olivelle (1998: 145) with the important exceptions that “uprooted” (Olivelle) has been replaced by “strongly pulled on” and similarly “would uproot” (Olivelle) by “would strongly pull on”. Wezler (1982/1983) has examined *samvṛh* in BĀU 6.1.13 and the parallel *samkhid* in ChU 5.1.12 in astounding detail. While Olivelle’s translation closely follows most previous translations, Wezler’s arguments against “uproot” are convincing. Among other arguments, Wezler discusses the meanings of the prefix *sam*. Importantly for this paper, breath does not leave the body or “uproot” the other vital functions, but just threatens to do so.

36 Olivelle 1998: 145.

offer their tributes in ChU 5.1.13–14 similar to BĀU 6.1.14. Śaṅkara (ChU\_Ś: 165, l. 8) comments:

*atha hainaṃ vāgādayaḥ prāṇasya śreṣṭhatvaṃ kāryeṇāpādayanta āhur  
balim iva haranto rājñe viśaḥ ...*

Speech and the rest, establishing, by their action, the *superiority* of Breath, said to him – making offerings like the people to their King ...<sup>37</sup>

Indeed, the tribute (*bali*) offered to the best (*śreyas*) is a familiar topic. As ŚB 11.2.6.14 (p. 842) states:

*... śreyase pāpīyān baliṃ hared vaiśyo vā rājñe baliṃ haret ...*

... an inferior brings tribute to his superior, or a merchant brings tribute to the king ...

Thus, the reason behind the tribute may lie in the fact that the competition of the vital functions serves as a “political allegory where the superiority of *prāṇa* in relation to the other vital functions is likened to the supremacy of the king among his rivals and ministers” (Black 2007: 122). While this is certainly true,<sup>38</sup> the tribute can also be seen as serving a specific purpose in the context of the approach taken in this paper (see Section 3.3).

Now, turning to the main topic of the current paper, with the last two sentences of the above commentary on BĀU 6.1.13 (*ayaṃ ca prāṇasamvādaḥ ... iti parīkṣaṇaṃ karoti*), Śaṅkara explains the agreed-upon withdrawal. He makes abundantly clear that he considers the threat of withdrawal a generalizable procedure. In particular, he talks about a test (*parīkṣaṇa*, see (a) in the introduction) and a method that is teachable (*prakāropadeśa*, see (b)).

Similarly (also with the words *kalpito viduṣaḥ*), Śaṅkara comments on the purpose of this method in his Chāndogya-Upaniṣad commentary (ChU\_Ś: 167, ll. 3–4):

*vāgādīnāṃ ceha samvādaḥ kalpito viduṣo ’nvayavyatirekābhyāṃ  
prāṇasreṣṭhatānirdhāraṇārtham*

*yathā loke puruṣāḥ anyonyam ātmanaḥ śreṣṭhatāyai vivadamānāḥ kañcid  
guṇaviśeṣābhijñāṃ prcchanti ko naḥ śreṣṭho guṇair iti*

And this agreement by speech and so on is imagined on the part of a learned person in order to determine the superiority of breath with the method of concomitant presence and concomitant absence,<sup>39</sup> as people in the world who mutually argue about their own superiority ask

37 Jha 2005: 225.

38 See also Rau (1957: 34) and Bodewitz (1992: 57).

39 Jha (2005: 227) translates *anvayavyatirekābhyāṃ* as “by means of negation and affirmation”. Halbfass (1991: 162–77) analyses Śaṅkara’s use of this term. For the application at hand, “concomitant presence” (p. 170) means that the presence of breath goes together with the presence of life. In contrast, “concomitant absence” refers to the simultaneous absence (or rather threat of absence) of breath and (threatening) loss of life.

somebody who is knowledgeable about special qualities: “who of us is the best in terms of qualities”?

The second sentence in the quotation above (*yathā loke . . .*) points to the wider applicability of the approach involving alternating withdrawal, just as suggested by (c) in the introduction. Consider a second piece of evidence where Śāṅkara (ChU\_Ś: 165, ll. 16–17) presents the following objection against this method:

*nanu katham idaṃ yuktaṃ cetanāvanta iva puruṣā ahaṃśreṣṭhatāyai viva-  
danto 'nyonyaṃ spardherann iti |  
na hi cakṣurādīnāṃ vācaṃ pratyākhyāya pratyekaṃ vadaṇaṃ sambhavati  
tathā 'pagamo dehāt punaḥ praveśo brahmagamaṇaṃ prāṇastutir vopa-  
padyate |*

How could this be logical, namely that [the vital functions] can compete against each other by arguing about who among them is the greatest, as conscious humans would. For speaking one by one is not possible for sight and so on, excepting speech. Likewise, departing from the body, entering again, going to Brahman, or praising breath are not reasonable.

While Śāṅkara’s reply is not helpful for the present purpose, it needs to be noted that he considered conscious humans (*cetanāvantaḥ puruṣāḥ*) the most obvious contenders in such fights for superiority, in line with (c) in the introduction. Thus Śāṅkara presupposes a wider applicability of this method.

### 3. The Shapley value

#### 3.1. Basic definitions

Before linking the Shapley value to the pre-modern Indian contest of the vital functions, a short tutorial is called for. The Shapley value belongs to the realm of cooperative game theory.<sup>40</sup> This theory presupposes  $n$  players, collected in a set  $N = \{1, 2, \dots, n\}$ , and a so-called coalition function  $w$ . The players are supposed to “cooperate” in any economic, political or social venture. Coalition functions are meant to reflect the “production” possibilities of groups of players. Production is to be understood in a wide sense and may refer to economic production in a narrow sense, but also point to other social or political frameworks.

A subset  $K$  of  $N$  is called a coalition.  $N$  itself is called the grand coalition. To each coalition, the coalition function attributes a “worth”  $w(K)$ . The worth stands for the economic, social, political or other gain that the particular group of players can achieve (“create”) by cooperating. A worth can only be created if at least one player is present, i.e. the empty set  $\emptyset$  creates the worth zero,  $w(\emptyset) = 0$ . To simplify the notation, I write  $w(i)$  instead of  $w(\{i\})$  for the worth created by player  $i$  (or for the worth of the one-man coalition that hosts only player  $i$ ),  $w(1, 2)$  instead of  $w(\{1, 2\})$  for the worth created by the two players 1 and 2, and  $w(K \cup i)$  instead of  $w(K \cup \{i\})$ .

40 See, for example, ch. 9 in the textbook by Myerson (1991).

The aim of cooperative game theory is to specify payoffs for the players. These specifications are called “solution concepts”. Several solution concepts, i.e. possibilities of how to determine the payoffs, have been explored. For each solution concept, cooperative game theory uses two different approaches to arrive at payoff vectors from coalition functions: (i) The algorithmic approach applies some algebraic manipulations of the coalition functions in order to derive payoff vectors; (ii) The axiomatic approach suggests general rules of distribution. The most famous solution concept is the Shapley value. The two different approaches are presented in Sections 3.2 and 3.3.

**3.2. The algorithmic approach**

The algorithm of the Shapley value builds on the players’ “marginal contributions”. A player’s marginal contribution is the worth of a coalition with him minus the worth of this coalition without him, i.e. the difference he would make. In the following I will focus on two players; for further details and the general case, the reader is referred to the footnotes and the appendix. Player 1 has two marginal contributions, the first with respect to the empty set  $\emptyset$ , where his marginal contribution is  $w(1) - w(\emptyset)$ , the second with respect to the other player, where his marginal contribution is  $w(1, 2) - w(2)$ .<sup>41</sup>

Player 1’s Shapley value is the average of his marginal contributions, taken over all sequences of the two players. For two players, there are just two sequences, player 1 may be first, amounting to sequence (1, 2), or second, amounting to sequence (2, 1). Thus, the players’ Shapley values<sup>42</sup> are

$$Sh_1 = \frac{1}{2}(w(1) - w(\emptyset)) + \frac{1}{2}(w(1, 2) - w(2)) \tag{1}$$

and

$$Sh_2 = \frac{1}{2}(w(2) - w(\emptyset)) + \frac{1}{2}(w(1, 2) - w(1)) \tag{2}$$

The procedure of the vital functions’ singly leaving the body or entering into it (see Section 2.2) is closely related to the algorithmic approach of defining the Shapley value. In AĀ 2.1.4, the sequence of the vital functions that enter the body is speech (*sp*), sight (*si*), hearing (*h*), mind (*m*), and finally breath (*b*).

One might now consider the player set  $N = \{b, sp, si, h, m\}$  and worths for each coalition consisting of one or several vital functions. These worths can in principle be summarized in a coalition function. While numerical values are not mentioned in the examined Indian texts, it seems clear from the text that the “worths” increase the more vital functions are involved. A body with speech, sight, and hearing would be “more alive” than a body with just two

41 For a general player set  $N$ , the marginal contribution ( $MC$ ) of player  $i$  with respect to a coalition  $K$  that does not contain that player is defined by the difference  $MC_i(K) = w(K \cup i) - w(K)$ .

42 For a general player set  $N$ , let  $S$  be the set of sequences. For  $n$  players, there exist  $n! = 1 \cdot 2 \cdot \dots \cdot n$  different sequences. Let  $K_i(s)$  denote the set of players in the sequence  $s$  up to but not including player  $i$ . Then, player  $i$ ’s Shapley value is  $Sh_i = \frac{\sum_{s \in S} MC_i(K_i(s))}{n!}$ .

of these functions. Furthermore, breath’s superiority can be reflected in a coalition function. A specific example is given in the appendix.

In general, the payoffs involved in entering and leaving differ for a given sequence of the vital functions. In the special case of just breath (*b*) and speech (*sp*) as players, consider the sequence (*sp*, *b*). In the entering case, speech’s marginal contribution has to be calculated with respect to the empty set. In the case of leaving, one calculates speech’s marginal contribution with respect to breath. Compare A) and B) in the appendix. Therefore, AĀ 2.1.4 does not mention both the entering and the leaving sequence without effect. In general, AĀ 2.1.4 and the KauU 2.14 do not reflect the Shapley value. Instead, they hint at the payoffs relating to one specific entering sequence, starting from the empty set, and one specific leaving sequence, starting from the grand coalition. The Shapley value for the case of all five vital functions is calculated in C) in the appendix.

**3.3. The axiomatic approach**

For two players, the Shapley value fulfills the following axioms:

Additivity axiom: The sum of the Shapley values equals the worth of the grand coalition, i.e.

$$Sh_1 + Sh_2 = w(1, 2) \tag{3}$$

This property means that (i) all the players “work together”, i.e. the grand coalition forms, and that (ii) the Shapley value distributes the worth of the grand coalition among the players.

Equal-damage axiom: If player 1 withdraws<sup>43</sup> from the game, the damage to player 2 in terms of his Shapley payoff equals the damage that player 1 suffers should player 2 withdraw, i.e.

$$Sh_2 - w(2) = Sh_1 - w(1) \tag{4}$$

Consider the left side of the equation. If player 1 withdraws, player 2 does not obtain the Shapley value *Sh*<sub>2</sub> anymore, but the Shapley value of the game of which he is now the only player. In that game he obtains the worth *w*(2) of his one-man coalition.

Equations (3) and (4) lead to the Shapley values in equations (1) and (2) above where *w*(∅) = 0 should be noted.<sup>44</sup> Cooperative game theorists then say that the axioms expressed by equations (3) and (4) axiomatize the Shapley value. This means that the Shapley value in its algorithmic form (see Section 3.2) fulfills these axioms and that there is no value different from the Shapley value that also fulfills these axioms. This particular axiomatization has been introduced by Myerson (1980).

43 Withdrawal means that the player set is reduced by withdrawing players and that the worths of the remaining players stay the same.

44 For more than two players, we have  $\sum_{i \in N} Sh_i = w(N)$  (compare equation (3)) and the following version of equation (4): Consider any subset *K* of *N*. On the basis of *K* as the new grand coalition, a *K*-game can be defined where the coalitions in that game have the same worth as in the original game. Consider two players *i* and *j* that are members of *K*. If player *i* withdraws from the *K*-game, player *j*’s change in his Shapley payoff equals the change that player *i* endures should player *j* withdraw.

Myerson's axiom is related to the threat of withdrawal addressed in Section 2.3. One may object that the threat uttered by breath ( $b$ ) is more serious than the threat uttered by speech ( $sp$ ). Indeed, BÄU 6.1<sup>45</sup> and ChU 5.1 can be expressed by the inequality

$$w(si, h, m, b) > w(sp, si, h, m) \quad 5$$

or, equivalently,

$$w(sp, si, \dots, b) - w(si, \dots, b) < w(sp, \dots, m, b) - w(sp, \dots, m) \quad 6$$

The first inequality says that the body can get up in the presence of breath even if speech is not present, but not the other way around. The second inequality is equivalent and says that the marginal contribution of speech (left side) is smaller than the marginal contribution of breath (right side). Or, differently put, the damage of withdrawal that breath can inflict in terms of worth is larger than the corresponding damage that speech or the other vital functions can inflict.

At first sight, this inequality seems to contradict equation (4), which can be rewritten in the following manner:

$$\begin{aligned} Sh_b(w \text{ with all players}) - Sh_b(w \text{ with all players except } sp) \\ = Sh_{sp}(w \text{ with all players}) - Sh_{sp}(w \text{ with all players except } b) \end{aligned} \quad 7$$

For the Shapley values after the withdrawal of one of the players, see D) and E) in the appendix. How can it be explained that breath's leaving the body exerts such great damage as seen on the right-hand side of equation (6), but that the threat of withdrawal is balanced by equation (7)?

This seeming puzzle is "solved" in BÄU 6.13 where breath tells the other vital functions: "If that's so [i.e. if I, leaving the body, can exert more damage than you would], offer a tribute to me." Apparently, the tribute is a positive entity. After they reply with "We will," breath's Shapley value includes the *bali*. Now, after having turned over the tribute to breath within the body, i.e. in the grand coalition, speech does not suffer more from breath's leaving the body than breath would suffer from speech's exit. For a concrete coalition function, the *bali* can be calculated (see F) in the appendix.

The mechanism that is at work here has been explained by the sociologist Emerson (1962). He presented a simple and intriguing theory of power and dependence. According to him, whenever a person is more dependent on another one, the relation is unbalanced and calls for "balancing operations". It is best to illustrate this by the following example taken from Emerson's paper.<sup>46</sup> Consider two children A and B. They take turns in playing their favourite games. Their relationship is balanced. Now, one of these two children (child B, say) finds another playing buddy C. B is therefore less dependent on A than before and the relationship of A and B has become unbalanced. As a consequence, B can impose her favourite game on A more often than before. While B still has available the buddy C, not

45 For the present purpose, it is a minor aspect that BÄU 6.1 enumerates six vital functions, among them also semen.

46 For other examples, see Wiese 2009.

available to A, the relationship between B and A has become balanced once again because A gives in to B's wishes more often than before. In sum, Emerson has convincingly argued that social-exchange situations tend to be "balanced" in the long run.

#### 4. Conclusion

While the *Āraṇyakas* and the utilized *Upaniṣads* (being post-Vedic, but pre-classical texts) are normally considered to deal with esoteric, religious and philosophical matters, Black (2007) focuses on the social questions and questions of power that are also involved. The thesis of this paper is that in some of its versions the ancient Indian motif of the contest among the vital functions employs generalizable procedures and that this was obvious to the commentators. In contrast, Aesop's related fable belongs to what I have termed idiosyncratic approaches. I am not aware of any pre-modern solutions to the problem of superiority that were developed outside India and proceed along these generalizable lines.

Turning to pre-modern Indian texts on the problem of superiority, the controversy about *daiva* versus *puruṣakāra* known especially from the Mahābhārata comes to mind. MBh XIII.6 deals with the question of whether divine or human activity is superior.<sup>47</sup> MBh XIII.6.7 presents the following simile:

*yathā bījaṃ vinā kṣetram uptaṃ bhavati niṣphalam  
tathā puruṣakāreṇa vinā daivaṃ na sidhyati*

Just as seed will be fruitlessly sown without a field, so "divine [power]" will not succeed without human activity.<sup>48</sup>

Here, the idea of "where would you be without me" is clearly present.<sup>49</sup> In this example, let  $N = \{b, kṣ\}$ , with  $b$  for *bīja* and  $kṣ$  for *kṣetra*, and let the one-player worths be given by  $w(b) = w(kṣ) = 0$ . Then the Shapley values for *bīja* and *kṣetra* are the same and reflect the idea that both 'divine [power]' and human activity are necessary for success.<sup>50</sup>

A second, but more difficult example, might be found in the Arthaśāstra. In the framework of the seven-member theory of a state, Kauṭilya (KAŚ 6.1.1) enumerates:

*svāmyamātyajanapadadurgakośadaṇḍamitrāṇi prakṛtayaḥ*

Lord, minister, countryside, fort, treasury, army, and ally are the constituent elements.<sup>51</sup>

47 Slaje (1998) presents a translation and detailed discussion of MBh XIII.6.

48 Slaje 1998: 31.

49 In YSm I.347 the fact that a chariot cannot move with only one wheel is adduced as a simile for the same problem.

50 However, according to YSm I.68–9, when a brother-in-law is employed to make a sonless married woman pregnant, a "son born according to this procedure belongs to the owner of the field" (Olivelle 2019), i.e. the husband.

51 Olivelle 2013: 271.

The constituent elements enumerated here are adduced in this specific order for a certain reason: Kauṭilya argues in detail why, in the order given above, “a calamity affecting each previous one is more serious”.<sup>52</sup> If we, somewhat loosely, identify “a member withdraws” with “a calamity affects a member”, Kauṭilya hints at the approach involving alternating withdrawal here.

It seems that the generalizable procedures advocated in the above *Āraṇyakas* and *Upaniṣads* were not so well understood by later readers that their use would automatically have come to (their) mind. Thus, further examples for the application of these generalizable procedures are difficult to find.

However, various superiority problems without the application of the generalizable procedures demonstrated in the late Vedic literature can be found easily. Just consider the Ṛgvedic Hymn of the Man (*puruṣasūkta*) or Manu on the rank order of creatures (MDh 1.96–7).<sup>53</sup>

Second, there is a whole class of superiority questions in the Upaniṣadic literature that are “solved” by similar mechanisms. For example, some item A is superior to another item B if

- A is “the essence of” B as in *pṛthivyā āpo rasah*<sup>54</sup> (“the essence of the earth is the waters”)<sup>55</sup>
- A is “higher than” B as in *manasas tu parā buddhiḥ*<sup>56</sup> (“higher than the mind is the intellect”)<sup>57</sup>
- B is “woven back and forth on” A as in *kasminn u khalu prajāpatilokā otāś ca protāś ca*<sup>58</sup> (“On what, then, are the worlds of Prajāpati woven back and forth?”).<sup>59</sup>

Although these mechanisms are similar in that B rests on A, B is lower than A etc., I argue that they are not truly generalizable. After all, rather specific arguments (not given in the text) would be needed in order to justify why “the worlds of Prajāpati ... are woven back and forth on ... the worlds of *brahman*”.<sup>60</sup> Similarly, what specific factor might make “the intellect ... higher than the mind”?<sup>61</sup> However, a certain closeness of the ideas presented here and those underlying alternating withdrawal must not be denied. In fact, if the worlds of Prajāpati are woven back and forth on the worlds of *brahman*, it seems that the former would be “nowhere” without the latter and in that sense the latter’s threat of withdrawal should indeed be very serious.

Returning to the main topic of this article, I have shown that the approach of singly leaving or entering described in pre-modern Indian texts gets close to the

52 Olivelle 2013: 331.

53 Taking the Indian case as a starting point, Dumont (1980) analyses hierarchy and considers man as “homo hierarchicus”. Halbfass (1991: 347–405) discusses the pre-modern Indian history of thought of *varṇa*.

54 ChU 1.1.2.

55 Olivelle 1998: 171.

56 KU 3.10.

57 Olivelle 1998: 389.

58 BĀU 3.6.

59 Olivelle 1998: 85.

60 Olivelle 1998: 85.

61 Olivelle 1998: 389.

algorithmic definition of the Shapley value. Furthermore, the approach involving alternating withdrawal is not far from Myerson’s axiomatic definition of the Shapley value. I have attempted to show in which respect the Indian thinkers would have had to take a few extra steps if they were to arrive at the Shapley value, defined algorithmically or axiomatically. The main difference is this: the Shapley value produces numerical figures, whereas in the Indian context superiority is only about rank order.

One may, of course, surmise that arguments of the sort “where would you be without me” are commonplace in mankind. In modern times, Emerson argued for balancing operations that bring initially unbalanced social situations into balance. In the contest of the vital functions, the *bali* serves as such a “balancing mechanism”. The balanced situation itself is implicit in the Shapley value. However, it was only Myerson who realized this property.

When, in 1980, the economist Myerson provided another axiomatization for the Shapley value of 1953, he was not aware of the paper by the sociologist Emerson published already in 1962. The latter, for his part, did not acknowledge the Shapley value. Not surprisingly, none of these modern-day scholars took their Indian forerunners into account.

### Appendix

For the player set  $N = \{sp, si, h, m, b\}$  and the coalition of vital functions other than breath  $V = \{sp, si, h, m\}$ , assume the coalition function  $w$  defined by  $w(v) \geq 0$  for all  $v \in N$  and

$$w(K) = \begin{cases} \sum_{v \in V \cap K} w(v), & b \text{ not in } K \\ w(b) + \alpha \sum_{v \in V \cap K} w(v), & b \text{ in } K \end{cases}$$

for every subset  $K$  of  $N$ . Let  $\alpha \geq 1$  which amounts to the superadditivity of  $w$ , i.e.  $w(N) \geq w(K) + w(N \setminus K)$  for every subset  $K$  of  $N$ . For this coalition function, the following assertions hold:

A) Along the entering sequence  $(sp, si, h, m, b)$  the marginal contributions are

- $w(v)$  for each vital function  $v$  from  $V$  and
- $w(b) + (\alpha - 1) \sum_{v \in V} w(v)$  for  $b$ .

B) Along the leaving sequence  $(sp, si, h, m, b)$  (or: along the entering sequence  $(b, m, h, si, sp)$ ) the marginal contributions are

- $\alpha w(v)$  for each vital function  $v$  from  $V$  and
- $w(b)$  for  $b$ .

If  $\alpha$  takes the special value of 1, the payoffs are the same for the entering and the leaving sequence.

C) The Shapley values for the above coalition function are

- $Sh_v = \frac{1+\alpha}{2} w(v)$  for the vital functions  $v \in V$
- $Sh_b = w(b) + \frac{\alpha-1}{2} \sum_{v \in V} w(v)$  for breath.

Proof:

Speech (and any other vital functions from  $V$ ) has the same chance of entering before breath (with the marginal contribution being  $w(sp)$ ) or entering after breath (with the marginal contribution being  $\alpha w(sp)$ ). This explains the Shapley values for the vital functions from  $V$ . The Shapley value distributes the worth of the grand coalition among the players. Hence, breath gets the rest.

D) If speech withdraws from the game, the Shapley values for the remaining players are

- $Sh_v = \frac{1+\alpha}{2} w(v)$  for the vital functions  $v \in \{si, h, m\}$
- $Sh_b = w(b) + \frac{\alpha-1}{2} \sum_{v \in \{si, h, m\}} w(v)$  for breath.

Proof:

If speech has withdrawn, the other players' payoffs are derived as in C.

E) If breath withdraws from the original game, the Shapley values are  $Sh_v = w(v)$  for the vital functions  $v \in \{sp, si, h, m\}$ .

Proof:

If breath has withdrawn, the vital functions  $sp, si, h, m$  receive their one-man worth in each sequence and hence as the Shapley value.

F) Before the contest, each vital function has obtained  $\frac{1}{5}$  of the body's proper functioning of  $w(b) + \alpha \sum_{v \in V} w(v)$ . After the contest, breath obtains the *bali*, which is implicitly defined by

$$\frac{w(b) + \alpha \sum_{v \in V} w(v)}{5} + bali = Sh_b = w(b) + \frac{\alpha - 1}{2} \sum_{v \in V} w(v)$$

and hence explicitly by

$$bali = \frac{4}{5} w(b) + \left[ \frac{3}{10} \alpha - \frac{1}{2} \right] \sum_{v \in V} w(v)$$

By solving  $bali > 0$  for  $\alpha$ , the tribute is found to be positive if  $\alpha > \frac{5}{3} - \frac{8}{3} \frac{w(b)}{\sum_{v \in V} w(v)}$  holds.

### Abbreviations

AĀ	Aitareya Āraṇyaka (Keith 1909)
AĀ_Sā	Commentary on Aitareya Āraṇyaka by Sāyaṇa (Deo 1992)
BĀU	Bṛhadāraṇyaka Upaniṣad (Olivelle 1998)
BĀU_Ś	Commentary on Bṛhadāraṇyaka Upaniṣad by Śaṅkara (Shastri 1986)
ChU	Chāndogya Upaniṣad (Olivelle 1998)
ChU_Ś	Commentary on Chāndogya Upaniṣad by Śaṅkara (Shastri 1982)

KauU	Kauṣītaki Upaniṣad (Olivelle 1998)
KAŚ	Kauṭilya's Arthaśāstra (Kangle 1969)
KU	Kaṭha Upaniṣad (Olivelle 1998)
MBh	Mahābhārata (Sukthankar 1927–1959)
MDh	Mānava Dharmasāstra (Olivelle 2005)
PU	Praśna Upaniṣad (Olivelle 1998)
PW	Sanskrit-Wörterbuch in kürzerer Fassung (Böhtlingk 2009)
ŚĀ	Śāṅkhāyana Āraṇyaka (Apte 1922)
ŚB	Śatapatha Brāhmaṇa (Weber 1855)
YSm	Yājñavalkya Smṛti (Olivelle 2019)

## References

- Apte, V.G. 1922. *Śāṅkhāyana Āraṇyaka*. Poona: Ānandāśrama Press.
- Black, Brian. 2007. *The Character of the Self in Ancient India*. Albany: State University of New York Press.
- Bodewitz, Henk W. 1986. "Prāṇa, apāna and other prāṇa-s in Vedic literature", *Adyar Library Bulletin* 50, 326–48.
- Bodewitz, Henk W. 1992. "King Prāṇa", in Albert W. Van den Hoek and Jan C. Heesterman (eds), *Ritual, State and History in South Asia. Essays in Honour of J.C. Heesterman*. Leiden: Brill, 50–64.
- Bodewitz, Henk W. 2002. *Kauṣītaki Upaniṣad*. Groningen: Egbert Forsten.
- Böhtlingk, Otto. 2009. *Sanskrit-Wörterbuch in kürzerer Fassung*. Delhi: Motilal Banarsidass.
- Bronkhorst, Johannes. 2007. *Greater Magadha. Studies in the Culture of Early India*. Leiden: Brill.
- Burnell, A.C. 1873. *The Vaṃśabrāhmaṇa of the Sāma Veda*. Mangalore: Stolz & Hirner, Basel Mission Press.
- Daly, Lloyd W. 1961. *Aesop without Morals*. London: Thomas Yoseloff.
- Deo, Munishwar. 1992. *Aitareyāranyaka with the Commentary of Sāyaṇa*. Hoshiarpur: Vishveshvaranand Vedic Research Institute.
- Dumont, Louis. 1980. *Homo Hierarchicus. The Caste System and its Implications*. Chicago: University of Chicago Press.
- Emerson, Richard M. 1962. "Power-dependence relations", *American Sociological Review* 27, 31–41.
- Erman, Adolf. 1927. *The Literature of the Ancient Egyptians*. London: Aylward M. Methuen & Company.
- Frauwallner, Erich. 1997. *History of Indian Philosophy*, Vol. I. Delhi: Motilal Banarsidass.
- Halbfass, Wilhelm. 1991. *Tradition and Reflection. Explorations in Indian Thought*. Albany: State University of New York Press.
- Harvey, Arnold David. 2007. *Body Politic: Political Metaphor and Political Violence*. Newcastle: Cambridge Scholars Publishing.
- Ilsley Hicks, Ruth. 1963. "The body political and the body ecclesiastical", *Journal of Bible and Religion* 31, 29–35.
- Isayeva, Natalia. 1993. *Shankara and Indian Philosophy*. New York: State University of New York Press.
- Jha, Ganganatha. 2005. *The Chāndogyaopaniṣad*. Delhi: Chaukhamba Sanskrit Pratishthan.
- Kangle, R.P. 1969. *The Kauṭilya Arthaśāstra. Part I: Sanskrit Text with a Glossary*. 2nd edn. Bombay: Bombay University.

- Keith, Arthur Berriedale. 1908. *The Śāṅkhāyana Āraṇyaka*. London: Royal Asiatic Society.
- Keith, Arthur Berriedale. 1909. *The Aitareya Āraṇyaka*. Oxford: Clarendon Press.
- Myerson, Roger B. 1980. "Conference structures and fair allocation rules", *International Journal of Game Theory*, 9, 169–82.
- Myerson, Roger B. 1991. *Game Theory. Analysis of Conflict*. Cambridge: Harvard University Press.
- Nederman, Cary J. 2004. "Body politics: the diversification of organic metaphors in the Later Middle Ages", *Pensiero Politico Medievale* 2, 59–87.
- Olivelle, Patrick. 1998. *The Early Upaniṣads. Annotated Text and Translation*. New York: Oxford University Press.
- Olivelle, Patrick. 2005. *Manu's Code of Law. A Critical Edition and Translation of the Mānava-Dharmaśāstra*. New York: Oxford University Press.
- Olivelle, Patrick. 2013. *King, Governance, and Law in Ancient India. Kauṭilya's Arthaśāstra*. New York: Oxford University Press.
- Olivelle, Patrick. 2019. *Yajñavalkya: A Treatise on Dharma*. Cambridge: Cambridge University Press.
- Preisendanz, Karin. 2005. "Soul, body and person in Ancient India", in Hans-Dieter Klein (ed.), *Der Begriff der Seele in der Philosophiegeschichte*. Würzburg: Königshausen & Neumann, 119–75.
- Rau, Wilhelm. 1957. *Staat und Gesellschaft im alten Indien nach den Brāhmaṇa-Texten dargestellt*. Wiesbaden: Otto Harrassowitz.
- Ruben, Walter. 1947. *Die Philosophen der Upanishaden*. Bern: A. Francke Verlag.
- Shapley, Lloyd S. 1953. "A value for n-person games", in Harold W. Kuhn and A. W. Tucker (eds), *Contributions to the Theory of Games*. Princeton: Princeton University Press, 307–17.
- Shastri, S. Subrahmanya. 1982. *Chāndogyopaniṣad*. Varanasi: Mahesh Research Institute (Upaniṣadbhāṣyam, 2).
- Shastri, S. Subrahmanya. 1986. *Bṛhadāraṇyakopaniṣad*. Varanasi: Mahesh Research Institute (Upaniṣadbhāṣyam, 3).
- Shogimen, Takashi. 2008. "Treating the body politic: the medical metaphor of political rule in Late Medieval Europe and Tokugawa Japan", *Review of Politics* 70, 77–104.
- Slaje, Walter. 1998. "Nāsti daive prabhutvam. Traces of demythologisation in Indian Epic thought", *Journal of Indian Philosophy* 26, 27–50.
- Slaje, Walter. 2010. "Sāyaṇa oder Mādḥava? Verfasserschaft und Reihenfolge der Saṃhitā-Kommentare aus Vijayanagara", *Zeitschrift der Deutschen Morgenländischen Gesellschaft* 160, 383–414.
- Sukthankar, Vishnu Sitaram (ed.). 1927–59. *The Mahābhārata*. Poona: Bhandarkar Oriental Research Institute.
- Weber, Albrecht. 1855. *Śatapatha Brāhmaṇa*. London: Williams and Norgate.
- Wezler, Albrecht. 1982/1983. "Zum Verständnis von Chāndogya Upaniṣad 5.1.12", *Studien zur Indologie und Iranistik* 8/9, 147–68.
- Wiese, Harald. 2009. "Applying cooperative game theory to power relations", *Quality and Quantity* 43, 519–33.
- Zysk, Kenneth G. 1993. "The science of respiration and the doctrine of the bodily winds in Ancient India", *Journal of the American Oriental Society* 113, 198–213.