## Introduction

The problems of power corrections have been intensively discussed during the last few years [1,3,329–344,356–398].<sup>1</sup> By power corrections to the parton model, one means terms of the order of:

$$\left(\frac{\Lambda}{Q}\right)^n \sim \exp\left(\frac{n\pi}{\alpha_s(Q^2)\beta_1}\right),$$
 (26.1)

where Q is a typical momentum much larger than the QCD scale  $\Lambda$ , and  $\beta_1 = -1/2(11 - 2n_f/3)$  in our notation. A priori, this is problematic as these contributions are exponentially small in the inverse of the running coupling, and can be related to many orders of the perturbative expansion. In order to develop a phenomenology of these power corrections, one then assumes that they are numerically important and are responsible for the breaking of asymptotic freedom at intermediate energies. This fact has been firstly indicated in the QCD spectral sum rules phenomenology [1,2,3,356–365] and in the analysis of renormalons [329–380] and instantons [382–398]. However, as noticed in [162], the idea of power corrections are not quite new but can be traced back to an old paper [416]. It has been considered an  $e^+e^-$  pair at distance r placed into a centre of a conducting cage of size L. Assuming that  $L \gg r$ , the potential energy of the pair can be approximated by:

$$V_{e^+e^-}(r) \simeq -\frac{\alpha}{r} + \text{const} \frac{\alpha r^2}{L^3} \quad \text{for} \quad L \gg r ,$$
 (26.2)

where the second term can be viewed as a power correction to the Coulomb potential. In classical electrodynamics, this correction corresponds to the interaction of a dipole with its image, or can be also derived in terms of one-photon exchange. From this example, one can derive, by analogy, the heavy quark potential of QCD, which at short distances looks as [417]:

$$V_{\bar{Q}Q}(r \to 0) \simeq \frac{C}{r} + \text{const } \Lambda^3 r^2 ,$$
 (26.3)

where *C* is calculable in perturbative QCD as series in  $\alpha_s$ , and where one should notice the absence of a linear term at short distance. For deriving this expression, one has replaced *L* by  $1/\Lambda$ , assuming that the gluon propagator is modified by IR effects at the scale  $1/\Lambda$ . The shift

<sup>&</sup>lt;sup>1</sup> It is a pleasure to thank Valya Zakharov for reading this Part.

of the atomic levels in the cage are sensitive to a local characteristic of the non-perturbative fields, which on dimensional grounds reads [416,162]:

$$(\delta E)_{NP} \sim \frac{\langle 0|\mathbf{E}^2|0\rangle}{m_e^3},$$
 (26.4)

where  $\langle 0|\mathbf{E}^2|0\rangle$  corresponds to the difference of the mean value of  $\mathbf{E}^2$  in one photon approximation evaluated without and with the cage, and which is UV finite. Translated in QCD, one can have the picture of the  $\bar{Q}Q$  bound states, in terms of the density of the colour field strengths, or more popularly known as gluon condensate  $\langle 0|\alpha_s(G^a_{\mu\nu})^2|0\rangle$ , first discussed in [418]:

$$\delta E_{nl} \sim C_{nl} \frac{\langle 0|\alpha_s \left(G^a_{\mu\nu}\right)^2 |0\rangle}{M_O^4} + \cdots$$
(26.5)

Considering now the QCD correlation function, which generically reads:

$$\Pi_{H}(-q^{2}) \equiv i \int d^{4}x \; e^{iqx} \langle 0|\mathcal{T}J(x)_{H} \left(J_{H}(0)\right)^{\dagger}|0\rangle \;, \tag{26.6}$$

where  $J_H(x)$  is the hadronic current of quark and/or gluon fields, a use of the standard OPE leads to [1]:

$$\Pi_{H}(-q^{2}) \sim \Pi_{H}(-q^{2})_{\text{parton}} \left[ 1 + a_{H}\alpha_{s}(-q^{2}) + \frac{b_{H}}{q^{4}} + \cdots \right]$$
(26.7)

for massless quarks, and, where  $a_H$  and  $b_H$  are constants which depend on the hadron quantum numbers. One should notice the absence of the  $1/Q^2$  power term in the standard OPE as we shall discuss in detail in the next chapter. We shall also see in the following chapters how the resummation of the QCD asymptotic series and its phenomenological picture can induce such a term.