This report is organized according to twelve subjects of interest in modern celestial mechanics. Emphasis is placed on the results of an exhaustive literature search in which assistance was received from several members of Commission 7. The impetus on celestial mechanics by space projects, by new observational techniques and by the use of high speed electronic computers continued to be of considerable importance. The fact that celestial mechanics became an experimental science is still too recent to be fully evaluated. The high accuracy of radar and laser observations, the new analytical and numerical techniques, the recent sophisticated statistical orbit determination methods and many other results achieved during the past three years suggest that celestial mechanics is an exploding field of science in which new and exciting results emerge. Celestial mechanics is influencing other fields of scientific endeavor and is intimately connected and embedded in almost all areas of modern astronomy and space research.

The main topics are: (1) orbits of artificial satellites and the Earth's gravitational field; (2) natural satellites of the Solar system; (3) lunar theory and lunar probes; (4) planetary theory and planetary probes; (5) orbits of asteroids, comets and minor planets; (6) the problem of two bodies; (7) the problem of three bodies and periodic orbits; (8) the many-body gravitational problem; (9) integrals and stochasticity of dynamical systems; (10) resonance in celestial mechanics; (11) varying gravitational constant; (12) analytical and numerical techniques.


(1) The subject of finding the Earth's potential from observations of artificial satellites was still very much in the forefront, and the recognition of some fundamental problems concerning the non-uniqueness and the persistent divergence of the solution of this problem resulted in several new descriptions of the potential. Modifications of classical elements, the use of the extended phase space, elimination of the parallax allowed integration of the main problem to the first order. Luni-solar perturbations, drag, aerodynamic lift and radiation effects received analytical treatment but prediction of the long-time behavior of artificial satellites and of their re-entry still challenge the profession. The detection of tectonic plate movements, of ocean surface motions, of relativity effects and of gravitational waves became subjects of satellite technology. The problem of relative motion of non-interacting satellites and the effect of rigid body motion on the orbital motion of satellites received considerable interest. See: Aksenov, E., Vashkoviak, S. N. and Emelianov, N. V. (1978), TSAI 49; Aksnes, K., 17.052.002; Alfriend, K. T. (1977), CM 16, 441; Amin, M., 17.081.002; Andrus, J. F.
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Richardson, J. A., 18.081.038; Wagner, C. A. and Klocko (1977), CM 16, 143;
Walker, D. M. C., 19.081.019; Yurkina, M. I., 17.081.034; Zagars, J., 19.052.025;
Zlatoustov, V. A., 18.052.004.

(2) The natural satellites of the solar system received considerable attention. The orbits of Jupiter's satellites were studied in detail and a second order theory for the construction of quasi-periodic solutions of the Galilean satellites was presented. The dynamics and stability of Saturn's and Uranus' rings were investigated. Improved ephemerides of the satellites of Mars, Jupiter and Saturn have been presented and Saturn's oblateness parameter was studied. Theories of the rigid body (and not-so-rigid-body) motion of planets and satellites were developed and unified theories of spin-orbit coupling were offered. An example of success along these lines was the establishment of Mercury's 3:2 spin-orbit resonance. Some of the results discussed under topics (1) and (10) are applicable to the above subject. For references see: Aksnes, K. and Marsden, B. G., 18.099.212; Brown, B. (1977), CM 16, 229; Davis, D. R., 18.042.054; Demchenko, B. J. (1978), SSAI 208, 209; Ferraz-Mello, S. (1978), Proc. IAU Coll. 41, 209; Horedt, G. P., 18.042.029; Kozai, Y. (1976), PASJ 28, 675; Lieske, J. H., 18.042.049; Orlov, A. A., Solovaya, N. A. and Chepurova, V. M. (1978), TSAI 49 and SSAI 208, 209; Seldemann, P. K. (1975), CM 12, 59; Singer, S. F. (1977), Proc. IAU Coll. 39; Szebehely, V. (1978), CM 18, 383; Szebehely, V. and McKenzie, R. (1978), CM 18, 391.


(4) In the field of planetary theories new numerical and analytical results allowed the preparation of high accuracy ephemerides, secular terms to high orders were studied analytically, secular trends were investigated numerically, and long-time numerical integrations were performed. Indications that the solar system is stable according to Hill's definition, attempts to apply a modified KAM theory and the finding that secular terms in the semi-major axes of the planets do not appear

(5) The orbits of asteroids, comets and minor planets received considerable attention since it was pointed out that because of their large number they may be considered more representative of the solar system than the planets are. New theories of capture were proposed, the still unsolved problem of the Kirkwood gaps was treated extensively (see also item 10) and dynamical explanations of the origin of these members of the solar system were offered. Secular perturbations of asteroids and comets were studied with semi-analytical methods. New minimum distances between asteroids and their relative motions were determined. The effect of Jupiter's oblateness on cometary orbits was evaluated by means of the two fixed force center problem. Improved orbits for 110 long-period comets were computed. The long term motion of 1976 AA and 1976 UA were studied. These seem to be the first two minor planets with semi-major axes smaller than 1 a.u. See: Banfi, V., 18.042.031; Belbruno, E. A. (1977), CM 15, 467; Bien, R., 19.042.022, 20.042.077 and (1978), AA 68, 295; Chepurova, V. M. (1977), Vestnic Mosk. Gos. Univ. No. 1 & 2; Duboshin, G. N. (1976), CM 14, 239; Erdi, B., 20.042.049; Froeschle, C. and

(7) The fundamental unsolved and non-integrable problem of celestial mechanics, the problem of three bodies received the same attention as it has ever since Newton and Euler attacked it. Similarly to Ellis Ståmgren's work in the early nineteen hundreds on the restricted problem of three bodies, the recent efforts concentrated on establishing families of periodic orbits of the general problem of three bodies. The classification and systematic survey of these families and of their stability is far from complete at the present time. New families of symmetric, asymmetric, two and three-dimensional periodic orbits were found. Bifurcations were used to generate new families of the general problem. The method of the surface of section was utilized to study quasi-periodic orbits. The plane planetary three-body problem \((m_1 >> m_2=m_3=km, 0<k<1/5)\) and its stability received considerable attention. The existence of mono-parametric families with fixed masses was shown. It was demonstrated that such orbits are linearly stable if resonance conditions of the type \((2n-1)/(2n+1)\) are avoided. Triple stellar systems and their stability were studied by establishing zero velocity surfaces for the general problem of three bodies and by using classical perturbation methods. Integrals and particular solutions of the problem of three finite rigid bodies under non-Newtonian forces were investigated. Motion and stability of orbits near equilibrium points received special attention. See: Aarseth, S. J. and Heggie, D. C. (1976), AA 53, 259; Agekyan, T. A. and Anosova, Z. P., 19.042.034; Anderle, P., 17.052.005; Antonov, V. A. and Choleshevnikov, U. V. (1978), AN 299, 131; Arenstorf, R. F. and Bozeman, R. E. (1977), CM 16, 179; Arenstorf, R. F. (1978), CM 17, 331; Ash, M. E., 18.052.035; Barkham, P. G. D., Modi, V. J. and Soudack, A. C., 19.042.040; Benest, D., 17.042.044; 18.042.055 and 19.042.018; Bhatnagar, K. B. and Chawla, J. M. (1977), CM 16, 129; Bhatnagar, K. B. and Hallan, P. O. (1978), CM 18, 105; Bozis, G., 18.042.019; Bozis, G. and Hadjilemetriou, J. D., 17.042.041; Breakwell, J. V., 18.042.050; Broucke, R., 17.042.021; Choudhry, R. K. (1977), CM 16, 411; Contopoulos, G. (1976), Proc. NATO Adv. Study Inst., 43; Contopoulos, G. (1978), AA 64, 323; Delibaltas, P., 18.042.063; Demin, V. G. and Kurchanova, M. V., 20.042.021; Duboshin, G. N. (1978), CM 17, 357 and

(8) The increased speed and storage capacity of high-speed electronic computers allowed the efficient study of gravitational n-body problems. New approximate techniques were introduced to integrate numerically the differential equations of motion of 1000 gravitationally interacting point masses. On the other end of the spectrum, the motion and stability of four and five-body problems, resembling the solar system, were studied (see item 7). Systems of interacting finite non-spherical bodies and systems with several ‘fixed and one free body were analyzed. First order perturbations were derived concerning the motion of a star in an ellipsoidal cluster. See: Arazov, G. T., 17.042.070, 19.042.019 and (1977), CM 16,
(9) The fundamental problem of integrability was attacked with new vigor. Since
the pertinent differential equations of celestial mechanics are, in general, non-
integrable, the establishment of local (or "third") integrals offer a never-
ending challenge to workers in this field. On the other hand, because of the non-
integrability of the system, the long-time behavior in the classical deterministic
sense, is an unsolved problem. Relations between high order resonant periodic
orbits, characteristic exponents and integrability were studied. The range of
validity of local integrals in the elliptic restricted problem was established.
The disappearance of integrals, the onset of stochasticity and the existence of
ergodic seas was demonstrated in various dynamical systems. The utilization of
techniques of statistical mechanics and the study of bundles of trajectories
(corresponding to not precisely defined initial conditions) has been proposed.
cago Univ. Press, 93, (1978), Stochastic Behavior in Hamiltonian Syst., Springer
CM 17, 267; Hameen-Anttila, K. A., 20.042.056 and 20.042.061; Losco, L. 20.042.053;
Nahon, F., 18.042.095; Olson, P., 20.081.007; Prigogine, I., Grecos, A. and George,
71; Sarris, E. (1977), Thesis, Univ. Athens; Sergyels, R. (1976), AA 48, 257 and

(10) The motion of Trojan asteroids, the Kirkwood gaps, locked in planetary and
some special satellite motions have the common property of being associated with
resonance problems. The non-linear dynamical systems occurring in celestial
mechanics show peculiar behavior in resonance conditions, especially when more
than one low order commensurability occurs. The dynamical topology of this pro­
blem in the circular as well as in the elliptic restricted problem of three
bodies received considerable attention. See: Bhatnagar, K. B. and Gupta, B.,
19.042.033; Bien, R. (1978), AA 68, 295 and 19.042.022; Blitzer, L. (1977), CM 16,
87; Contopoulos, G. (1978), CM 18, 195; Contopoulos, G. and Mertzanides (1977),
AA 61, 477; Dallas, S. S., 20.052.020; Froeschlé, C. and Scholl, H., 17.098.016
and 19.042.045; Garfinkel, B. (1976), CM 14, 301, 17.042.046 and 18.042.045;
Giacaglia, G. E. O., 18.042.079; Greenberg, R., 20.042.016; Jefferys, W. H.,
17.042.038; Khoront, A. A., 18.042.092; Klokocnik, J., 18.054.005; Lewin, L. and
Vagners, J., 19.021.005; Osorio, J., 17.042.031; Peale, S. J., 20.042.019;
Reigber, C. and Baltimo, G., 18.081.021; Roels, J., 17.042.032; Romanowicz, B.,
18.054.009; Sanders, J. (1977), CM 16, 421; Scholl, H. and Froeschlé, C.,

(11) A special subject in celestial mechanics, known as the problem of varying
gravitational constant also received renewed interest and attention during the
past three years. See: McVittie, G. C. (1978), MN 183, 749; Reasenberg, R. D.
and Shapiro, I. I., 18.043.005; Saari, D. (1977), CM 16, 407; Van Fladern, T. C.
(1976), Sc. Am. 234, 44; Vinti, J. (1976), CM 14, 363 and (1977), CM 16, 391;
Walter, H. G., 17.043.001.

(12) New analytical and numerical techniques always abound in celestial mechanics
since this field of astronomy strongly attracts mathematicians and numerical
Some of the major books published during the past three years were:


In summary, it may be stated that a significant progress made during the past three years in celestial mechanics, indeed in modern dynamics, was the recognition of the limitations of present day analytical and numerical techniques. When artificial earth satellite observations offer centimeter accuracy our theories seem to be quite unacceptable in comparison, even for high altitude orbits. The accuracy of our lunar theories are in order of magnitude below modern observations. But some limitations, in principle, are even more significant. Long-time predictions, both of qualitative and quantitative levels encounter serious difficulties. The often divergent series solutions of general perturbation methods are inadequate for long-time predictions or for stability research and even the most sophisticated high-order numerical integration techniques have similar limitations. These limitations offer challenges and an unquestionably bright future for workers in the field of celestial mechanics. A field which was certainly not closed in Newton's or Poincaré's time and is ready to enter one of its most exciting periods of expansion.

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President of the Commission.