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RATE DETERMINING PROCESSES OF SEA ICE GROWTH

by

T. KURODA

Institute of Low Temperature Science, Hokkaido University, Sapporo, Japan 060

ABSTRACT

We have derived an analytical expression for the growth rate of sea ice by taking account of the processes relevant to the growth, eg heat conduction, diffusion of salt molecules, radiation, sensible heat transport, evaporation and so on. We discuss the role of each process as rate determining processes under various environmental conditions. It is shown that because of coupling of salt diffusion and heat conduction, the growth rate feeds back to the heat flux Q_w from water to ice which controls the growth rate and that Q_w decreases with the thickness I of sea ice, even if the environmental conditions are kept constant.

INTRODUCTION

Growth of sea ice is the growth of ice crystals from the melt which contains salt of high concentration. The following processes are relevent to the growth: (i) Conduction process through ice of the latent heat of freezing generated at the growing ice/water interface and of the heat coming from water to ice, (ii) Diffusion process of salt molecules rejected by ice at the interface, and (iii) Incoming and outgoing processes of various heats at the upper ice surface, eg the radiation heat, the sensible heat, the latent heat of sublimation, and the conductive heat through ice. These processes are coupled to each other in a very complicated manner.

The purposes of this paper are firstly to derive an analytical expression for the growth rate of sea ice from which we can physically picture the phenomenon by using a simple model (Figure 1) and taking into account the above processes, and secondly to discuss the role of each process as rate determining process under various environmental conditions.

A MODEL OF GROWING SEA ICE

Let us pursue the growth of sea ice using a simple model illustrated in Figure 1. The model is based on the following assumptions:

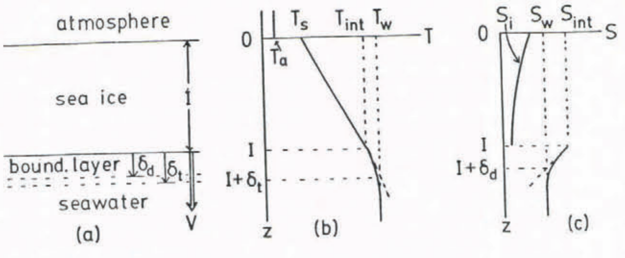

Fig.1. Schematic illustration of a model of growing sea ice. Temperature profile (b) and salinity profile (c).

1) Atmosphere, ice and water are horizontally homogeneous so that temperature T and salinity S depend on only vertical direction z (Figure 1).

2) The temperature profile in ice is governed by the solution of the equation of heat conduction under steady state conditions, i e

$$\frac{\partial T}{\partial t} = \frac{\kappa_i}{c\rho}\frac{\partial^2 T}{\partial_z{}^2} = 0, \qquad (1)$$

where κ_i is the thermal conductivity, ρ the density and c the specific heat of sea ice. Therefore, T is linearly proportional to z:

$$T = T_s + \frac{T_{int} - T_s}{I} z \qquad (0 \leqslant z \leqslant I), \qquad (2)$$

where T_s is the temperature of the upper surface of ice (z = 0), T_{int} the temperature of the ice/water interface (z = I), ie the lower surface of ice, and I the thickness of ice. However, the change in temperature at a fixed position in ice is taken into account through an increase in I with time. Such a solution is called a quasi-steady state solution. In the Equation 1, internal heating due to incoming short-wave radition is neglected. The conductive heat flux through ice is given by

$$Q_i = \kappa_i \frac{T_{int} - T_s}{I} . \qquad (3)$$

3) A diffusion boundary layer with a thickness δ_d is formed in the seawater near the lower surface of ice (Figure 1a and 1c). Since only a part of the salt molecules in water can be incorporated into ice, the remainder is rejected at the growing lower surface of ice and the salinity S_{int} in water adjacent to the lower surface (z = I) becomes larger than the salinity S_w of water at Z >> I. The rejected salt molecules are carried away by diffusion in the boundary layer with a flux

$$J_d = D \frac{S_{int} - S_w}{\delta_d} , \qquad (4)$$

where D is the diffusion constant of salt molecules in water.

4) The salinity S_i of ice determined by incorporation kinetics at z = I is proportional to S_{int}:

$$S_i = k^* S_{int} , \qquad (5)$$

where k* is the so-called distribution coefficient at the cellular ice/water interface (Weeks and Lofgren 1967).

5) The temperature T_{int} at z = I is equal to the freezing temperature which is determined by the salinity S_{int}:

$$T_{int} = 273 - \alpha S_{int} , \qquad (6)$$

where α is the slope of the liquidus line of the phase diagram of a water-salt system. Therefore, T_{int} is lower than T_w by

$$T_w - T_{int} = \alpha(S_{int} - S_w) . \qquad (7)$$

6) A thermal boundary layer with a thickness δ_t exists in water near the lower surface of ice (Figure 1a and 1b). Therefore, the conductive heat flux from water to ice is given by

$$Q_w = \kappa_w \frac{T_w - T_{int}}{\delta_t} , \qquad (8)$$

$$Q_w = \kappa_w \frac{\alpha(S_{int} - S_w)}{\delta_t} , \qquad (8)'$$

where κ_w is the thermal conductivity of seawater.

7) There are several incoming and outgoing heat fluxes at the upper surface ($z = 0$) of ice. Incoming fluxes include long-wave radiation Q_ℓ^{in} from the atmosphere, short-wave radiation Q_s^{in} and conductive heat Q_i through the ice towards the upper surface. Outgoing fluxes consist of long-wave radiation Q_ℓ^{out}, sensible heat Q_{sen} and latent heat of sublimation Q_{sub}.

Q_ℓ^{out} is given by

$$Q_\ell^{out} = \epsilon\sigma T_s^4$$

$$Q_\ell^{out} \simeq \epsilon\sigma T_a^4 + 4\epsilon\sigma T_a^3 (T_s - T_a), \qquad (9)$$

where ϵ is the long-wave emmisivity, σ the Stefan-Boltzmann constant and T_a the temperature of the atmosphere.

Q_{sen} in W/m^2 is expressed as

$$Q_{sen} = 3.5\ U_a\ (T_s - T_a)\ , \qquad (10)$$

where U_a is the wind velocity in m/s (Kuz'min 1972).

Q_{sub} is given by

$$Q_{sub} = 1.64\text{x}10^{-11}\ L_s\rho\ U_a\left[p_e(T_s) - sp_e(T_a)\right] \qquad (11$$

$$Q_{sub} \simeq 1.64\text{x}10^{-11}\ L_s\rho\ U_a\left[(1\text{-}s)p_e(T_a) + \left(\frac{\partial p_e}{\partial T}\right)_{T_a} \left(T_s - T_a\right)\right], \qquad (11)'$$

where $L_s\rho$ is the latent heat of sublimation in J/m^3, s the humidity, $p_e(T)$ the saturation vapour pressure of ice at T in Pa and $(\partial p_e/\partial T)_T$ the temperature coefficient of the saturation vapour pressure at T given by the Clausius-Clapeyron equation. Equation 11 was derived from the empirical relation obtained by Ono and others (1980).

DETERMINATION OF THE GROWTH OF SEA ICE

Heat balance at the lower surface of ice

The process which directly controls the growth rate V of sea ice is the conduction process of the latent heat of freezing. From the heat budget equation at z = I, we obtain an expression

$$V = \frac{1}{L_f\rho}\left[\kappa_i \frac{T_{int} - T_s}{I} - \kappa_w \frac{T_w - T_{int}}{\delta_t}\right], \qquad (12)$$

where $L_f\rho$ is the latent heat of freezing in J/m^3. Now determine T_{int} and T_s which are closely tied to the processes (i) to (iii) mentioned above.

Equilibrium condition for T_{int}

As shown in Equation 6, T_{int} is assumed to be equal to the equilibrium freezing temperature corresponding to the salinity S_{int} which is determined by the following mass conservation conditions of salt molecules at z = I.

Mass balance of salt molecules at the lower surface of ice

Since only a portion of the salt molecules in water is incorporated into ice at z = I (see Equation 5), the salt molecules are rejected there by amount of $V(S_{int} - S_i) = V(1 - k^*)S_{int}$ per unit time and unit area. The quantity should be equal to the diffusion flux J_d given by Equation 4 under steady state conditions:

$$V\ (1 - k^*)S_{int} = D\ \frac{S_{int} - S_w}{\delta_d}. \qquad (13)$$

Therefore, we obtain a relation between S_{int} and V

$$S_{int} = S_w\left[1 + \delta_d(1 - k^*)V/D\right]. \qquad (14)$$

Heat balance at the upper surface of ice

The temperature T_s is determined by the balance equation of the heat fluxes at z = 0 mentioned in section 2:

$$Q_\ell^{in} + (1\text{-}\partial)\ Q_s^{in} + \kappa_i \frac{T_{int} - T_s}{I} - \epsilon\sigma\left[T_a^4 + 4T_a^3(T_s - T_a)\right] -$$

$$- 3.5U_a(T_s\text{-}T_a) - 1.6\ \text{x}\ 10^{-11}L_s\rho U_a\left[(1\text{-}s)p_e(T_a) + \right.$$

$$\left. + \left(\frac{\partial p_e}{\partial T}\right)_{T_a} (T_s - T_a)\right] = 0\ , \qquad (15)$$

where ∂ is the surface albedo. From this equation, we obtain

$$T_s = \frac{T_a - B/A + (\kappa_i/IA)T_{int}}{1 + \kappa_i/IA}\ , \qquad (16)$$

where

$$A = 4\epsilon\sigma T_a^3 + 3.5U_a + 1.6\text{x}10^{-11}L_s\rho U_a(\partial p_e/\partial T)_{T_a} \qquad (17)$$

and

$$B = \epsilon\sigma T_a^4 + 1.6\text{x}10^{-11}L_s\rho U_a(1\text{-}s)p_e(T_a) - $$

$$- \left[Q_\ell^{in} + (1 - \partial)Q_s^{in}\right]. \qquad (18)$$

$A = 23.4\ W/m^2K$ and $B = 60.7\ W/m^2$, if we assign $T_a = 253$ K, $\epsilon = 1$, $\sigma = 5.67\text{x}10^{-8}\ W/m^2K^4$, $U_a = 5$ m/s, $L_s\rho = 2.6\text{x}10^9\ J/m^3$, $(\partial p_e/\partial T)_{T_a} = 10.4$ Pa/K, $p_e(T_a) = 1.03\text{x}10^2$ Pa, $s = 0.8$, $Q_\ell^{in} = 1.76\text{x}10^2\ W/m^2$ and $Q_s^{in} = 0$.

From Equation 16 one can see that $T_s = T_a$ for $A\to\infty$ and $T_s = T_{int}$ for $\kappa_i\to\infty$. These results are plausible.

It should be noted that T_{int} and T_s which control the growth rate V (Equation 12) themselves depend on V (see Equations 6, 14 and 16).

Growth rate

By solving Equations 12, 6, 14 and 16 self-consistently, we can obtain an expression for the growth rate V as a function of temperature T_a of the atmosphere and T_w of sea water:

$$V = \zeta\ \frac{\kappa_i}{L_f\rho}\ \frac{T_w - T_s'}{I}, \qquad (19)$$

where

$$T_w = 273 - \alpha\ S_w\ , \qquad (20)$$

T_s' is the temperature at the upper surface of ice (z = 0) when $T_{int} = T_w$ in Equation 16. ie

$$T_s' = \left[T_a - B/A + (\kappa_i/IA)\ T_w\right]/(1 + \kappa_i/IA), \qquad (16)'$$

and

$$\zeta = 1/\left[1 + \frac{\alpha S_w(1-k^*)\delta_d}{L_f\rho D}\ \left\{\frac{\kappa_i}{I}\left[1 - \frac{\kappa_i/IA}{1-\kappa_i/IA}\right] + \frac{\kappa_w}{\delta_t}\right\}\right]. \qquad (21)$$

The obtained growth rate includes the physical quantities relevant to the processes of freezing, conduction of the latent heat of freezing, incorporation of salt into ice, diffusion of salt molecules, incoming and outgoing radiation, transport of sensible heat and latent heat of sublimation as described above.

DISCUSSION

Growth rate

Using Equations 19, 20, 16' and 21, let us discuss the influence of the environmental conditions on the growth rate of sea ice.

In the most extreme case that $S_w = 0$ and $A \to \infty$, $T_{int} = T_w = 273$ K, $T_s = T_a$ and $\zeta = 1$. Therefore, we obtain the simplest Stefan equation $V^{(0)}$ from Equation 19:

$$V^{(0)} = \frac{\kappa_i}{L_f \rho} \frac{273 - T_a}{I} . \qquad (19)'$$

If $S_w > 0$ and $D \to \infty$, S_{int} is equal to S_w because of infinite rate of diffusion of rejected salt molecules, then $T_{int} = T_w$ and $\zeta = 1$. In this case, the growth rate is expressed as

$$V^{(1)} = \frac{\kappa_i}{L_f \rho} \frac{T_w - T_s'}{I} . \qquad (19)''$$

In the general case that $S_w > 0$ and D is finite, $S_{int} > S_w$ because of finite rate of diffusion of salt molecules, consequently $T_{int} < T_w$ and $\zeta < 1$. Therefore, the growth rate V in the general case given by Equation 19 is smaller than $V^{(1)}$ by a factor of ζ. It should be noted that $1/\zeta$ corresponds to the interface resistance to the growth due to the coupling processes of material and heat transports in the diffusion and thermal boundary layers (Figure 1).

With increasing S_w the growth rate decreases because of a decrease in $V^{(1)}$ and ζ in this model. The first effect is due to a decrease in T_w with S_w. On the other hand, the second effect is rather complicated. Since an increase in S_w makes the diffusion of salt molecules slower (the right side of Equation 13) and consequently raises S_{int} (Equation 14), T_{int} falls with S_w (Equation 6). Therefore, V decreases with S_w (Equation 12). A decrease in ζ with S_w represents this effect.

In this paper, we assume that the thickness δ_d of diffusion boundary layer is given. The value of δ_d/D reported by Weeks and Lofgren (1967) is 5.09 x 10^5 s/m and that by Nakawo and Sinha (1981) is 4.2 x 10^6 s/m. If D is assumed to be 1 x 10^{-9} m^2/s, $\delta_d \approx 0.51$ mm in the former case and $\delta_d \approx 4.2$ mm in the latter case. The thickness δ_d is expected to decrease with flow velocity U_w of water, since the thickness of velocity boundary layer decreases with U_w. A decrease in δ_d leads to an increase in ζ or V because of faster diffusion of salt molecules. However, the influence of U_w on V through the thickness δ_t of thermal boundary layer is different. With increasing U_w, δ_t may also decrease. And ζ decreases with decreasing δ_t (Equation 21), since the heat flux Q_w from water to ice increases with decreasing δ_t (the second term of the right side in Equation 12).

The dependence of the growth rate on the thickness I of sea ice is shown in Table 1. The numerical values used for calculation are as follows: $T_a = 253$ K, $\kappa_i = 2.26$ W/m K, $\kappa_w = 0.52$ W/m K, $\alpha = 0.055$ K/°/oo, $S_w = 32.9$°/oo, $L_f\rho = 3.08$ x 10^8 J/m³, $k^* = 0.12$, $\delta_d/D = 4.2$ x 10^6 s/m, $\delta_t = 2$ x 10^{-2}m, A = 23.4 W/m² K and B = 60.7 W/m². With increasing I, $V^{(1)}$ decreases because of a decrease in conductive heat flux in ice. On the other hand, ζ slightly increases with I, since the flux of rejected salt moelcules decreases with decreasing growth rate (see the left side of Equation 13). However, this is a secondary effect, so that $V = \zeta V^{(1)}$ decreases with I.

Effective distribution coefficient k_{eff}

From Equations 5 and 14, we obtain the so-called effective distribution coefficient $k_{eff} = S_i/S_w$ of salt molecules.

$$k_{eff} = k^*\left[1 + \delta_d(1 - k^*)V/D\right], \qquad (22)$$

where V is given by Equation 19. The salinity S_i in ice is expected to decrease with z (Figure 1c), since the growth rate V in Equation 22 decreases with I. The Equation 22 coincides with the expression for k_{eff} derived by Weeks and Lofgren (1967), if $\delta_d V/D << 1$.

Heat flux Q_w from water to ice

Using Equations 8' and 14, the heat flux Q_w from water to ice is given by

$$Q_w = \kappa_w (1 - k^*) \alpha S_w (\delta_d/\delta_t) V/D. \qquad (8)''$$

This equation means that the growth rate V feeds back to the heat flux Q_w which controls V because of coupling of salt diffusion in the diffusion boundary layer and heat conduction in the thermal boundary layer.

TABLE 1. THE DEPENDENCE ON THE THICKNESS I OF SEA ICE OF THE GROWTH RATE V AND THE HEAT FLUX Q_w FROM WATER TO ICE.

I [m]	0.1	0.5	1.0
$V^{(1)}$ [m/s]	7.7 x 10^{-7}	2.5 x 10^{-7}	1.4 x 10^{-7}
ζ	0.56	0.62	0.63
$V = \zeta V^{(1)}$ [m/s]	4.3 x 10^{-7}	1.6 x 10^{-7}	8.7 x 10^{-8}
Q_w [W/m²]	71.4	26.1	14.4

As shown in Table 1, Q_w decreases with the thickness I of sea ice, even if the environmental conditions are kept constant. An increase in I leads to a decrease in S_{int} (Equation 14), since V decreases with I. Therefore, Q_w falls with I (Equations 8' and 14). On calculation, we assumed $\delta_d/\delta_t = 0.2$ and D = 1 x 10^{-9} m^2/s.

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