Glasgow Math. J. **48** (2006) 483–490. © 2006 Glasgow Mathematical Journal Trust.
doi:10.1017/S0017089506003235. Printed in the United Kingdom

INTERPOLATION BY ANALYTIC FUNCTIONS ON PREDUALS OF LORENTZ SEQUENCE SPACES

M. L. LOURENÇO
Departamento de Matemática, Universidade de São Paulo, CP 66281, CEP: 05311-970 São Paulo, Brazil
e-mail: mllouren@ime.usp.br

and L. PELLEGRINI*
Escola de Artes Ciências e Humanidades, Universidade de São Paulo, CP 66281, CEP: 03828-000 São Paulo, Brazil
e-mail: leonardo@ime.usp.br

(Received 10 February, 2006; revised 6 June, 2006; accepted 2 July, 2006)

Abstract. Let (e_n) be the canonical basis of the predual of the Lorentz sequence space $d_*(w, 1)$. We consider the restriction operator R associated to the basis (e_i) from some Banach space of analytic functions into the complex sequence space and we characterize the ranges of R.

2000 *Mathematics Subject Classification.* 46G20.

1. Introduction. Let E be a complex Banach space with a Schauder basis (e_i). Let $\mathcal{F}$ be a space of continuous complex valued functions on a subset of E which contains the Schauder basis (e_i). We are interested in an interpolation problem formulated as follows. Let us consider the *restriction operator* R associated to the basis (e_i) of E defined by

$$R : \mathcal{F} \to \mathbb{C}^{\mathbb{N}}$$
$$f \mapsto (f(e_i))_{i\in\mathbb{N}},$$

and then ask about the range of R for some spaces $\mathcal{F}$ of analytic functions. The motivation for studying these ranges is based in the papers of Aron-Globevnik [**1**], Llavona-Jaramillo [**6**], and Gomes-Jaramillo [**5**]. Indeed, Aron and Globevnik have characterized the range of R for several nice spaces $\mathcal{F}$ of analytic functions on the space c_0. And Llavona-Jaramillo have studied the relationship between reflexivity of the space $\mathcal{F}$ and the range of R, where $\mathcal{F}$ is the space of real valued infinitely differentiable functions.

We are interested here in the Banach space $\mathcal{F} = \mathcal{A}^\infty(B_E)$ of all bounded and continuous functions on the closed unit ball of E which are analytic on the open unit ball of E and in the subspace $\mathcal{A}_U(B_E)$ of $\mathcal{A}^\infty(B_E)$ of all uniformly continuous functions on the closed unit ball of E, in the case where $E = d_*(w, 1)$ is the predual of Lorentz sequence space. Also we are interested in the spaces given by n-homogeneous polynomials on $d_*(w, 1)$. In spite of the canonical basis on the predual of Lorentz

*Supported by FAPESP, Brazil, Research Grant 01/04220-8.

space having properties similar to the canonical basis of c_0, the ranges of R from these spaces above mentioned are totally different when $E = c_0$.

Now we fix some notation. Given a decreasing sequence $w = (w_i)_{i\in\mathbb{N}}$ of positive real numbers which satisfies $w \in c_0 \setminus l_1$, $w_1 = 1$, the complex Lorentz sequence space $d(w, 1)$ is given by

$$d(w, 1) = \left\{ x = (x_n) : \sup \left\{ \sum_{n=1}^{\infty} |x_{\pi(n)}| w_n : \pi \text{ is a permutation of } \mathbb{N} \right\} < +\infty \right\}.$$

The norm is given by

$$\|x\|_{d(w,1)} := \sup_{\pi\in\Pi} \sum_{i=1}^{\infty} |x_{\pi(i)}| w_i < \infty.$$

where Π is the set of all permutations of the natural numbers. It is well known and easy to verify that the above supremum is attained for the decreasing rearrangement of x. The usual vector basis (e_n) is a Schauder basis of $d(w, 1)$. The canonical predual $d_*(w, 1)$ of $d(w, 1)$ is given by

$$d_*(w, 1) = \left\{ x = (x_i)_{i\in\mathbb{N}} \in c_0 : \lim_{k\to\infty} \frac{\sum_{n=1}^{k} [x]_i}{\sum_{i=1}^{k} w_i} = 0 \right\},$$

where $([x]_i)$ is the decreasing rearrangement of $(|x|_i)$. This space is a Banach space endowed with the norm

$$\|x\| = \sup_k \frac{\sum_{n=1}^{k} [x]_i}{\sum_{i=1}^{k} w_i}.$$

and it has a Schauder basis (e_n) whose sequence of biorthogonal functions is the canonical basis of $d(w, 1)$.

2. Polynomials. In this section we are interested in characterizing the range of restriction operator R when $\mathcal{F}$ is the space of all m-homogeneous polynomials on the predual of Lorentz space $d_*(w, 1)$.

For a complex Banach E with dual E', B_E denotes the closed unit ball of E. $P(^m E)$ denotes the Banach space of all continuous m-homogeneous polynomials on E with the norm $\|P\| = \sup_{x\in B_E} |P(x)|$.

In [**1**], Aron and Globevnik showed that if $E = c_0$ the range of R for $\mathcal{F} = P(^n c_0)$ is the space $l_1 = c_0'$, for all $n \in \mathbb{N}$. The natural question here is the following: if the Banach space E has a Schauder basis with similar properties to the canonical basis of c_0 (for example shrinking or unconditional) is it possible that the range of $R(P(^n E)) = E'$?

We are going to show that in spite of the Schauder basis of $d_*(w, 1)$ having the properties mentioned above, the restriction operator R is totally different in the predual of Lorentz space.

We recall (see [**1**]) that for every natural number $n \geq 2$, the generalized Rademacher functions (s_j) are defined inductively as follows. Let $\alpha_1 = 1, \alpha_2, \ldots, \alpha_n$ be the complex n-th roots of unit. For $j = 1, \ldots, n$ let $I_j = (\frac{j-1}{n}, \frac{j}{n})$ and let I_{j_1,j_2} denote the j_2-th open subinterval of lenght $\frac{1}{n^2}$ of I_{j_1} $(j_1, j_2 = 1, 2, \ldots, n)$. Proceeding like this, it is clear how to

https://doi.org/10.1017/S0017089506003235 Published online by Cambridge University Press

define the interval $I_{j_1,\dots,j_k}$ for any k. Now $s_1 : [0, 1] \to \mathbb{C}$ is defined by setting $s_1(t) = \alpha_j$ for $t \in I_j$, where $1 \le j \le n$. In general, $s_k(t)$ is defined to be α_j if t belongs to the subinterval $I_{j_1,\dots,j_k}$ where $j_k = j$. There is no harm in setting $s_k(t) = 1$ for all endpoints t.

The next lemma gives the main properties of the sequence (s_k) of generalized Rademacher functions which we will need. The verificatiom of these properties follows exactly the same lines as the corresponding result for the classical Rademacher functions.

LEMMA 2.1. *For each $n = 2, 3, \dots,$ the associated Rademacher's functions $\{s_k\}_{k\in\mathbb{N}}$ satisfy the following properties*:

(a) $|s_k(t)| = 1, \forall k \in \mathbb{N}, \forall t \in [0, 1]$.

(b) *For any $k_1, \dots, k_n$,*

$$\int_0^1 s_{k_1}(t)\cdots s_{k_n}(t)dt \begin{cases} 1, & \text{if } k_1 = k_2 = \cdots = k_n; \\ 0, & \text{otherwise.} \end{cases}$$

PROPOSITION 2.2. *For each $n \in \mathbb{N}$, let $P \in P(^n d_*(w, 1))$. Then*

$$\|(P(e_i))_i\|_{d(w^n,1)} \le \|P\|,$$

where w^n denotes the sequence $(w_i^n)_i$.

Proof. Let $P \in P\big(^n d_*(w, 1)\big)$ and let (e_i) the canonical basis of $d_*(w, 1)$. We define

$$\lambda_i = \begin{cases} \dfrac{|P(e_i)|}{P(e_i)}, & \text{if } P(e_i) \neq 0; \\ 1, & \text{if } P(e_i) = 0. \end{cases}$$

Hence, $\lambda_i P(e_i) = |P(e_i)|$. We take $\beta_i \in \mathbb{C}$ such that $\beta_i^n = \lambda_i$. Let $\check{P}$ denote the symmetric n-linear mapping associated to P and (s_j) be the sequence of generalized Rademacher functions corresponding to n. For each permutation $\pi : \mathbb{N} \to \mathbb{N}$ and each $m \in \mathbb{N}$ we get

$$\begin{aligned}
\sum_{i=1}^m w_{\pi(i)}^n |P(e_i)| &= \sum_{i=1}^m w_{\pi(i)}^n \lambda_i P(e_i) \\
&= \sum_{i,i_2,\dots,i_n=1}^m \left(\int_0^1 s_i(t)\cdots s_{i_n}(t)dt \right) w_{\pi(i)}^n \lambda_i \check{P}(e_i, \dots, e_{i_n}) \\
&= \int_0^1 \left(\sum_{i,i_2,\dots,i_n=1}^m \beta_i^n w_{\pi(i)}^n s_i(t)\cdots s_{i_n}(t) \check{P}(e_i, \dots, e_{i_n}) \right) dt \\
&= \int_0^1 \check{P}\left(\sum_{i=1}^m \beta_i w_{\pi(i)} s_i(t) e_i, \dots, \sum_{i_n=1}^m \beta_{i_n} w_{\pi(i_n)} s_{i_n}(t) e_{i_n} \right) dt \\
&= \int_0^1 P\left(\sum_{i=1}^m \beta_i w_{\pi(i)} s_i(t) e_i \right) dt. \quad (*)
\end{aligned}$$

For $t \in [0, 1]$, we define $z(t) = \sum_{i=1}^{m} \beta_i w_{\pi(i)} s_i(t) e_i$. So, $|z(t)_i| = |\beta_i w_{\pi(i)} s_i(t)| = 1 \cdot w_{\pi(i)} \cdot 1 = w_{\pi(i)}$, if $i \leq m$, and $|z(t)_i| = 0$, if $i > m$.

Hence,

$$\|z(t)\|_{d_*(w,1)} = \sup_l \frac{\sum_{i=1}^{l} [z(t)]_i}{\sum_{i=1}^{l} w_i} \leq \sup_l \frac{\sum_{i=1}^{l} w_i}{\sum_{i=1}^{l} w_i} = 1,$$

In the last inequality we used the fact that the sequence (w_i) is decreasing. Consequently, for each $t \in [0, 1]$, $|P(z(t))| \leq \|P\|$. Then, for $(*)$, we get

$$\sum_{i=1}^{m} w_{\pi(i)}^n |P(e_i)| = \int_0^1 P(z(t))\, dt \leq \|P\|.$$

Since m is arbitrary, $\sum_{i=1}^{\infty} w_{\pi(i)}^n |P(e_i)| \leq \|P\|$; therefore

$$\|(P(e_i))_i\|_{d(w^n,1)} = \sup_{\pi} \sum_{i=1}^{\infty} w_{\pi(i)}^n |P(e_i)| \leq \|P\|.$$

□

From this proposition we conclude that $R(P(^n d_*(w, 1))) \subset d(w^n, 1)$. Our aim is to determine $R(P(^n d_*(w, 1)))$. In order to do that we establish the following lemma.

LEMMA 2.3. *Let $p \geq 1$ and let $k \in \mathbb{N}$. Given positive real numbers, $\alpha_1, \ldots, \alpha_k$ then there exists π^0 in the group S_k of permutations of k such that for every $x \in B_{d_*(w,1)}$ we have*

$$\sum_{j=1}^{k} \alpha_j |x_j|^p \leq \sum_{j=1}^{k} \alpha_j w_{\pi^0(j)}^p.$$

Before we prove the lemma we need the next proposition, for which the proof is in [**2**]. Let us recall that a point e of a convex subset A of the space E is called an *extreme* point of A if when $e = tx + (1 - t)y$ for some $t \in (0, 1)$ then, it has to be $e = x = y$. We denote by $ext(A)$ the set of all extreme points of A and $B^k_{d_*(w,1)}$ denotes the closed unit ball of the k-dimensional subspace $d_*(w, 1)$ spanned by $\{e_1, e_2, \ldots, e_k\}$.

PROPOSITION 2.4. [**2**] *The extreme points of $B^k_{d_*(w,1)}$ are the points with coordinates $|x_i| = w_{\pi(i)}$, $1 \leq i \leq k$ and $x_i = 0$ otherwise, for some permutation $\pi \in S_k$.*

Proof of Lemma 2.3. Among all the permutations $\pi \in S_k$ we choose π^0 such that the sum $\sum_{j=1}^{k} \alpha_j w_{\pi(j)}^p$ is maximum. Let $x \in B_{d_*(w,1)}$ and consider $\tilde{x} = (x_1, \ldots, x_k, 0, 0, \ldots)$. It is easy to see that $\tilde{x} \in B^k_{d_*(w,1)}$. By Krein-Milman's theorem, we have $B^k_{d_*(w,1)} = \overline{\text{co}}(ext B^k_{d_*(w,1)})$. So, $\tilde{x} \in \overline{\text{co}}(ext B^k_{d_*(w,1)})$. Firstly, we suppose that $\tilde{x} \in \text{co}(ext B^k_{d_*(w,1)})$. Hence, $\tilde{x} = \sum_{i=1}^{m} \lambda^i x^i$, where $\lambda^i > 0$, $\sum_{i=1}^{m} \lambda^i = 1$, and for each i, x^i is an extreme point of $B^k_{d_*(w,1)}$. So, for each $1 \leq i \leq m$, there exists $\pi^i \in S_k$ such that $|x_j^i| = w_{\pi^i(j)}, \forall j \leq k$. Then, for each $j \leq k$ we have that

$$\sum_{j=1}^{k} \alpha_j |x_j|^p = \sum_{j=1}^{k} \alpha_j \left| \sum_{i=1}^{m} \lambda^i x_j^i \right|^p \leq \sum_{j=1}^{k} \alpha_j \sum_{i=1}^{m} \lambda^i |x_j^i|^p = \sum_{i=1}^{m} \lambda^i \sum_{j=1}^{k} \alpha_j w^p_{\pi^i(j)}$$
$$\leq \sum_{i=1}^{m} \lambda^i \sum_{j=1}^{k} \alpha_j w^p_{\pi^0(j)} = \sum_{j=1}^{k} \alpha_j w^p_{\pi^0(j)}.$$

Secondly, in the case $\tilde{x} \in \overline{\text{co}}(extB^k_{d_*(w,1)}) \setminus \text{co}(extB^k_{d_*(w,1)})$, we can consider a sequence in $\text{co}(extB^k_{d_*(w,1)})$ which converges to $\tilde{x}$. □

In the proof of the next theorem we use Lemma 2.3 in order to determine $R(P(^n d_*(w, 1)))$.

THEOREM 2.5. *For each* $n \in \mathbb{N}$, $R(P(^n d_*(w, 1))) = d(w^n, 1)$.

Proof. Using Proposition 2.2 we get $R(P(^n d_*(w, 1))) \subset d(w^n, 1)$. On the other hand, let $y = (y_i) \in d(w^n, 1)$ and define the n-homogeneous polynomial on $d_*(w, 1)$ by

$$P(x) = \sum_{i=1}^{\infty} y_i x_i^n, \quad x = (x_i) \in d_*(w, 1).$$

For each $x \in B_{d_*(w,1)}$, by Lemma 2.3 we have

$$\sum_{i=1}^{k} |y_i||x_i|^n \leq \sum_{i=1}^{k} |y_i| w^n_{\pi^0(i)} \leq \|y\|_{d(w^n,1)},$$

for all $k \in \mathbb{N}$. So, P is well defined. Obviously $R(P) = y$. □

REMARK 2.1. Lemma 2.3 could be used to give another proof for the well-known result: if $w \in l_p$, for $p > 1$, then $d_*(w, 1) \subset l_p$. This could be done just taking the α_j equal to 1 and so get

$$\sum_{j=1}^{k} |x_j|^p \leq \sum_{j=1}^{k} w^p_{\pi^0(j)} \leq \|w\|_p^p < \infty.$$

3. Analytic functions. In this section, we discuss the behaviour of the range of the restriction operator R for the following Banach spaces of analytic functions:

$$\mathcal{A}^\infty(B_E) = \{f : B_E \to \mathbb{C} : f \text{ is analytic on } \overset{\circ}{B}_E, \text{ continuous and bounded on } B_E\}$$

and

$$\mathcal{A}_U(B_E) = \{f \in \mathcal{A}^\infty(B_E) : f \text{ is uniformly continuous}\}.$$

We remark that these spaces are the natural generalization in infinite dimensional of the disc algebra.

In [**1**], Aron and Globevnik have proved that any sequence of 0 and 1 can be interpolated by a function in $\mathcal{A}^\infty(B_{c_0})$ with norm 1. More precisely, if $S \subset \mathbb{N}$ is an arbitrary set, then there exists a function with norm 1 in $\mathcal{A}^\infty(B_{c_0})$ such that $f(e_n) = 1$

if $n \in S$, and $f(e_n) = 0$ if $n \notin S$. Besides, if S is finite, f can be taken in $\mathcal{A}_U(B_{c_0})$. An analogous result in $d_*(w, 1)$ holds, since for each $x \in d_*(w, 1)$ we have

$$\|x\|_\infty = [x]_1 \leq \sup_k \frac{\sum_{n=1}^{k} [x_n]}{\sum_{n=1}^{k} w_n} \|x\|_{d_*(w,1)},$$

which means the canonical inclusion $i : d_*(w, 1) \to c_0$ is continuous and, consequently, uniformly continuous and analytic. More precisely, we have the following lemma.

LEMMA 3.1. *Let S and S' be disjoint subsets of $\mathbb{N}$.*
(i) *There exists a function $f \in \mathcal{A}^\infty(B_{d_*(w,1)})$ with $\|f\| \leq 2$ such that*

$$f(e_n) = \begin{cases} 1, & \text{if } n \in S; \\ -1, & \text{if } n \in S'; \\ 0, & \text{otherwise.} \end{cases}$$

(ii) *If both of the sets S and S' are finite, then the function f above can be taken in $\mathcal{A}_U(B_{d_*(w,1)})$.*

Using the previous lemma, we obtain the following properties of $R(\mathcal{F})$ for the spaces $\mathcal{F}$ mentioned above.

PROPOSITION 3.2. (i) *Given $x \in l_\infty$, there exists $f \in \mathcal{A}^\infty(B_{d_*(w,1)})$ such that $R(f) = x$ and $\|f\| \leq 4\|x\|_\infty$. Consequently, $R(\mathcal{A}^\infty(B_{d_*(w,1)})) = l_\infty$.*
(ii) *Given $x \in c$, there exists $f \in \mathcal{A}_U(B_{d_*(w,1)})$ such that $R(f) = x$ and $\|f\| \leq 10\|x\|_\infty$. Hence $c \subset R(\mathcal{A}_U(B_{d_*(w,1)}))$.*

Proof. (i): If $x = 0$, it is enough to take $f \equiv 0$. Let $x \neq 0$. First assume that for each n, $x_n \in \mathbb{R}$. So, for each $n \in \mathbb{N}$, $\frac{x_n}{\|x\|} \in [-1, 1]$ and we write $\frac{x_n}{\|x\|}$ in its binary representation, so that $\frac{x_n}{\|x\|} = \sum_{j=1}^{\infty} 2^{-j} \alpha_{n_j}$, where each α_{n_j} is 0, 1 or -1. For each j, let $S_j = \{n \in \mathbb{N} : \alpha_{n_j} = 1\}$ and $S'_j = \{n \in \mathbb{N} : \alpha_{n_j} = -1\}$, and let F_j be the function obtained using Lemma 3.1(i). Let $f \sum_{j=1}^{\infty} 2^{-j}\|x\|F_j$. Then

$$f(e_n) = \sum_{j=1}^{\infty} 2^{-j}\|x\|F_j(e_n) = \|x\| \sum_{j=1}^{\infty} 2^{-j}\alpha_{n_j} = \|x\| \frac{x_n}{\|x\|} = x_n,$$

and for this case

$$\|f\| \leq \sum_{j=1}^{\infty} 2^{-j}\|x\|\|F_j\| \leq 2\|x\|_\infty.$$

In the general case, for each $n \in \mathbb{N}$ take $x_n = p_n + iq_n$, where $p_n, q_n \in \mathbb{R}$. Hence using the proof of the real case we get f_p and f_q and we consider $f = f_p + if_q$ with $\|f\| \leq \|f_p\| + \|f_q\| \leq 4\|x\|_\infty$. So, f is the required function.

(ii): Let $x \in c$. We assume that for each $n \in \mathbb{N}$, $x_n \in \mathbb{R}$. Let $l = \lim_n x_n$ and define $\beta_n = x_n - l$ for each n and $\beta = (\beta_n)_n$. Hence, $x_n = l + \beta_n$ and $\|\beta\| \leq 2\|x\|_\infty$. Now using the argument of (i) for β, we obtained the functions F_j in $\mathcal{A}_U(B_{d_*(w,1)})$; since $\beta_n \to 0$ we have that the sets $S_j = \{n \in \mathbb{N} : \alpha_{n_j} = 1\}$ and $S'_j = \{n \in \mathbb{N} : \alpha_{n_j} = -1\}$ are finite. Hence,

$f = \sum_{j=1}^{\infty} 2^{-j} \|\beta\| F_j + l$ is the function we were looking for, and in this case

$$\|f\| \leq 2\|\beta\|_\infty + |l| \leq 4\|x\| + \|x\| = 5\|x\|.$$

The general case it is similar to (i). We write each β_n in the form $p_n + iq_n$, where $p_n, q_n \in \mathbb{R}$ and we get f such that $\|f\| \leq 10\|x\|_\infty$. □

The above proposition gave us $c \subset R(\mathcal{A}_U(B_{d_*(w,1)}))$ and in the next theorem we characterize, under some hypothesis on w, the range of the restriction operator R associated to the usual basis of $d_*(w, 1)$.

THEOREM 3.3. *Let $w \in c_0 \setminus l_1$ be a decreasing sequence of positive real numbers. Then, $R(\mathcal{A}_U(B_{d_*(w,1)})) = c$ if and only if $w \notin l_p$ for all $p > 1$. If $w \in l_p$ for some $p > 1$, then $R(\mathcal{A}_U(B_{d_*(w,1)})) = l_\infty$.*

Proof. Let us assume that $w \notin l_p$ $\forall p > 1$. In view of Proposition 3.2 it suffices to show $R(\mathcal{A}_U(B_{d_*(w,1)})) \subset c$. Let $f \in \mathcal{A}_U(B_{d_*(w,1)})$. As f is uniformly continuous, given $\varepsilon > 0$, there exists $\delta > 0$ such that $\|x - y\| < \delta \Rightarrow |f(x) - f(y)| < \varepsilon/2$. Hence, taking $1 - \delta < r < 1$ we have that for all $x \in B_{d_*(w,1)}$, $\|x - rx\| < 1 - r < \delta$ and therefore,

$$|f(rx) - f(x)| < \varepsilon/2, \quad \forall x \in B_{d_*(w,1)}.$$

The function $x \mapsto f(rx)$ is analytic and bounded on $\frac{1}{r} \overset{\circ}{B}_{d_*(w,1)}$. Thus, the power series of $f(r\,\cdot\,)$ at zero converges uniformly on $\frac{1}{r} \overset{\circ}{B}_{d_*(w,1)}$ (see [**7**, Theorem 7.13]). Then, there exist $m \in \mathbb{N}$ and $P_k \in P(^k d_*(w, 1))$, $k = 0, 1, \ldots, m$, such that

$$\left| f(rx) - \sum_{k=0}^{m} P_k(x) \right| < \varepsilon/2, \quad \forall x \in B_{d_*(w,1)}.$$

Therefore, for all $x \in B_{d_*(w,1)}$, we have

$$\left| f(x) - \sum_{k=0}^{m} P_k(x) \right| \leq |f(x) - f(rx)| + \left| f(rx) - \sum_{k=0}^{m} P_k(x) \right| < \varepsilon,$$

in particular, $|f(e_n) - \sum_{k=0}^{m} P_k(e_n)| < \varepsilon$.

As $w \notin l_p$, $\forall p \geq 1$, by a result of Payá and Sevilla in [**8**] it follows that the polynomials P_k are weakly sequentially continuous for each $k = 1, \ldots, m$; that means P_k maps weakly convergent sequences into convergent sequences. As (e_n) converges weakly to zero, we have, for each $k = 1, \ldots, m$, $P_k(e_n)$ converges to zero and so

$$\lim_n \left| f(e_n) - \sum_{k=0}^{m} P_k(e_n) \right| = \left| \lim_n f(e_n) - f(0) \right| \leq \varepsilon.$$

Therefore, $f(e_n) \to f(0)$ and $R(\mathcal{A}_U(B_{d_*(w,1)})) \subset c$.

In the case $w \in l_p$ for some $p > 1$, by Remark 2.1 we have that $d_*(w, 1) \subset l_N$, for $N > p$. Hence, given any sequence $y = (y_n) \in l_\infty$, we can define a N-homogeneous polynomial on $d_*(w, 1)$ by

$$P(x) = \sum_{n=1}^{\infty} y_n x_n^N, \quad x = (x_n).$$

Therefore $R(P) = y$. The proof is complete. □

https://doi.org/10.1017/S0017089506003235 Published online by Cambridge University Press

REFERENCES

1. R. M. Aron and J. Globevnik, Analytic functions on c_0, *Rev. Mat. Univ. Complut. Madrid* **2** (1989), 27–33.

2. Y. S. Choi, K. H. Han and H. G. Song, Extensions of polynomials on preduals of Lorentz sequence spaces, *Glasgow Math. J.* **47**(2) (2005), 395–403.

3. S. Dineen, *Complex analysis on infinite dimensional spaces*, Springer Monographs in Mathematics (Springer-Verlag, 1999).

4. D. J. H. Garling, On symmetric sequence spaces, *Proc. London Math. Soc. (3)* **16** (1966), 85–106.

5. J. Gomes and J. A. Jaramillo, Interpolation by weakly differentiable fucntions on Banach spaces, *J. Math. Anal. Appl.* **182** (1994), 501–515.

6. J. G. Llavona and J. A. Jaramillo, Homomorphisms between algebras of continuous funcions, *Canad. J. Math.* **41** (1989), 132–162.

7. J. Mujica, *Complex analysis in Banach spaces*, North Holland Math. Studies, **120** (North Holland, 1986).

8. R. Payá and M. J. Sevilla, Norm attaining multilinear forms and polynomials on preduals of Lorentz sequence spaces, *Studia Math.* **127** (1998), 99–112.

https://doi.org/10.1017/S0017089506003235 Published online by Cambridge University Press