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The Noisy Ageing of Slow Pulsars: New Thoughts on the Evolution of the Pulsar Population

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Abstract. Over the last decade or so, it has become clear that pulsars exhibit sudden and significant changes in their spin properties. At the same time, a better understanding of the geometry of young and older pulsars, is providing clues about the long-term evolution of the magnetic inclination angle. In this talk, we present a simple simulation of the pulsar population that takes into account current observational facts. We show how, with very few assumptions, the observed $P - \dot{P}$ diagram can be reproduced for a synthesized population. The implications are interesting and testable.

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1. Pulsar evolution on the $P - \dot{P}$ diagram

At this moment in time, despite the ever increasing number of known pulsars, we are not in a position to answer some basic questions regarding the birth and evolution of these stars. We have little understanding of their spin period at birth. We also see that the main population of pulsars, i.e. the stars that have not been recycled into millisecond pulsars, show a broad range of spin periods P and spin period derivatives $\dot{P}$. The question is, what is the path that each pulsar has taken to reach its current observed place on the $P - \dot{P}$ diagram. The answer to this question holds valuable information related to the physical processes that govern pulsar evolution, the true age of pulsars, and the size of the pulsar population, particularly those observable from Earth. Briefly, assuming pulsars are magnetic dipoles rotating in vacuum, B is given by

$$B = \sqrt{\frac{3c^3 I}{8\pi^2 R^6 \sin^2\alpha} P\dot{P}}, \tag{1.1}$$

the spin-down energy, $\dot{E}$, can be written

$$\dot{E} = 4\pi^2 I \frac{\dot{P}}{P^3} \tag{1.2}$$

and the characteristic age, τ_c, is

$$\tau_c = \frac{P}{2\dot{P}}. \tag{1.3}$$

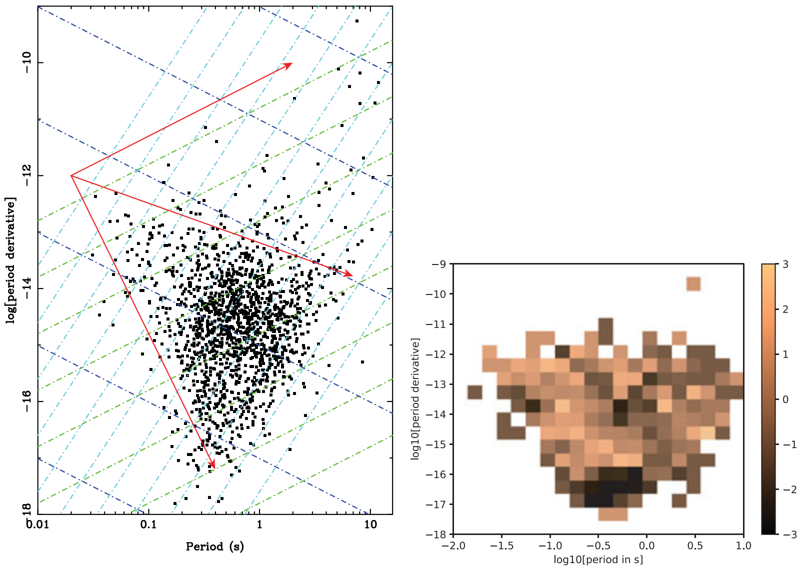

Figure 1. *Left:*. The $P-\dot{P}$ diagram for 1600 known pulsars. Shown are lines of constant B (negative slope), lines of constant τ_c (shallow positive slope) and lines of constant $\dot{E}$ (steep positive slope). From an initial position at $P = 20$ ms, $\dot{P} = 10^{-12}$, the red arrows show time-evolution through the diagram for $n = 1.0$, 2.7 and 6.0 from top to bottom. *Right:* The difference between the observed and simulated $P - \dot{P}$ diagram, colour coded using Eq. 3.2. Lighter signifies an over-abundance of simulated pulsars, darker an under-abundance.

In the above, c is the speed of light, I the moment of inertia, R its radius and α the inclination angle between the rotation and magnetic axes. Assuming constant I and R across the pulsar population, and assuming pulsars are orthogonal rotators, allows placing lines of constant B and $\dot{E}$ onto the $P - \dot{P}$ diagram (see Figure 1).

Lines of constant τ_c are also shown in Figure 1. The characteristic age is widely considered not to agree with the true age, as it assumes magnetic dipole braking as the only braking mechanism over the lifetime of a given pulsar. It is used however, to broadly separate the young, highly energetic pulsars at the top left of the diagram (and, to some extent, the magnetars in the top right) from the bulk of the pulsar population. Pulsars with $\dot{E}$ below 10^{30} ergs^{-1} are rare, indicating this as the minimum required $\dot{E}$ for radio emission.

It is useful to write the first derivative of the spin frequency as a function of the spin frequency itself,

$$\dot{\nu} = -K\nu^n \tag{1.4}$$

where K is a constant. This allows us to define n, the braking index,

$$n = \frac{\nu\ddot{\nu}}{\dot{\nu}^2} \tag{1.5}$$

The braking index is a useful way to parametrize the problem of pulsar evolution for two reasons. Firstly, a measurement of the braking index can, in theory, point to the dominant braking process. A particle wind corresponds to $n = 1$, magnetic dipole radiation corresponds to $n = 3$, and a magnetic quadrupole to $n = 5$. Secondly, the track on the $P - \dot{P}$ diagram that a pulsar with constant n will follow, is a straight line with slope $2 - n$.

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2. Evolution of n

For many reasons it is too simplistic to consider straight line tracks as being real evolutionary paths of pulsars. Johnston & Karastergiou (2017) (hereafter JK17) summarize this by dissecting the following equation, based on work in Tauris & Konar (2001).

$$n(t) = n_0 - \frac{3c^3 I\dot{B}(t)}{R^6 B^3(t)\sin^2\alpha(t)\Omega^2(t)} - \frac{3c^3 I\cos\alpha(t)\dot{\alpha}(t)}{R^6 B^2(t)\sin^3\alpha(t)\Omega^2(t)} + \mathrm{TN}(t) \tag{2.1}$$

Here, the evolution of n is described with four terms. An initial n_0, the time derivative of the magnetic field, $\dot{B}$, the time derivative of the inclination axis, $\dot{\alpha}$, and $\mathrm{TN}(t)$, a random component of the braking index. JK17 introduce the last term as a catch-all term for the noisy effects of state changes, timing noise, glitches, and/or intermittency.

The literature is inconclusive when it comes to understanding a possible decay of the magnetic field (Gunn & Ostriker 1970). There are several examples of work making the case for (e.g. Gonthier *et al.* 2004) and against (e.g. Lorimer *et al.* 1997) this. Theoretical studies also reach a variety of conclusions (Goldreich & Reisenegger 1992; Gullón *et al.* 2014; Igoshev & Popov 2015).

In terms of $\dot{\alpha}$, there are a few possibilities. It is easier to make the assumption that on long timescales, α changes monotonically. The case of pulsars moving towards alignment has been made in recent years based on observations of large populations of pulsars (Tauris & Manchester 1998; Weltevrede & Johnston 2008). The Crab pulsar offers a counter example Lyne *et al.* (2013). Over 22 years, α is increasing at a rate of 0.62 degrees per century. It is hard to claim that a single pulsar observed over a short timescale challenges the picture that emerges from studies of large populations.

3. Synthesizing a population of pulsars

The work in JK17 is motivated by studies of the birth and evolution of isolated radio pulsars, such as Faucher-Giguère & Kaspi (2006). In addition to concluding that magnetic field decay is not significant, they show the importance of the radio luminosity L of a pulsar being a function of the spin-down energy, more specifically $\sqrt{\dot{E}}$. To reproduce the observed population, Faucher-Giguère & Kaspi (2006) assume a wide distribution of the period at birth, centred on 300 ms, and a constant braking index per pulsar. In a different study, Ridley & Lorimer (2010) allowed for random n at birth, but again constant with time.

JK17 generate a population according to the following recipe:

(*a*) All pulsars born with Crab pulsar -like properties ($P = 20$ ms, $\dot{P} = 10^{-12}$).

(*b*) Pulsars are born with a random geometry.

(*c*) Pulsars are born at random distances from Earth, based on the Galactic stellar density distribution.

(*d*) One pulsar is born every R_b years, with R_b ranging between 30-100 as per Keane & Kramer (2008).

(*e*) The noise term $\mathrm{TN}(t)$ is given a new value every R_n years.

(*f*) Pulsar luminosity is a function of P and $\dot{P}$.

(*g*) Pulsars are only added to the diagram if they are detectable, i.e. they are beaming towards us, and they are sufficiently bright.

(*h*) Assume a function of B and α with time.

(i) Pulsars with an intrinsic $\log \dot{E} < 30$ do not produce detectable radio emission. It is generally accepted to infer the luminosity as a function of P and $\dot{P}$ using the following equation.

$$\log L = \log(L_0 \;\; P^{\epsilon_1} \;\; \dot{P}^{\epsilon_2}) + L_c. \tag{3.1}$$

Under the recipe provided above, the values of $\epsilon_1 = -1.5$, $\epsilon_2 = 0.5$ chosen by Faucher-Giguère & Kaspi (2006) and Gullón *et al.* (2014) result in an over abundance of young, short period pulsars. Conversely, setting both parameters to zero obviously results in too many old pulsars. JK17 set $\epsilon_1 = -0.75$, $\epsilon_2 = 0.25$.

For the braking index, JK17 assume pulsars are initially born with a braking index drawn out of a Gaussian distribution, with a mean of 2.8 and a standard deviation of 1. R_b is set to be 100 yr. Then, every $R_n = 1000$ yr, TN(t) is given a random value from a Gaussian distribution with a mean of the current n and a standard deviation of $n/3$. The magnetic field B is chosen to remain constant, while α is chosen to decay exponentially over a timescale of 10^7 yr.

A comparison between the JK17 population and the real pulsar population is shown on the right-hand plot of Figure 1. A 2D histogram with 20 bins in each dimension is formed for both populations, and the parameter R is computed for each bin, based on the number of simulated and observed pulsars $N_{\rm sim}$ and $N_{\rm obs}$.

$$R = \frac{N_{\rm sim} - N_{\rm obs}}{\sqrt{N_{\rm sim} + N_{\rm obs}}} \tag{3.2}$$

4. Implications

The simulations in JK17 have interesting implications. Firstly, old pulsars may be moving vertically on the $P - \dot{P}$ dot diagram. In general, smaller inclination angles imply a larger braking index. Contrary to previous works, it is possible for all pulsars to be born with short periods without a requirement for broad period distributions at birth. Importantly, the simulated pulsar population is significantly younger than the characteristic age suggets. Only 7% of the simulated population is older than 10^7 yr. The decay in α suggests the total observable population could be significantly smaller than previous estimates. This total estimated prediction of ∼20000 observable pulsars will be directly tested by telescopes such as the Square Kilometre Array.

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