THE PROFILE OF A JETSTREAM

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The jetstream concept was introduced by Alfvén in 1969. Since then, the subject has been studied from various aspects by Danielsson (1969), Arnold (1969), Alfvén and Arrhenius (1970), Lindblad and Southworth,[1] and Trulsen.[2] In an attempt to define a jetstream, we may say that it is a group of objects moving in space with almost identical orbits. The largest objects in the jetstream may have any size, but the group must include a vast number of very small objects and their density must be large enough for the objects to interact. This means that collisions between the particles give rise to viscosity in the stream. Other interactions (e.g., by electromagnetic forces) are not excluded a priori.

The meteor streams, or at least some of them, seem to have a constitution that is not in conflict with this definition. As far as asteroid streams are concerned, we know nothing. However, one might assume that the observed size spectrum of asteroids can be extrapolated to smaller objects. (We, of course, do introduce a great uncertainty if we extrapolate all the way to the size of micrometeoroids.) The best assumption we can make about the distribution of the orbital elements for the subvisual objects is that it is similar to that of the visual bodies.

With these ideas as a background, Alfvén (1969) studied the classical Hirayama families among the asteroids to see whether there existed any clustering in the two orbital parameters that were not included in the analysis by Hirayama. Alfvén thus claimed to have found three streams in the Flora family, which were called Flora A, B, and C.

By essentially the same principle, Arnold (1969) searched all of the main asteroidal belt for streams. An important difference was that Arnold considered all five orbital parameters at the same time; his technique was to enclose each asteroid in turn in a five-dimensional "rectangular" box with predetermined sides and to count the number of asteroids in each box. If the number was "large," a stream was considered located.

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[1]See p. 337.
[2]See p. 327.

Lindblad and Southworth[3] used a different and in principle better method to find streams. They employed a five-dimensional distance formula originally derived by Southworth and Hawkins (1963) to find the distances between meteor orbits. By this method they located a great number of asteroid streams; many of them, however, have very few members.

Unfortunately these three works, notably the last two, do not agree very well; i.e., they generally do not find the same streams. One may raise the question of the statistical significance of the observed streams. This is a very difficult problem. (See discussion in a later paragraph of this paper.) In an earlier investigation, it was claimed that the streams Flora A and C are statistically significant (Danielsson, 1969). The value of this work is limited, however, because it did not consider the full five-dimensional problem. The significance of Arnold's streams is impossible to determine. His method for finding groups gives, contrary to what he claims (Arnold, 1969, pp. 1235 and 1236: "probability . . . 10^{-100}"), nothing on which to base a judgment. It is likely, however, that at least the groups with many ($\geqslant 10$) members are significant. The same should be true for Lindblad's investigation. In both these works, the method used is tested on synthetic distributions of the orbital elements. The disagreement of the results also can be attributed to the difference in the methods.

SIMILARITY OF ORBITS

Two orbits are similar if their orbital elements differ little from each other or, in other words, if the distance between the points representing the orbits in the five-dimensional orbit space is short. The methods used so far are based on estimates according to this principle. A shortcoming of Arnold's method is that the parameters enter independently of each other. The formula used by Lindblad and Southworth is an empiric expression found to work well for meteor streams; i.e., by choosing a suitable value for the orbit "distance," the formula will include members of the stream and exclude nonmembers as determined by the classical technique. Because, however, the classical technique is four-dimensional, one cannot be sure that the five-dimensional formula tested in this way is appropriate. It is also well known that the individual objects of a meteor stream may be very far apart when they are far from the neighborhood of Earth. One can say that the formula is insensitive to variations of the eccentricity (whereas it is oversensitive to variations in the perihelion longitude). This may be well motivated for meteor streams because the uncertainty in the determination of the eccentricity is quite large. It seems doubtful whether this formula is the best possible for stream searches among main belt asteroid orbits.

To estimate an average distance D between two orbits one might calculate instead the actual distance between the intersections of two orbits with a

[3]See p. 337.

heliocentric meridian plane as a function of longitude $d(\lambda)$ to get the quantity

$$D^2 = \frac{1}{2\pi}\int_0^{2\pi} d^2(\lambda)\, d\lambda$$

$d(\lambda)$ is a good approximation of the shortest distance from a point on one of the orbits to the other orbit for moderate eccentricities and inclinations. If terms of the order $e^{4-q} \sin^q i$ and smaller are neglected, the result is

$$D^2 = a_1{}^2(1 - e_1{}^2) + a_2{}^2(1 - e_2{}^2) - a_1a_2(2 - e_1{}^2 - e_2{}^2)$$
$$+ \tfrac{1}{2}[e_1{}^2a_1{}^2 + e_2{}^2a_2{}^2 - 2e_1e_2a_1\, a_2 \cos(\lambda_{p_2} - \lambda_{p_1})]$$
$$+ \tfrac{1}{2}a_1a_2[\sin^2 i_1 + \sin^2 i_2 - 2 \sin i_1 \sin i_2 \cos(\lambda_{n_2} - \lambda_{n_1})] \quad (1)$$

where λ_p and λ_n are the longitudes of perihelion and ascending node, respectively. The terms are arranged so as to emphasize their geometrical interpretation. The advantages with this formula over the one Lindblad uses are mainly that it gives an average value of the distance between two orbits and that this distance is expressed in normal length units, AU. Admittedly, the averaging can be done in different ways. The method used here is probably the easiest.

THE FLORA STREAMS

The Flora A, B, and C streams now can be redefined according to formula (1). Let us specify that all objects in Flora A with mutual distances less than 0.15 AU according to equation (1) be retained and let us in addition include all other asteroids that fulfill the same requirements. Present elements are used throughout. Four objects will then be added and six excluded to make Flora A contain asteroids 244, 703, 827, 836, 1037, 1120, 1335, 1422, 1494, and 1536. A mean orbit of these 10 orbits is defined by the mean values of each orbital element; the average distance to this mean orbit is less than 0.1 AU for all the members. It might seem that the average distance 0.1 AU is quite large, but it must be remembered that this is a distance in a five-dimensional space and that the probability of finding some neighboring orbit within this distance of a random orbit depends on the five-dimensional density n_5 of the asteroids:

$$P(D_{1,2} \leqslant d) = 1 - e^{-n_5 d^5}.$$

With the present definition, all three Flora streams appear as clusters of orbits with 10, 9, and 10 members, respectively. So far nothing is known about the statistical significance of these clusters.

The quantity n_5d^5 is best estimated through experiment; it is found to be 1.0 for $d = 0.10$ AU in the inner region of the main belt.

GEOMETRICAL PROPERTIES OF SOME ASTEROID STREAMS

We are not only interested in the statistical significance of a certain pattern in the distribution of the orbital parameters but even more in the geometrical properties of a group (stream). Figures 1 and 2 show the geometrical profile of Flora A. Figure 1 shows the intersections of the individual orbits with a heliocentric meridional plane as this plane makes one cycle around the ecliptic polar axis. The four groups of curve symbols show the intersection points for the longitudes 90°, 180°, 270°, and 355°/360°. Figure 2 shows the same curves but now in relation to the intersection of the mean orbit, which is stationary at the origin of this plot.

From the phase markings in figure 1, it is concluded that the orbits remain rather well collimated through the cycle and that they seem to have two "focusing" points at 110° and 290°. Figure 2 shows that at the extremes of the orbit a stream member can be as much as 0.11 AU from the mean orbit. The average distance of a stream member from the mean orbit according to equation (1) varies between 0.046 and 0.082 AU.[4]

In studying the evolution of the asteroids, their mutual collisions are of fundamental importance. The focusing points may be of particular interest because the probability for collisions is largest in these regions. At the longitude 290°, for example, seven members of Flora A intersect the plane within an area $\Delta r \times \Delta z = 0.070 \times 0.035$ (AU)^2, where r is the distance from

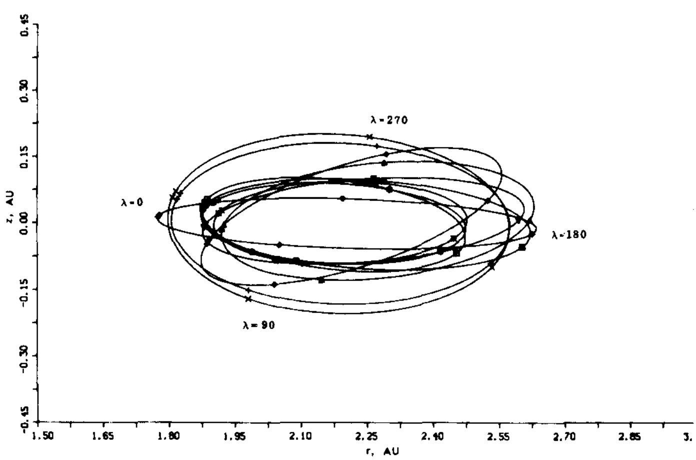

Figure 1.–Intersections of the 10 individual orbits of Flora A with a heliocentric meridional plane as this plane makes one cycle around the ecliptic polar axis. A curve symbol for each asteroid is plotted for λ = 90°, 180°, 270°, and 355°/360°.

[4]It is not known whether it is possible to find an orbit with the average distance to the other orbits always smaller than 0.5 × 0.15 = 0.075 AU by means of the approximate formula used here.

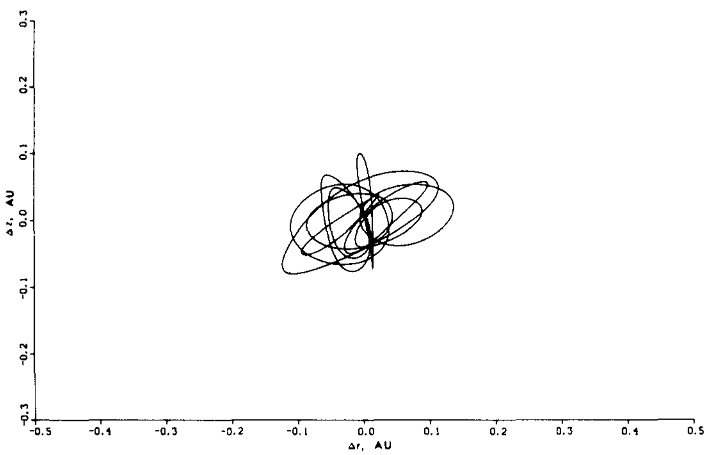

Figure 2.–Flora A orbits in relation to their mean orbit. The scale of the plot is chosen the same as in figures 3 and 4 for comparison.

the Sun and z is the distance from the ecliptic plane. It can be estimated that a random area of this size should be intersected by two or three orbits out of the total 1700; this particular area is in fact intersected by nine orbits (i.e., seven Flora A orbits and two others). Their relative velocities at the focus range from 0.2 to 1 km/s, which is 1 to 5 percent of the orbital velocities. The relative velocity between asteroids that by chance come close to each other is typically in the range 5 to 8 km/s.

Approximately the same holds for the other focusing point in Flora A and for the two focusing points in Flora C, whereas the Flora B stream is not as well focused anywhere.

This demonstrates that there are regions in space where the density of orbits is considerably larger than expected and where the relative velocities are substantially smaller than expected.

In the investigations made so far, the Flora A stream is unique because it is the only stream that can be recognized in a comparison between Arnold's and Lindblad's works. However, the three versions of Flora A do not contain exactly the same members. A comparison of the three corresponding stream profiles may then reveal something about the geometrical properties of the methods used in selecting them. Plots analogous to those in figures 1 and 2 have been prepared for these streams; namely, Arnold's stream J-1 with 32 members and Lindblad's stream 21 with 15 members (Lindblad, 1970, private communication).[5] Similar plots have also been made for two other streams of

[5]The stream numbers used in this paragraph refer to Lindblad's preliminary results. He later used a larger rejection level for $D(M, N)$ than in the work presented elsewhere in this volume.

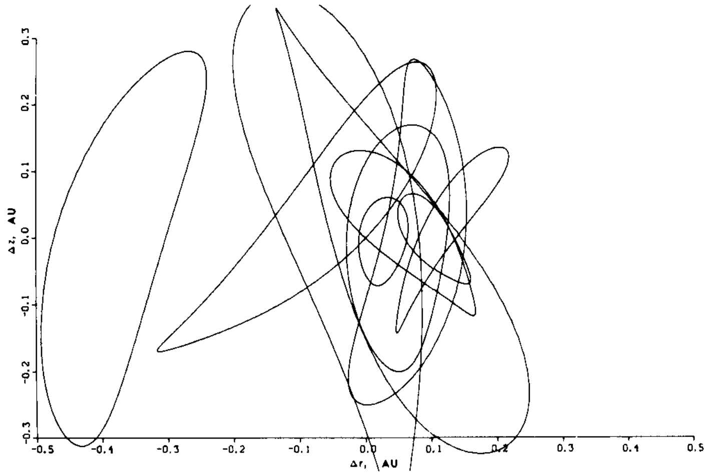

Figure 3.–The 10 orbits of stream J-6 (Arnold) in relation to their mean orbit.

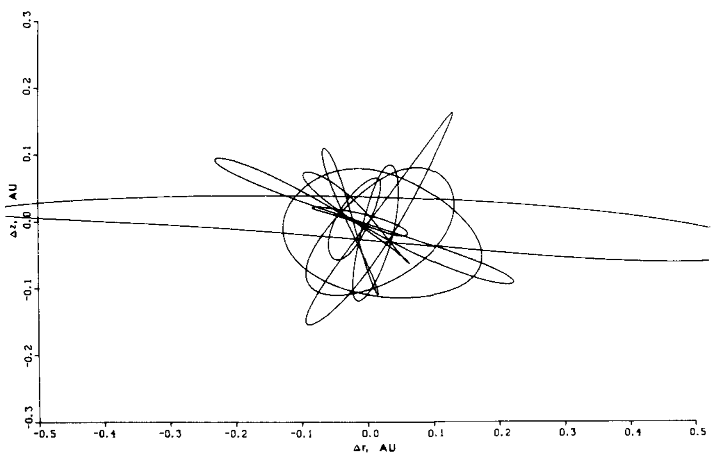

Figure 4.–Ten orbits of stream 2 (Lindblad) in relation to their mean orbit. (The four members most distant to the mean orbit are excluded.)

the same size as Flora A; namely, stream J-6 (Arnold) and stream 2 (Lindblad). (The latter stream was reduced from 14 to 10 members by omitting the four members with the largest value of $D(M, N)$ according to the formula used by Lindblad.) The plots of the orbits relative to their mean orbits for the two latter streams are shown in figures 3 and 4. The distance to the mean orbit is about twice as large for Lindblad's streams and about 10 times larger for Arnold's streams compared to Flora A, B, or C. Further, this investigation does not show any focusing regions, either in Lindblad's or in Arnold's streams.

COMET GROUPS

Another observed phenomenon might be included in a survey of jetstreams; namely, what is often called comet groups. From a statistical point of view these are probably insignificant because very few members (2 to 4) are included in each group (except for the Sun-grazing group). The only reason for mentioning them here is that if comets are considered to accrete from jetstreams (meteor streams) one could as easily imagine a stream developing several condensations.

STATISTICAL REMARKS

An important problem as far as the statistics are concerned is to decide whether "observed streams" are real or not. Hence, we want to estimate the probability (risk) that a certain property of the observed distribution is a result of a Poisson process. This probability is the level of significance of our conclusions concerning, for example, jetstreams. The problem thus formulated is a very difficult one (see the appendix for a simple example), which has never been solved in an analytic way (with the exception of the example given in the appendix). For general references on this type of problem, see Kendall and Moran (1963, chs. 2 and 5) and Roach (1968, ch. 4). Analytical methods described in the first of these works could possibly be employed, but this would be quite difficult and it is not at all certain that the result would be useful. A remaining possibility is to test synthetic distributions for the property under consideration (Roach, 1968; Danielsson, 1969). This test has to be done, of course, on a substantial number of synthetic distributions because the significance of such a test only can be determined from the distribution of the studied property among these synthetic distributions. In the present case, even making synthetic distributions is a complicated task.

Thanks to the Palomar-Leiden survey (PLS) (van Houten et al., 1970), which represents an additional, independent sample of asteroids, we can get an indication concerning the reality of our jetstreams if we find them here also. The value of this test is limited because of the observational selection of the PLS; essentially the test has to be confined to streams of low inclination. Nine hundred and thirty-one well-determined orbits (class I) have been investigated. The three Flora streams do appear also in the PLS material; however, these clusters of orbits are much less noticeable here. Within a distance $D = 0.10$ AU of the mean orbits of Flora A, B, and C, there are four, five, and three objects in the new material. At the same time, the density in this region of the five-dimensional space is twice as large in the new material as in the old. (This fact is found by experiment.) Because the mean orbits of Flora A, B, and C can be regarded as random points in relation to the PLS sample, one would expect them to have two (experimentally found average) neighbors within 0.10 AU if the distributions were random. It is obvious that the significance of each individual stream tested in this way is not overwhelming. If the streams are tested together, one finds that the risk that they all are a result of a Poisson process is about 1 percent.

CONCLUSIONS

By means of the new definition of an average distance between celestial orbits (eq. (1)) asteroid streams can be defined. So far only the three streams in the Flora family, Flora A, B, and C (Alfvén, 1969), have been studied (and redefined) by this method. It is found that the orbits of the members in these streams are well collimated everywhere along their path in contrast to previously defined streams. Furthermore, two of the streams show marked focusing regions where a majority of the orbits come very close together and where the relative velocities are an order of magnitude smaller than between randomly coinciding asteroid orbits.

From the point of view of jetstream physics, the best definition of a jetstream might be connected more closely with regions where the density of orbits is high and at the same time the relative velocity is low. This argument is not quite in line with the one leading to the distance formula used here. Maybe a weight function, giving more weight to those parts of two orbits where the distance is smallest, should be included in the integration leading to equation (1). In view of this argument, the classical way to determine a meteor stream would be quite good. According to this, a meteor stream is defined by the geocentric quantities of radiant, velocity, and date.

The statistical significance of the studied streams, admittedly, is shown far from satisfactorily. More work is required on this problem.

APPENDIX–PROPERTIES OF A POISSON PROCESS

The need to estimate the probability that a certain property of an observed distribution can be expected to appear in one realization of a Poisson process arises frequently in works of the present type. Because this is a very difficult task and misconceptions concerning the fundamental character of this problem are not rare in the literature of nonspecialized disciplines, this comment is considered worthwhile.

Any of the above discussed methods for finding clusters of similar orbits among the asteroids can serve as an example. In some way, the number of neighbors to an orbit (a point in a five-dimensional space) is determined; and if this number is "large," an orbit cluster is considered located. By "large" number is meant that the probability of finding the same cluster in a random distribution should be small. However, one has to be very careful as to what can be expected in a random (Poisson) distribution. It gives an entirely false result to regard an observation of a certain large cluster of this kind as a random observation. Thus probabilities according to the formula

$$P(X = k) = \frac{n^k e^{-n}}{k!} \qquad \text{(A-1)}$$

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are completely irrelevant in our case. (n and k are the uniform average and actually observed number of members in the cluster.)

As earlier pointed out (see above and Danielsson, 1969) the problem of finding an analytical expression for the probability of coming across a certain cluster in a random distribution is in reality a very difficult one. It seems to have been solved only for a very special one-dimensional case (Ajne, 1968). The formulation of the problem should be as follows: Given a random distribution with n members, what is the probability of observing a cluster of k members in some volume of suitably chosen size and location (k being considerably larger than the uniform average)?

The problem will be illustrated by two examples:

(1) Let five points be randomly distributed on the perimeter of a circle. The probability that all of them occur on one side of a given (in advance) diameter is of course $2^{-5} = 0.031$. The probability that all of them can be located on one side of a suitably chosen diameter can be calculated according to a formula deduced by Ajne from straightforward combinatorial analysis: for $2k-n > 0$,

$$P(X \geqslant k) = 2^{1-n}(2k-n)\sum_{j=0}^{\infty}\binom{n}{j(2k-n)+k}$$

With $k = n = 5$, then $P(X = 5) = 5 \times 2^{-4} = 0.31$; i.e., 10 times more likely than in the first case.

(2) Consider the alleged asteroidal cluster Flora B as studied by Danielsson (1969). In a two-dimensional area, where only one point would be found on an average, seven were observed. If the area had been randomly located, the probability for this occurrence in a Poisson distribution would be $(e \cdot 7!)^{-1} = 7 \cdot 10^{-5}$.

To estimate the actual probability under the proper formulation of the problem, 100 synthetic random distributions were made to simulate the observed population. Seven points were observed in the given area, suitably located, 26 times. Thus the probability was estimated to be 0.26. More than seven points were observed three times so that the probability of finding seven or more points was 0.29.

It is clear that formula (A-1) can be wrong by very many orders of magnitude when the number of points is large. For example, the probability 10^{-100} mentioned by Arnold (1969, p. 1236) may very well be wrong by a factor of 10^{90} or more.

ACKNOWLEDGMENTS

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DISCUSSION

WILLIAMS: Were observational selection effects considered in judging the significance of jetstreams?

DANIELSSON: The problem of observational selection certainly needs to be investigated very carefully to determine whether the asteroid streams are real or not. One can probably assume that asteroids of absolute magnitude (visual) $g < 12$ are unbiased with respect to observational selection. In a paper examining the Flora family (Danielsson, 1969) I have shown that if the asteroids with $g \geqslant 12$ are excluded, one of the streams (Flora C) remains statistically significant. Selecting the largest asteroids of the family in this way, of course, meant a substantial reduction of the number of members.

UREY: Are the jetstream particles the result of a collision in which the components that were produced remained in neighboring orbits?

DANIELSSON: The appearance of focusing points could possibly be the result of a collision, but this must then have been a very recent (10^4 to 10^5 yr) event because the phases of these orbits are very quickly spread out.

UREY: Do the geometrical properties you describe support a model based on fragmentation?

DANIELSSON: The geometrical properties that I have described do not tell you anything directly about accretion or fragmentation. However, as far as I can see, the well-collimated streams with focusing regions would have a very short lifetime unless there were some viscous force in the stream producing and maintaining these properties. Thus, if these geometric characteristics are found to be common for most of the streams, it would indicate the existence of such a force. This in turn would probably favor an accretion model.

DISCUSSION REFERENCE

Danielsson, L. 1969, Statistical Arguments for Asteroidal Jet Streams. Astrophys. Space Sci. **5,** 53.