# SOME THOUGHTS ON MULTIPLE STARS 

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#### Abstract

RESUMEN Se ha hecho una reseña de los métodos para el descubrimiento de estrellas múltiples por medio de ocultación y cspectroscopía y de la técnica de mediciones espectroscópicas. Se propone una notación para estrellas múltiples. Un examen de los niveles de interacción gravitacional sugieren un valor promedio de $\Delta \log \mathrm{P}$ para un paso jerárquico cercano a 3 y para $\Delta \log$ a cercano a 2.0 , consistente con una razón de amarre cerca de 1:10000. La distribución para las estrellas $B$ parece ser diferente a la de otros tipos y se conjetura que ellas no pueden ser lo suficientemente viejas para ser residuos de cúmulos. En general, pocos sistemas múltiples parecen mostrar evidencia de una edad suficiente, como para que presenten una evolución avanzada las componentes.


#### Abstract

The occultation and spectroscopic approaches to the discovery of multiple stars are reviewed, and the technique of spectroscopic measurement recapitulated. A notation for multiple stars is proposed. An examination of the gravitational interaction levels suggests an average value of $\Delta \log P$ for one hierarchical step close to 3.0 and $\Delta \log$ a near 2.0 consistent with a binding ratio near $1: 10000$. The distribution for $B$ stars seems to differ from other types and it is conjectured that they cannot be old enough to be residues of clusters. In general few multiple systems seems to show evidence of age sufficient for advanced evolution of the components.


Discussions of multiple stars seem in the past to have been concerned mainly with dynamical problems in which the components are regarded as mass points, on which the whole armory of celestial mechanical techniques is brought to bear. My personal chances of survival in such a world are minimal and if I am to say anything useful it must be along the lines of considering multiple star components as real objects, capable, for example, of evolution. I shall take some of the systems which actually exist, and even though I may not be able to discern the rules of the game from this empirical standpoint, perhaps I can suggest some problems which could usefully be studied. While I was preparing these remarks I became aware of a publication in press by Dr. Helmut Abt and Dr. Saul Levy which
assembles a vast array of observational data and has some conclusions which I think are rather similar to my own (Abt and Levy 1975).

We find multiple stars by patiently looking at the sky. One technique which I have practised for a good many years is the observation of lunar occultations in which the discovery of binary stars is a common occurrence and of triple stars not rare. The example in Figures 1 and 2 (Dunham et al. 1973), shows the remarkable angular resolution readily achievable. It is doubtful whether any other astronomical technique at present known could demonstrate the existence of the faint component.

Observations of this kind have the great advantage of giving accurate relative luminosities of the components which has always been a problem in


Ftg. 1. Intensity plot of 8.5 magnitude F8 triple star SAO 118786 . The separations along P.A. $310^{\circ}$ are 22 arcmin and 186 arcmin with the faintest star having $\mathrm{m} \sim 10.5$.
double star astronomy, and Dr. David Dunham has privately circulated lists of double stars found in this way. A single observation from one station does not however provide the classical datum of angular separation and position angle. In principle multiple observations of the same phenomenon from different stations can provide this. It has rarely been achieved and is difficult to organize but we should like to encourage attempts especially since the work can be done with small telescopes. We have succeeded in resolving Atlas as a spectroscopic binary with separations down to 4 arc min (Nather and Evans 1971; McGraw et al. 1974; Bartholdi 1975). Although this is unusual it does demonstrate the power
of the method. There is no more guarantee of physical connection between components than in ordinary visual observation. Indeed in some of our most striking cases there is no spectroscopic evidence of duplicity. The stars observed are a random selection, but because of the inclination of the lunar orbit to the ecliptic and the motion of the lunar node a ten degree band or one sixth of the sky is eventually accessible to occultation observation. So far we have not tried to discuss statistically the incidence of duplicity among occulted stars since there are severe problems of observational selection to be faced. Although I may not seem too sanguine about the possibilities of discovering profitable multiple stars


Fig. 2. Integral plot of same trace.
by occultation observations, certain special cases where known multiple stars are occulted are extremely important. I want particularly to alert you to the occultation of $\beta$ Cap which will take place on the evening of December 6, 1975 (December 7 UT) and to the occultation of $\beta$ Sco on the night of July 7, 1976 (July 8 UT).
At the present time I believe that the most profitable field is the spectroscopic study of bright stars believed to be spectroscopic binaries involved in systems known to be visual doubles or multiples. In some cases the visual components are so close that they cannot be observed separately with the spectroscope. Some years ago I devised a method (Evans
1968) for analysing spectra of multiple stars which seemed especially adapted to the study of stars with types near F , on the grounds that these provided enough stellar lines to be recognizable on high dispersion but not so many that the composite spectrum would degenerate into hopeless confusion. The method is a laborious exercise in pattern recognition and, unlike some other methods, can actually be used by someone other than its inventor, as the excellent paper on $\psi$ Sagittarii to be described here later by Mr. Francis Fekel will demonstrate (Fekel 1975). There is undoubtedly a selection in favor of systems having components of similar luminosity since this minimizes the difficulty that each component spectrum
is seen diluted by the continua of all the other components. The method cries out for adaptation to computer techniques and to judge by a remark in the 1974 Annual Report of the Dominion Astrophysical Observatory, something along these lines may already have been achieved there by Mr. Chris Morbey.

One important topic in the study of multiple stars has always seemed to me to be the comparison and contrast of such systems with clusters. Globular clusters have membership in the range of $10^{3}-10^{6}$ with velocity dispersions possibly near $10 \mathrm{~km} \mathrm{~s}^{-1}$ and cluster rotation velocities of the same order and so it would appear, no identifiable system of stable orbits for cluster members. Galactic clusters have memberships up to, say, $10^{3}$, with velocity dispersions, possibly as much as $0.5 \mathrm{~km} \mathrm{~s}^{-1}$, but below the limit of definitive measurement. Apart from the occurrence in certain clusters of numerous binaries (say $40 \%$ of the membership in the Pleiades, (Bartholdi 1975)), the gravitational binding is weak.

Apart from the case represented to me in discussion by Dr. Victor Szebehely which contemplates a group of stars in temporary interaction-and I do not believe we have any method now for identifying such groups -multiple stars have well defined orbits with strong gravitational binding and a low numerical membership. Is there common ground among these different kinds of aggregate and can they be transformed one into another? In particular what is the highest degree of multiplicity which can occur in a multiple star?

In my Quarterly Journal article '(Evans 1968) I introduced the idea of a mobile diagram which contemplated a hierarchical organization such that different levels of gravitational binding were to a large extent independent. To save drawing little diagrams I now suggest a one-line structure designation notation and as an example I take the case of $\beta$ Scorpii (Van Flandern and Espenschied 1975) in which some of us have an intense interest. Van Flandern and Espenschied raise the possibility that there may be more components than are now recognized, but if we confine ourselves to those for which there is already some observational evidence we have the structural formula:

HR 5984/5 $\beta$ Sco Multiplicity: ?5+ \{[(3.0:B0, 4.0:B0, 6.828d), 5.0:X, 0"5:, X] $2.63 \mathrm{~B} 0.5 \mathrm{~V},\left(4.9 \mathrm{~B} 2 \mathrm{~V}, ? 6: \mathrm{X}, 0.097,308^{\circ}\right)$
4.92B2V, 13 ". $\left.6,21^{\circ}\right\} \quad \Delta \log \mathrm{a}=1.4:, 2.147$

This indicates a close spectroscopic pair with magnitude difference near 1.0 and similar spectral types with a known orbital period of 6.828 days. This has a visual companion possibly of magnitude 5.0 and unknown spectral type, possibly (see Van Flandern and Espenschied 1975) at 0.5 arc seconds in an unknown position angle. This seems like a catalogue of ignorance but it is often useful to specify which pieces of information are missing. These three stars together make up the 2.63 B 0.5 V assigned by the Bright Star Catalogue to $\beta^{1} \mathrm{Sco}$. The system is physically connected with $\beta^{2}$ Sco, 4.92 B 2 V at 13.6 arc seconds, position angle $21^{\circ}$. And $\beta^{2}$ Sco may, problematically have a faint companion thought to be near magnitude 6 at a separation of 0.097 arc seconds in position angle $308^{\circ}$. If we had orbital elements for any of the visual systems we should write in semiaxis major and period in place of the spot values of separation and position angle. This system of nesting brackets shows the structure of our multiple star. I will come to the use of the logarithmic statement later.
The Castor system is a sextuple of hierarchy 3 for which the structural formula is HR 2890/1 $\alpha$ Gem Multiplicity: 6
\{[(XX, XX, 2.928d) 1.99A1V, (XX, XX, 9.213d) 2.85Am, 6 "295, 420.07y],
(9.82MOVe, $\left.9.82 \mathrm{MOVe}, 0.814 \mathrm{~d}), 72.15,164^{\circ}\right\}$
to which we add for use below $\Delta \log P=4.719$, 4.222 and $\Delta \log \mathrm{a}=1.061$.

The Mizar-Alcor system gives
HR 5054/55/62 $(\xi+80)$ UMa Multiplicity: 5
\{[(3.2X, 3.2X 0". $0115,0.05621 \mathrm{y}) 2.27 \mathrm{~A} 2 \mathrm{~V}$, (XX, XX, 175.55d) $\left.3.95 \mathrm{Am}, 14.18,150^{\circ}\right] 2.04$, $\left.4.01 \mathrm{~A} 5 \mathrm{~V}, 709^{\prime \prime}, 72^{\circ}\right\} \quad \Delta \log \mathrm{a}=3.110,1.680$

Adopting this concept of a mobile diagram and this way of writing out a structure description, let us now try to build a multiple star, in particular
a multiple star of the highest possible degree of multiplicity. We could consider making the lowest level in the hierarchy out of pairs of stars in contact. Two white dwarfs in contact would have an orbital period of about 30 seconds, orbital speeds near 3000 $\mathrm{km} \mathrm{s}^{-1}$ and presumably equatorial rotational speeds of the same order. It is hard to see how such a system could originate since the white dwarf components would have passed through evolutionary stages of greater radii and separation. For contact pairs of main sequence stars the period is near 1.1 days for B 0 stars, 0.5 days for A 0 , with a slow decline from 0.28 days for F 0 to 0.17 days for M0. If we look at actual pairs one never seems to find periods shorter than some 3 to 10 times these limits. ER Vul at 0.70 days seems to have the shortest period for a pair of G dwarfs and YYGem, 0.81 days the shortest for a pair of M dwarfs. These are unevolved stars and presumably were able to come to these configurations without having encountered severe interaction problems in their contraction phases. One never seems to be able to make a general statement in astronomy without immediately turning up a contradiction. The August (1975) issue of the Astronomical Journal contains a report by John Hershey that VW Cephei, a W UMa star with a period of 6.7 hours has a 30.5 year companion now seen by W. D. Heintz. This will put an extra point at 4.60, G5 on my Figure 3.


Fro. 3. Values of $\Delta \log P$ us type of brightest component for multiple stars,

It would seem that for relatively long term stability the gravitational force at any level in the hierarchy of a multiple star ought to be much greater than that at the next higher level. Perhaps by considering some actual systems we may see whether this idea has any validity and if so what kind of force ratio applies.

For a pair of stars of masses $M_{1}, M_{2}$, the gravitational force $\mathrm{F} \sim \mathrm{M}_{1} \mathrm{M}_{2} \mathrm{a}^{-2}$, and $\mathrm{P}^{2} \sim \mathrm{a}^{3}\left(\mathrm{M}_{1}+\mathrm{M}_{2}\right)^{-1}$ so that $P \sim M_{1}^{3 / 4} M_{2}^{3 / 4}\left(M_{1}+M_{2}\right)^{-1 / 2} \mathrm{~F}^{-3 / 4}$ or $\mathrm{P} \sim \mathrm{fF}-\frac{1}{4}$. For $M_{1}=M_{2}, f=M / 2$ and ranges from about $5 \mathrm{M}_{\odot}$ for B dwarfs to $\mathrm{M}_{\odot} / 4$ for M dwarfs. If we take $M=1$, then $f=1 / 2$ and since we can expect masses approximately to double at each hierarchical step, then if $f_{1}=1 / 2, f_{2}=1, f_{3}=2$ etc. If the shortest period were, say, 10 days, $f_{1}=1 / 2,10=$ $\mathrm{k} \times 1 / 2 \times \mathrm{F}_{1}^{-1 / 2}$ and at subsequent higher levels we should have $\mathrm{P}_{2}^{\prime}=\mathrm{k} \times \mathrm{F}_{2}^{-5 / 2}, \mathrm{P}_{3}=\mathrm{k} \times 2 \times \mathrm{F}_{3}^{-3 / 4}$ where k is a constant. If the force ratio in a hierarchical step were 10000 , then for this system successive periods would be 10 days, 54 years, and 110000 years. This would result from the assumption that $\Delta \log \mathrm{F} \sim-4$ whence $\Delta \log \mathrm{P} \sim+3$ and $\Delta \log$ $\mathrm{a} \sim+2$.

In Table 1 I have gathered some more multiple stars, mostly from the Batten Catalogue of spectroscopic orbits (Batten 1967), and the Finsen and Worley Catalogue of Visual Binary Orbits. We also have $\xi$ Tauri (Bolton and Hurkens 1974), HD 100018 (Batten and Petrie 1970), DL Vir (Schöffel and Popper 1974), HD 14817 ( $=$ ADS 1833AB) (Frazier and Hall 1974), ADS 14839 (Wilson and Joy 1950), and 37 Peg (Abt and Levy 1975).
The list is short and certainly suffers from incompleteness of data. However acknowledging the dangers of such a situation we may plot the observed values of $\Delta \log \mathrm{P}$ in Figure 2 where it has seemed reasonable to select as abscissa the spectral type of the brightest component in each system. The average value of $\Delta \log \mathrm{P}$ is 2.54 suggesting that the gravitational binding ratio is quite close to the illustrative value of 10000 arbitrarily taken earlier. The figure suggests that among the B stars there is a much greater range of values. It hardly seems possible that a young B-type multiple star can have lived long enough to have been derived from the break-up of a cluster and the suggestion by Abt



and Levy that many B-star binaries are fission products seem very plausible. The occurrence of below average values of $\Delta \log P$ among early type stars casts doubt on their long term stability. Most of the multiple systems on our list do not include any far evolved stars. The situation suggests that when components of multiple stars begin to evolve, as contemplated by Fekel (1975) in the near future for $\psi \mathrm{Sgr}$, then some kind of interaction must take place which causes a change in the character of the system -possibly for example the fusion of two components into one. In short one does not seem to see any very old multiple systems.

The observed $\Delta \log \mathrm{P}$ both for very young and not so young systems is near 3.0 with much less scatter for the latter systems. One would suppose this ratio to be adhered to as one mounts the hierarchical steps since everything scales, except that instability ejection is a slower motion process. We do not have periods for wide separations, but, swallowing a great many difficulties concerned with projection and eccentricity, we can at the price of increased scatter accept spot measures of angular separations as a substitute for semi-axes major.

If $\Delta \log \mathrm{P} \sim 3$, then $\Delta \log \mathrm{a} \sim 2$ and for our pitiful sample of eight cases we have (Figure 4) a value of 1.83 , which is at least not discouraging. If all these speculations are valid we can now try to construct the multiple star of highest multiplicity which can exist. If the closest pair has, say, a period of 2 days, then the period at the next level will be 7 years, the next 7000 years and then we must stop since the next period at 7 million years is comparable with the galactic orbital period and the system could no longer preserve its identity. We have thus three hierarchical levels capable of containing eight components. Perhaps by squeezing ratios we might build a star of multiplicity 16 but that seems to be the limit. It is interesting that it is quite distinct in its organization both from a cluster and a planetary system. One might add that adopting $\Delta \log a=2$


Fig. 4. Values of $\Delta \log$ a $v s$ type of brightest component for multiple stars.
ought to give us a way of estimating parallaxes of classical triple systems.

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## DISCUSSION

Harrington: I agree that multiples are probably not the same things as small clusters, but I question the argument, particularly the assumption of a ratio of binding forces of 10000 . When talking about limits of multiplicity, you must consider minimum limits of $\Delta \log$ a or $\Delta \log$ P. I know of three examples where $\Delta \log$ a is less than 1 , so the factor of 1000 in $\Delta \log \mathrm{P}$ must be conservative. While in practice the limit to multiplicity may be around 8, I don't think this can be taken as a true limit.

Evans: The result I gave is of course an average based on all the observations of bright stars that Mr. Fekel and I could get hold of. I did assume the binding ratio to be the same at all hierarchical levels and this might not be true for large separations. However, instability will be much slower to take effect at large separations than at closer ones. I would not be surprised if my numbers had to be modified in the light of more extensive data, but I think statistical study is the way to discover the parameter values most likely to occur in practice.

