THE DEVELOPMENT OF THE OVERLAPPING-PLATE METHOD

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ABSTRACT. The principles of focal-plane astrometry are described, as well as the development of the overlapping-plate method. In particular, the advantages and pitfalls of this method are discussed. An extensive bibliography is appended.

1. INTRODUCTION

Photographic astrometry emerged, as a competitive and efficient method for obtaining relative star positions, essentially in the last quarter of the nineteenth century. The ability of the photographic plate, when attached to an appropriate telescope, to generate and preserve a record of the relative positions of the stars, complete to a certain brightness in the field of the telescope-camera strongly suggested the use of this medium for obtaining accurate and precise estimates for the positions and eventually the proper motions of stars. The international community of astrometrists recognized (but by no means unanimously[2]) this potential, witness the meeting in 1887 which we are celebrating, in which the plans for the Astrographic Catalogue were drawn.

The principle of focal-plane astrometry could most generally be formulated as follows.

Suppose an optical device (telescope) of focal length s

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[2]K. Graff occasionally mentioned in his lectures at the University of Vienna that the influential German astrometrist, Artur Auwers held that "the application of photography to astronomy was against nature".

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generates an image of a star field on its focal plane strictly following the geometry of gnomonic projection. Consider a normal coordinate system k (i.e., one which is rectangular cartesian with equal units of length on each axis) whose origin is in the optical center of the telescope and whose z-axis points toward the tangential point on the plate. In this system, the location vector of the tangential point is $\mathbf{X}_o^{k\mathsf{T}} = (0,0,s)$, and the coordinates of each image in the focal plane are $\mathbf{X}^{k\mathsf{T}} = (x,y,s)$; the orientation of the x- and y- axes is arbitrary. The focal plane is obviously parallel to the x-y plane at distance s. The unit vector in the direction of the image whose coordinates in the focal plane, with respect to the tangential point as origin, are (x,y) is obviously $\hat{\mathbf{X}}^{k\mathsf{T}} = (x^2+y^2+s^2)^{-1/2}(x,y,s)$.

The system k may be rotated into any other "normal" system κ by an orthogonal matrix $\mathbf{R}$ which is completely determined by three parameters:

$$\mathbf{X}^{\kappa} = \mathbf{R}\mathbf{X}^{k}. \tag{1}$$

In the earlier theoretical developments, this system κ was always the system which was related to the Ω-system by the matrix

$$\mathbf{M}(\Omega,\kappa) = \mathbf{R}_1(90^\circ-\delta_o)\mathbf{R}_3(\alpha_o+90^\circ),$$

where α_o,δ_o are in the direction whose image would be projected onto the tangential point. In this system κ, a star with coordinates α,δ has the position vector

$$\boldsymbol{\Xi}^{\mathsf{T}} = \begin{pmatrix}\Xi\\ \mathrm{H}\\ \mathrm{Z}\end{pmatrix} = \begin{pmatrix}\cos\delta\sin(\alpha-\alpha_o)\\ \sin\delta\cos\delta_o-\cos\delta\sin\delta_o\cos(\alpha-\alpha_o)\\ \sin\delta\sin\delta_o+\cos\delta\cos\delta_o\cos(\alpha-\alpha_o)\end{pmatrix}.$$

Since the z-axes of the systems k and κ are, by definition, identical, we have $\mathrm{Z} = s/(x^2+y^2+s^2)^{1/2}$ and introducing the "standard coordinates" (sometimes called "normal coordinates") $\xi = \Xi/\mathrm{Z}$ and $\eta = \mathrm{H}/\mathrm{Z}$, we have $(x,y)^{\mathsf{T}} = s\mathbf{R}(\varphi)(\xi,\eta)^{\mathsf{T}}$, the classical simplest relationship between measured and standard coordinates; note that this presumes that (a) the location of the tangential point on the sphere and (b) that of its image on the focal plane (plate) is known. It was, as we shall see below, realized only later (first apparently by Murray (1967)) that the system κ need not be oriented as specified above, but that a common system for all plates, e.g., the Ω-system, could be chosen to which the vector $\mathbf{X}^{k}$ can always be transformed by a general rotation. In this case, standard coordinates are no longer meaningful or germane.

Astrometry could always be practiced only with a huge effort of arithmetic, witness the fact that each original catalogue typically contains thousands of star position estimates, each of which requires a considerable fraction of an hour for its computation. Meridian observations require the calculation of mean places and if traditional mean Q-coordinates [3] are to be published from photographic observations, they must be computed from standard coordinates, which are themselves obtained from measured rectangular coordinates by means of "plate constants".

To calculate Q-coordinate estimates from all measured rectangular coordinates and to compile from them a definitive AC which lists unified Q-coordinate estimates for all stars was a task which one could then not seriously consider. Most of the AC was therefore published in the form of rectangular coordinates which were measured on overlapping plates, together with such "plate constants" as could be produced by a manageable effort which allowed the user to calculate needed Q-parameters himself from each pair of published measured coordinates.

The "reduction of a plate", meaning the performance of all the arithmetic between measuring the rectangular coordinates of the images and the publication of the results, initially consists of computing (more accurately: estimating) the plate constants (more properly: plate parameters), usually by matching the measured rectangular coordinates to standard coordinates calculated from available Q-coordinate estimates (usually extracted from a catalogue) of a selection of stars whose images were found on the plate, the so-called reference stars. In many cases, the task was considered finished with the calculation and publication of the standard coordinate estimates of the field stars which resulted from the estimated plate parameters and the measured coordinates, e.g., in the case of most photographic astrometric work carried out at the Bonn observatory under the direction of Friedrich Küstner.

2. UNAVOIDABLE SYSTEMATIC ERRORS: THE PARAMETER VARIANCE

A. Donner (1896) realized even in the nineteenth century that the unavoidable accidental errors in the estimated plate parameters would inexorably generate systematic errors in the standard coordinates (or, for that matter, any other quantities) which depended functionally on them and were calculated by means of these estimates. It is therefore clear that series of standard coordinate estimates of the same stars but obtained from data derived from different

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[3] right ascension and declination, which are longitude and latitude angle, respectively, in the Q-system, i.e., the system of the equator.

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plates and therefore with a different set of plate parameter estimates must show systematic differences against each other even if the plate parameter estimate sets of the several plates were obtained by immaculately correct adjustments, and even then if for the estimation of each of these sets the same set of reference star Q-coordinate estimates was used. Much later, Eichhorn and Williams (1963) provided a complete theory of this effect, the "parameter variance". To minimize the systematic error thereby introduced into the finally obtained star position estimates, Donner applied his technique of "attaching" plates to each other (*rattachement*) to the estimation of the plate parameters in the Helsingfors zone of the AC (Donner and Furuhjelm 1929, pp. 52-56). He could prove that this technique did indeed succeed in reducing the formal errors of the plate parameter estimates to about one quarter of their original value. This appears to have been the first systematic application of an overlapping-plate technique.

After the parameters of the plates of the Helsingfors AC zone had been estimated from comparison with reference star coordinate estimates alone, essentially using an affine model, Donner and Furuhjelm improved the parameters as follows. Typically (the exceptions are the plates at the boundaries of the zone) each plate is "overlapped" by four others, following the corner-in-center pattern. In each plate quarter, these authors selected a number (≤20) of field stars and calculated (and this can be done routinely) the standard coordinates which they would have had with respect to the center plate which they overlap. For the economy and ease of the calculations, the data pertaining to each plate quadrant were combined by forming two fictitious normal-point stars which connected the target plate with the four overlapping ones. The halved differences between the standard coordinates of the fictitious normal stars were next adjusted by rotation, shift and expansion (the classical four-constant model) of the target plate, thus fitting, in a sense, each target plate to its four neighbors. After four iterations on the procedure, there were no longer any perceptible changes. Donner and Furuhjelm claim (on the basis of a somewhat crude estimate) that this plate attaching reduced the errors of the plate constants by a factor of four.

3. FURTHER DEVELOPMENT OF THE OVERLAPPING-PLATE TECHNIQUE

Donner's procedure was the utmost which one could manage toward a rigorous block adjustment in the days of logarithm- and other tables and even mechanical desk calculators, and one must indeed admire his and his collaborators' industry and determination in carrying out this ambitious enterprise.

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It was perhaps the vast amount of tedious and repetitive arithmetic that kept those responsible for the production of the other AC zones from following his example.

In a rigorous block-adjustment, the condition equations provided by the reference stars would have had to be carried during each overlap iteration, each of the field stars would have had to be involved and no normal positions should have been formed, all impossible tasks without a computer!

It appears that W. Dieckvoß (1955) carried out the next -- successful -- attempt to improve the accuracy of the plate parameters by an overlap procedure. He computed proper motions of the stars in the field of the galactic cluster Messier 34 by comparing positions derived from plates taken at the 60cm refractor of the Bergedorf observatory with early epoch positions derived from the measured coordinates published in the AC Helsingfors. Since he realized that plate constant estimates, found from treating each of the old AC plates as a separate entity, would be quite inaccurate (because of the low precision of the Bonn AGK1 which was the only realistic source for reference position estimates), he constructed a rigid complex of plates which cover the area, by using the one-quadrant-overlap of one plate whose center coincides with the corners of four adjoining plates, and to this complex attached -- again, rigidly -- another, sixth plate by regarding all available stars in the first complex as reference stars for the sixth plate. He then adjusted this rigid complex to the system of the reference stars; the parameter estimates resulting from this fit were, of course, considerably more accurate than what could have been achieved had the several plates been adjusted independently. Dieckvoß used rigorous formulas to prepare for the "welding" of the several plates into a rigid complex, such as the transfer of the coordinates on all plates to a common tangential point (in this case a projective transformation).

This constitutes substantial progress over Donner's procedure who used series developments, and reflects the transition from working with logarithms and tables to calculating on mechanical desk calculators.

4. RIGOROUS BLOCK-ADJUSTMENT

4.1. Eichhorn's Original Proposal

It appears that H. Eichhorn (1960) was the first to publish a rigorous formalism for the block-adjustment of overlapping plates. There, the following situation is considered:

Let an extended region of the sky be covered by plates such that each plate covers, at least partially, a region

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also covered by another plate. (Plate overlap). In this situation, the parameter variance causes the accidental errors in the plate parameters to propagate as systematic errors into the field positions which were calculated from them (and the measurements, of course), thus unavoidably creating systematic differences between positions computed from different plates which were reduced independently of each other.

In principle, one could express the positions in terms of spherical coordinates directly, as Eichhorn (1971a) did later, but this causes further formal complications which are avoided if the positions are expressed in terms of standard coordinates, as they are in Eichhorn's (1960) paper. The difficulty here is that standard coordinates, referred to a tangent point off the (center of the) plate are in a projective relationship to the measured rectangular coordinates and thus lead to distortions which will be the more serious the further the tangential point with respect to which the standard coordinates were calculated is from the actual geometrical tangential point of the plate. In practice, this is more a nuisance than an intrinsic difficulty.

In essence, Eichhorn assumed (as is -- was -- customary, even though not quite correct) a relationship

$$\boldsymbol{\xi}_{\nu} = \boldsymbol{\Xi}_{\mu\nu}\boldsymbol{a}_{\mu} \tag{2}$$

between the standard coordinates $\boldsymbol{\xi}_{\nu}^{T} = (\xi_{\nu},\eta_{\nu},\zeta_{\nu})$ of the ν-th star, a model matrix $\boldsymbol{\Xi}_{\mu\nu}$, pertaining to the ν-th star on the μ-th plate and the vector $\boldsymbol{a}_{\mu}$ of plate parameters on the μ-th plate. The elements of $\boldsymbol{\Xi}_{\mu\nu}$ typically are products of the powers of the measured coordinates $x_{\mu\nu}, y_{\mu\nu}$ of the ν-th star on the μ-th plate and of measures m_{ν} and c_{ν} for magnitude and color, respectively, of the ν-th star. Estimates of the Q-coordinates will be found in some catalogue of positions for some of the stars, the *reference stars*. From these positions, "observed" $\boldsymbol{\xi}_{\nu o}$ can be calculated, allowing us to set up the equations of condition

$$\boldsymbol{\xi}_{\nu} = \boldsymbol{\xi}_{\nu o} \tag{3}$$

for those ν which belong to reference stars. One may generalize the developments to where proper motions as well are calculated in addition to the standard epoch positions but there, the principles are the same as in the case in which all plates and all reference positions refer to the same epoch.

The overlapping-plate method now regards all $\boldsymbol{\xi}_{\nu}$ and all $\boldsymbol{a}_{\mu}$ simultaneously as unknowns (adjustment parameters) in all sets of condition equations (2) and (3). Therefore, the system of normal equations formed from these several sets now contains *all* plate parameters and *all* star coordinates (not only those of the reference stars) as unknowns, in contrast to the traditional treatment in which only systems of the type (2), namely

$$\boldsymbol{\xi}_{\nu o} = \mathbf{S}_{\mu\nu}\mathbf{a}_{\mu}, \tag{2a}$$

which are available for reference stars only, were considered as condition equations for the parameters $\mathbf{a}_\mu$ on the μ-th plate. The coordinates of the field stars are not involved in a classical adjustment, but are computed only later from equations of the type (2a), after the plate parameters have been estimated. A traditional solution thus ignores the condition that the estimates for the same coordinate (at the same epoch, of course) cannot be different, regardless on whatever different sources they were based. Just for setting up the normal equations, the consequent enforcement of this constraint leads to an arithmetic effort which exceeds that required for a traditional reduction at least by a factor equal to the ratio of the number of field stars to the number of reference stars.

The matrix $\mathbf{N}$ of the resulting system of normal equations is naturally partitioned as follows

$$\mathbf{N} = \begin{pmatrix} \mathbf{P} & \mathbf{C} \\ \mathbf{C}^{\mathrm{T}} & \mathbf{S} \end{pmatrix} , \tag{4}$$

provided the adjustment parameters are ordered in the following sequence: $\mathbf{a}_1, \mathbf{a}_2, \ldots \mathbf{a}_m; \boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \ldots, \boldsymbol{\xi}_n$ where m and n are the total numbers of plates and stars, respectively. $\mathbf{P}$ is then square and block diagonal; the μ-th block is of the same order as the corresponding vector $\mathbf{a}_\mu$ of plate parameters. $\mathbf{S}$ is likewise block diagonal, the ν-th block is of the same order as the corresponding $\boldsymbol{\xi}_\nu$, that is $2\cdot2$ if coordinates only are calculated, $4\cdot4$ if proper motions are estimated as well, and even $1\cdot1$ if ξ and η are calculated separately. The columns and rows which contain elements of $\mathbf{P}$ are "plate columns" and "plate rows", respectively, the elements of $\mathbf{S}$ are analogously located on "star columns" and "star rows". $\mathbf{C}$ is a sparse matrix, it has nonzero terms in the star columns only in those rows which correspond to plates on which the corresponding star actually occurs.

Eichhorn (1960) recommended the solution of the system of normal equations by a Gauß-Seidel iteration, initially with $\mathbf{C} = 0$. Jefferys (1963) could later prove that this iteration converges always, albeit agonizingly slowly, especially when there are many more field stars than reference stars and when the variance of the field star position estimates is large compared to that of the measured

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rectangular coordinates of the stellar images.

4.2. Lacroute's Method

P. Lacroute (1964, 1964a) proposed an independent approach to the overlap problem and applied it later, with some variations, to an independent reduction of the coordinates that had been measured on plates taken at the Hamburg-Bergedorf observatory for the construction of the AGK3.

His suggestion essentially amounts to a consequent and complete execution of Donner's scheme, taking advantage of the availability of electronic computers and using not only a selection of connecting stars, but *all* field stars. In essence, Lacroute's scheme was to start by calculating estimates of spherical coordinates of all stars from the measured coordinates of their images on the plates, with plate parameters that had been estimated conventionally, then to average for each star the coordinate estimates derived from each plate on which it was measured, and to use these averages as reference positions for a reestimation of all plate parameters. The process is iterated until further iterations no longer produce any noticeable changes. This scheme will always converge, but convergence may be slow.

The mathematical problem underlying the iterations will be singular unless the original reference star positions are worked into the means each time the averages are taken. However, in practice this is no disadvantage, because the iterations will converge even if the problem has no *unique* solution. There is always a unique *minimum length solution* (cf. Lawson and Hanson 1974, p.7) in such a situation and it would be interesting to investigate whether this is the one toward which the iterations converge. Lacroute (1968), and Lacroute and Valbousquet (1970, 1970a, 1972, 1974, 1977) actually applied this method to construct a catalogue from the above mentioned Bergedorf material.

Particularly noteworthy are Lacroute's (1968) investigations concerning the influence of the parameter variance on the computed position estimates. He attempted to estimate this for various models of the relationship between measured and standard coordinates. For this purpose, he utilized the fact that -- at least differentially -- the position estimates which result from a particular set of initial data may be expressed in terms of linear functions of the measured coordinates of the stars' images as well as the coordinates of the reference stars.

The coefficients in this relationship are the well-known so-called "dependences" which were important for astrometry before the advent of electronic computers. The variance of an estimated standard coordinate may, following Lacroute, be split up into the sum of the variances of the appropriate measurement plus a parameter variance, expressed in the form $\Sigma(D_x^2+D_y^2)(\sigma_{xx}+\sigma_{yy})$, where D_x and D_y, respectively, are the dependences in x and y, and σ_{xx} and σ_{yy} are the variances of the x- and the y- measurements. These may be regarded as fairly constant, but $D_x^2+D_y^2$ depends on the position of the star concerned on the plate. Numerical experiments then give the value of the average over the plate of the expression $\Sigma(D_x^2+D_y^2)$. Lacroute calls this alternate way of attacking the problem of estimating the parameter variance the (square of) the "*systematic error of random origin*".

4.3. The Elimination of Star (Or Plate) Parameters

A significant simplification was introduced by W. D. Googe (1967). The unknowns in the normal equations (cf. eq. (4)) are the plate parameters and the star parameters. After eliminating the latter, the matrix of the remaining system in the plate parameters becomes $\mathbf{P} - \mathbf{C}\mathbf{S}^{-1}\mathbf{C}^T$. Since $\mathbf{S}$ is, as mentioned before, block diagonal with maximum dimensions of the individual blocks 4·4, it is extremely simple to find its inverse. Furthermore,the symmetrical matrix $\mathbf{C}\mathbf{S}^{-1}\mathbf{C}^T$, in which now rows *and* columns are generated by plates, has nonzero elements only in those positions which correspond to plates that have star images in common, because $\mathbf{C}$ has elements different from zero only in those positions that correspond to the intersection of star columns with plate rows at those places where the image of the star which belongs to the column was actually measured on the plate which generates the row. In extended regions of the sky which are typical for those observed with the aim of constructing a catalogue, stars will be common only to plates whose centers are not too far apart, and it will therefore always be possible to number the plates in such a way that the matrix $\mathbf{P} - \mathbf{C}\mathbf{S}^{-1}\mathbf{C}^T$ of the remaining system in the plate parameters, from which the star parameters have been eliminated, is banded or, at worst, banded-bordered. Such matrices are sparse and special methods for inverting them efficiently, and for finding their eigenvalues have been developed, cf. Brown (1971).

Note that a system may well contain more plate parameters than star parameters, for example when a series of plates is taken for the determination of parallaxes and proper motions. In such a case it will be appropriate to eliminate the plate parameters first and solve the remaining system in the star parameters, whose matrix is $\mathbf{S} - \mathbf{C}^T\mathbf{P}^{-1}\mathbf{C}$.

4.4. The Direct Use of Spherical Coordinates

One of the major nuisances in the overlapping-plate algorithms was "the problem of different tangential points", that is the fact that standard coordinates, even for identical Q-coordinates, differ, though predictably, for different tangential points. To check whether the standard coordinates referred to different tangential points actually derive from the same pair of Q-coordinates requires a small computation. Even though this is a matter of established routine, there is another problem: It is not quite correct, from the standpoint of error theory, to consider the condition equations provided by the reference stars in the form of eqs. (2) and (3) under the assumption that ξ_ν and η_ν are not correlated. ξ_ν and η_ν will indeed be correlated because both ξ and η depend on α as well as on δ. One could still use eqs. (2) and (3) as they are and set up the adjustment algorithm assigning to them an appropriately calculated covariance matrix, but this does not help the fact that the ν-th star will produce numerically different pairs $\xi_{\mu\nu},\eta_{\mu\nu}$ on plates whose tangential points are not the same. The most straightforward approach would obviously be to set up the equations of condition such that the Q-coordinates α and δ themselves are the star parameters in the adjustment; the frame parameters would be the a_μ, as before.

Eichhorn (1971a) actually set up the formalism for this approach. Originally, these formulas had the disadvantage that they depend on each star's Q-coordinates only through their trigonometric functions. It is somewhat inefficient to use the computer to calculate these for each star. Later, however, Eichhorn (1985) avoided this problem by replacing in all expressions the trigonometric functions of the stars' Q-coordinates by their standard coordinates on the appropriate plates, and the standard coordinates themselves relate to the measured coordinates x_ν,y_ν (to a very good approximation) by $(\xi,\eta) = (x,y)/s$, provided that the plate was appropriately inserted into the measuring machine before the coordinates were measured. s, the focal length, is, of course, always known quite accurately. The ξ,η pairs required in the formulas can therefore be computed from the direct x,y measurements without knowing the Q-coordinates. The standard coordinates are now no longer the adjustment parameters, but play the rôle of auxiliary quantities. It is thus no longer necessary that they (a) be precisely known and (b) have the same numerical values for each star.

Another, very ingenious way to avoid the problem of different tangential points was first brought to the author's attention by C. A. Murray (1967) during the IAU general assembly at Prague and later published as the appendix to another paper (Murray & *al*. 1971); see also S. V. M. Clube (1968, 1971). Independently, J. Stock (1981) proposed the same procedure. It is based on the fact mentioned in the Introduction of this paper, that the coordinate system κ need not have its z-axis directed toward the tangential point of the plate under consideration but that a common system κ could be chosen for all plates, as long as the position unit vector toward the star is calculated from the measured rectangular coordinates of its image on the plate and the focal length. If $\hat{\mathbf{X}}_{\mu\nu}$ is this vector for the ν-th star on the μ-th plate and κ is the Q-system, so that $\hat{\mathbf{Z}} = \mathbf{X}(\alpha, 90^\circ - \delta)$, we have, for the ν-th star on the μ-th plate

$$\hat{\mathbf{Z}}_{\nu} = \mathbf{R}(\mathbf{a}_{\mu})\hat{\mathbf{X}}_{\mu\nu}, \tag{5}$$

where $\mathbf{a}_{\mu}$ is a set of three independent parameters, characteristic for the μ-th plate (e.g., three Eulerian angles) which are the arguments in the orthogonal matrix $\mathbf{R}_{\mu}$.

Stock now considers condition equations of type (5), which are available for all ν which belong to reference stars, as well as condition equations of the type

$$\mathbf{R}(\mathbf{a}_{\mu})\hat{\mathbf{X}}_{\mu\nu} = \mathbf{R}(\mathbf{a}_{\lambda})\hat{\mathbf{X}}_{\lambda\nu}, \tag{6}$$

which are available whenever the image of the ν-th star was measured on the plates No. μ and λ.

In the practical implementation of these correct equations of condition, Stock adopts procedures which could be criticized for several reasons from the standpoint of modeling and error theory. He minimizes the sum of the squares of the components of the vectors $\mathbf{Z} - \mathbf{RX}$, which cannot be assumed to be normally distributed -- after all, it is the α, δ, x and y which are observed directly. He avoids the problem of nonlinearity by allowing $\mathbf{a}_{\mu}$ to consist of nine independent parameters. One might argue that this is, in a sense, equivalent to a model with quadratic terms and that it would anyway not accurately model the actual projection geometry if $\mathbf{R}$ were constrained to be proportional to an orthogonal matrix.

Stock applied his procedure in practice on several occasions (Stock *& al.* 1984, Stock and Cova S 1983) and in order to judge whether the results of the specifics of this approach are really inferior to those one would have obtained with an algorithm that is based on rigorous error theory, one would have to reduce Stock's material by using such an algorithm and judge both results by comparing them to independently obtained material. This is obviously no trivial matter.

Jefferys (1987) has pointed out that eqs. (5) and (6) can also be written in terms of quaternions. While Jefferys and Stock describe in principle the same geometry, the way in which the plate parameters enter the quaternions leads to simpler derivatives than one would obtain with matrices, because the plate parameters -- in essence, the components of the quaternion -- enter the transformations directly and quadratically and not by way of their trigonometric functions as when the rotation is performed by a matrix. This means that one can expect faster convergence (should iterations be necessary) than with Murray's procedure as advocated by Stock.

Jefferys further sketches how the actual equations of condition must be set up to make sure that the adjustment is driven by the principle that those quantities the sum of whose squares is minimized are actually normally distributed.

Jefferys' work on plate adjustment, as well as his profound reevaluation of the least-squares adjustment procedure (Jefferys 1980, 1981) are part of the preparations for using the Hubble Space Telescope as an astrometric instrument.

5. EXAMPLES FOR THE USE OF THE OVERLAPPING-PLATE TECHNIQUE

5.1. Emphasis On Positions

Eichhorn and Jefferys, around 1962, applied the iterations suggested in Eichhorn's (1960) paper to a complex of plates in the Helsingfors AC zone in the region of the association Cygnus VI, using star position estimates from the (corrected) Bonn zone of the AGK1 as reference positions. There were many more field stars than reference stars in the area and the variances of the reference star position estimates are much higher than those of the coordinate measurements of the field stars. The iterations converged therefore ever so slowly and the process was terminated while the results were still "creeping" after more than 150 iterations. The calculations were performed on a paper-tape operated LGP 30 computer -- now a museum piece -- with a magnetic drum as memory and an optical reader that managed to read twenty characters per second. The results were not published.

Eichhorn and Gatewood (1967) calculated new plate parameters for the Northern Hyderabad AC zone. The plates which had been exposed for constructing this catalogue are centered on declinations +36°, +37°, +38° and +39°. Each 90^m there appears a group of six plates that form a regular complex which is particularly suited for an overlapping-plate solution, that is, a block adjustment of the parameters.

The computer then available was insufficient for a comprehensive block-adjustment of the whole zone. The parameters for the plates within sixteen of such six-plate-complexes were obtained through overlapping-plate solutions after the star parameters had been eliminated. The results of these computations, especially the comparison of parameters obtained in classical with those obtained in overlapping-plate solutions underscored again one of the pitfalls of overlapping-plate adjustments: They are extremely sensitive to deficiencies in the model for the relationship between the standard and the measured coordinates, as Eichhorn (1971) pointed out later. In the case of the Hyderabad AC zone plate parameters, those obtained by Eichhorn and Gatewood *in the overlapping-plate complexes* are actually, on the whole, inferior to those obtained through a conventional solution because a magnitude dependent effect was improperly modeled. Likewise, de Vegt (1975) claimed that carrying magnitude terms in the reduction model for the Strasbourg version of the AGK2-3 (cf. Lacroute and Valbousquet 1974) has introduced rather than removed systematic errors in the final positions. The sensitivity of an overlapping-plate solution to overmodeling as well as undermodeling had already been pointed out by Eichhorn *& al.* (1967).

About the same time, the U.S. Army Map Service started a re-reduction of the measurements which had been the basis for several photographic star catalogues. This resulted eventually in the recomputation of two zones of the CPC (Lukac *& al.* 1971). The same agency also sponsored a new catalogue (Eichhorn *& al.* 1983) between declinations $-54°$ and $-48°$, based on plates taken with the Sydney Observatory catalogue camera and measured at the Department of Astronomy of the University of South Florida, at Tampa. Googe *& al.* (1970) published the details of the algorithm which was used for the computation of these catalogues.

The efforts of Lacroute and Valbousquet to rediscuss the Hamburg measurements undertaken for constructing the AGK2 and the AGK3 led to a recomputation of these catalogues (Lacroute and Valbousquet 1970, 1970a, 1972, 1974, 1977, 1977a). These authors also calculated new plate parameters for the "French" zones of the AC, cf. Lacroute (1981).

The most extensive, sophisticated and versatile applications of the overlapping-plate technique have been carried out by C. de Vegt and his collaborators. Particularly impressive is the construction of a repetition of the CPC on which de Vegt reports in another place in these Proceedings. He (de Vegt 1967) worked on the theory of overlapping-plate reductions through block adjustment and wrote (de Vegt 1968) a report on the subject which reflected the developments known to that date. Further reports were published by the same author (de Vegt 1978, 1979, 1981). Extensive additional research, in particular also with respect to the projects undertaken, *viz.* the Vatican AC zone, the AGK2 plate material and a newly planned fourfold coverage of the sky, was published by de Vegt and Ebner (1972). De Vegt and Ebner (1974) announced and described in detail (de Vegt and Ebner 1974a) the development of a versatile computer program which takes advantage of the structural peculiarities of the matrices which appear in the problem of constructing a catalogue by the overlapping-plate method. The powerful subroutine for the direct solution of the subsystem (whose matrix is banded-bordered) in the plate parameters is one of the principal features. Von der Heide (1978) used these authors' approach for writing the first version of the actual reduction program, which was used by Führmann (1979) for making a rigorous block-adjustment solution of the AGK2. Von der Heide (1977, 1977a, 1979, 1980) carried out further theoretical investigations concerning block-adjustment reductions, especially concerning the accuracy to be expected.

In 1984, Zacharias started at Hamburg work on a completely new block-adjustment reduction program, written entirely in FORTRAN 77, whose main objectives are applications to planned solutions on the entire sphere without any restrictions on the number of stars or the overlap pattern. The first application planned is the reduction of the whole CPC2 which contains 270000 stars and is based on 5800 doubly exposed plates in a fourfold overlap pattern, cf. Nicholson *& al*. 1984, Zacharias (1988, this volume) and de Vegt *& al*. (1988, this Volume).

5.2. The Central-Overlap Parallaxes and Proper Motions.

As in all other areas of astrometry, the reduction of the measurements toward obtaining parallaxes -- and proper motions -- were originally dominated by the need for economy in the arithmetic operations. The calculations were repetitive, time consuming and boring, and any efficient program had to pay careful attention to lightening the computational toil as much as possible. It had been toward this purpose that F. Schlesinger, the unsurpassed master of the efficient procedural shortcut, developed his celebrated method of dependences, which is essentially the expression of the coordinate of a target star as a linear function of the like coordinates of the reference stars.

In a typical parallax-proper motion field situation, the configuration of the stars whose positions are involved in the reductions remains essentially the same; the dependences, once computed, can therefore be used for all plates and the reductions are therefore considerably simplified. Another advantage of the dependences is that they are direct measures for the influence which an error in the coordinate of one of the reference stars has on the computed like coordinate of the target star. Thus they retain their usefulness for the analysis of errors even today, as was shown by Lacroute (1961, 1968) in his already mentioned estimation of the systematic errors to be expected in the coordinate estimates of stars obtained through an overlapping-plate solution. Mathematically, the coordinates of the target stars computed with the aid of dependences are identical with those one would obtain from a least-squares solution on the basis of a linear six-constant model.

As long as the computing effort was a significant consideration, the economy offered by the use of dependences more than counterbalanced their disadvantages, which are: Residuals for the reference stars are not available, other than linear reduction models are cumbersome (and were, to the author's knowledge, never used). In addition, dependences share the disadvantages of plate constants when each plate is regarded during the reduction as a separate entity, namely that the existing geometrical constraints on the reference star positions -- they must move uniformly -- are not enforced.

The advent of electronic computers has rendered considerations of computational parsimony of minor importance and has allowed one to implement an overlapping-plate solution for reducing parallax-proper motion fields by enforcing the condition that each reference star must move uniformly while the target star(s) display(s) uniform motion overlaid with the effects one looks for, namely parallax and occasionally orbital motion.

This is accomplished by setting up a system of equations in which the unknowns are not only -- as in a conventional plate parameter solution -- the plate parameters which would be used to convert the star images' measured coordinates to standard coordinates, but in addition to these, the stars' zero-epoch positions and their proper motions as well, and in the case of the target object, their other relevant astrometric parameters.

It is clear that the system which solves for relative positions and proper motions only, without tying the system of positions and proper motions to an external standard, will have a rank deficiency of typically 6. In case one solves for the parallaxes of the reference stars as well, the rank deficiency would grow to 9.

Eichhorn and Jefferys (1971) published the theoretical foundations for what was later to become known as the Central Overlap Method (Gatewood and Russell 1974). In their paper, the authors gave alternative possibilities for the additional constraints that would have to be enforced to make the problem nonsingular, although an iterative solution of even the singular equations also converges, possibly even toward the minimum length solution (cf. Lawson and Hanson 1974). Eichhorn and Russell (1976) published an explicit algorithm for the noniterative solution of the central overlap problem with the constraints enforced. A separate algorithm is necessary because the system becomes nonsingular only through the constraints. Jefferys (1979) improved this algorithm by making it more symmetrical. The central overlap algorithm is now widely applied for the reduction of parallax observatios. Murray and Corben (1979), Murray (1986) and Murray *& al*. (1986) used it to good advantage for the derivation of wholesale parallaxes of several thousand stars in the same field. Further examples, but probably not a complete list, are found in the bibliography.

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Discussion:

MURRAY Auwers fully supported Gill's photographic Durchmusterung and extension of the Argelander Bonner Durchmusterung.

EICHHORN I only reported what Graff repeatedly told in his lectures. Gill's CPD was, after all, not a precision catalogue.

HEMENWAY Jefferys' work has been motivated by the expected observations with the Fine Guidance Sensors of the Hubble Space Telescope. We will not have observations in a plane, but will measure angles directly on the sky, so that a direction cosine formalism is much more physically meaningful than the usual gnomonic projection.