Let (m, n) be a pair of positive integers satisfying $(*)$. If $m = n$, then $m = n = 1$. Suppose $m > n$ and let $t = m - n$. Then $t \geqslant 1$ and $m = t + n$. Substituting in $(*)$ gives:

$$\begin{aligned} & m^2 - n^2 = mn \pm 1 \\ \Leftrightarrow\ & (t+n)^2 - n^2 = (t+n)n \pm 1 \\ \Leftrightarrow\ & n^2 - t^2 = tn - (\pm 1) \\ \Leftrightarrow\ & n^2 - t^2 = nt \pm 1. \end{aligned}$$

So if (m, n) satisfy $(*)$, then so do (n, t). Furthermore (n, t) is a lower pair than (m, n). (For if $m^2 - n^2 = mn \pm 1$ as above, then $m = \frac{1}{2}(n + \sqrt{(5n^2 \pm 4)})$ and so $m \leqslant 2n$ and $t = m - n \leqslant n$.)

By replacing (m, n) by (n, t), this process can be repeated producing smaller pairs of integers satisfying $(*)$ until the pair $(1, 1)$ is reached. Reversing the process, the pair (m, n) must be one of the sequence $(1, 1)$, $(2, 1)$, $(3, 2)$, $(5, 3)$, $(8, 5)$, $(13, 8)$, …. Hence the original pair of integers satisfying $(*)$ must be two consecutive terms from the Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, …."

Correspondence

Looking for patterns

DEAR EDITOR,

Recent *Gazette* articles refer to the problem of how to avoid producing the result

$$\sum_{r=1}^{n} r^2 = \frac{1}{6} n(n+1)(2n+1)$$

like a rabbit from a conjuror's hat. Having always tried to encourage my students to look for patterns, I have found the following method simple but effective:

n	1	2	3	4	5	6	7	…
$\sum_{r=1}^{n} r$	1	3	6	10	15	21	28	…
$\sum r^2$	1	5	14	30	55	91	140	…
$\sum r^2/\sum r$	1	$\frac{5}{3}$	$\frac{7}{3}$	3	$\frac{11}{3}$	$\frac{13}{3}$	5	…,
i.e.	$\frac{3}{3}$	$\frac{5}{3}$	$\frac{7}{3}$	$\frac{9}{3}$	$\frac{11}{3}$	$\frac{13}{3}$	$\frac{15}{3}$	…

This suggests that

$$\sum r^2/\sum r = \frac{2n+1}{3} \quad \text{or} \quad \sum r^2 = \frac{n(n+1)}{2} \cdot \frac{2n+1}{3},$$

and it then seems quite natural to attempt to prove the result by induction.

Yours sincerely,

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A counter-example

DEAR EDITOR,

In answer to Robert Eastaway's question at the end of note **65.26**, Lander and Parkin discovered in 1966 that

$$27^5 + 84^5 + 110^5 + 133^5 = 144^5.$$

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As far as I am aware this is the only example known of r nth powers equal to an nth power, $n > r > 1$. Euler had conjectured that there were no such examples.

Yours sincerely,

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Reviews

100 years of mathematics, by George Temple. Pp 316. £32. 1981. ISBN 0-7156-1130-5 (Duckworth)

On reading the title, my thoughts were as follows. "An historical account, presumably. Which century? If the twentieth, as seems likely, is it really possible, in the space of 300 pages, to do justice to a period during which 90% of all known mathematics was discovered? And who is qualified to undertake such a formidable task?"

To begin with the final question, the distinguished author is Sedleian Professor Emeritus of Natural Philosophy at Oxford. His book is indeed an historical survey of major developments in mathematics over the last hundred years. The best way to indicate the immense scope of the book is to quote the chapter headings, which are Real numbers, Infinitesimals, Cantor and transfinite numbers, Finite and infinite numbers, Vectors and tensors, Geometry and measurement, The algebraic origins of modern algebraic geometry, The primitive notions of topology, The concept of functionality, Derivatives and integrals, Distributions, Ordinary differential equations, Calculus of variations, Potential theory and Mathematical logic. However, even Professor Temple had to draw the line somewhere: the major omissions are abstract algebra (to my regret!), numerical analysis, probability and statistics.

It will already be obvious that this book is not, and is not intended to be, a discursive 'history' in the style of, say, Boyer's well-known book; indeed, the recentness of the material precludes such an approach. Professor Temple begins where most histories end. Within each chapter he reviews briefly the 'pioneer work' in his chosen subject and then describes in more detail the important results discovered between 1870 and 1970. To quote from the introduction, the book is "essentially an account of the discovery or invention of mathematical concepts". The material is presented "neither chronologically nor biographically, but philosophically". Thus there is considerable overlap between chapters—intentionally so, for the author's aim is to show that "whereas in 1870 mathematics appeared to be diversified into distinct 'branches' . . . , at the present time the side-shoots of these branches are growing together and reuniting in a single trunk."

Potential buyers should be warned that this is a very demanding book; few concessions are made to the reader. Within each chapter the mathematics quickly becomes very technical. I found the first five chapters on number, some of the geometry and the final chapter on logic most accessible—but this selection merely reflects my personal tastes and previous knowledge. Although necessarily concise, the exposition is beautifully written. Aridity is avoided by the stylishness and occasional humour of the writing; for example, regarding the formalist–intuitionist controversy the author comments: "The other party, led by Weyl, accepted the new teaching with sorrow and repentance for earlier misdeeds". The book is very well produced and I noticed only one misprint—one of Gauss's initials wrong in the index! I was rather puzzled by the last sentence of paragraph (1) on p 282, which seems to bear little relation to what has gone before.

This book is an essential purchase for university libraries and many university lecturers will wish to have their own copies. I think that the author is justified in his belief that his book will be useful "to those embarking on reserach": indeed, one can envisage any of these chapters providing the framework for (at least) an M.Sc. thesis! But I think the potential readership is limited—not least in view of the exorbitant cost. Run-of-the-mill undergradu-