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PROJECTIVE AND AFFINE TRANSFORMATIONS OF A COMPLEX SYMMETRIC CONNECTION

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Let M be a compact complex manifold and ∇ an arbitrary complex (not necessarily Riemannian) connection. In this paper we study the relation between the geometry of (M, ∇) and the topology of M, that is, we are interested in the following problem: To what extent does the topology of M determine the relations between the group of holomorphically projective transformations, the group of projective transformations and the group of affine transformations on M? Under assumptions on the Ricci-type tensors of ∇ and Chern numbers of M we show that a holomorphically projective transformation and a projective transformation are in fact affine transformations on M. A family of interesting examples of connections of this kind are constructed. Also, the case when M is a Kähler manifold is studied.

0. INTRODUCTION

Let M be a manifold and ∇ an affine symmetric connection (not necessarily Riemannian). Then we can ask the following question: Is every projective transformation on M is an affine transformation? This question has been studied in a lot of papers. For example, for some manifolds where the Ricci curvature tensor is parallel or definite it has been shown that the group of projective transformations and the group of affine transformations coincide (see [**5, 10, 14**]).

Here we consider a compact complex manifold (M, J), equipped with a symmetric complex connection ∇, where J denotes the corresponding almost complex structure and $\nabla J = 0$. In this case we consider a similar question: To what extent does the topology of (M, J) determines the relations between the group of holomorphically projective transformations, the group of projective transformations and the group of affine transformations? We make some weaker assumptions (than in the above mentioned papers) for the Ricci-type tensors of ∇ but we additionally assume the Chern numbers of (M, J) satisfy some conditions.

In Section 1 we introduce notions and state the results of Ishihara which we use. In Section 2 we express the Chern number $c_1^n(M)$ in terms of the curvature tensor R

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of ∇, that is, in terms of the Ricci-type tensor $\overline{\mu}$ of ∇. We study here connections where the Ricci curvature tensor ρ is symmetric. These connections appear naturally in the study of holomorphic projective transformations [**15**], affinely conformal invariant connections [**13**], conformal motions et cetera.

In Section 3 we consider the group of holomorphically projective transformations, the group of projective transformations and the group of affine transformations of the affine connection ∇. The main result is stated and proved. Particularly, we have a corrollary concerning a surface of a general type. Then we construct a family of nontrivial examples of symmetric complex connections ∇ on $\mathbf{C}P^1 \times \mathbf{C}P^1$ which satisfy the first two conditions of Theorem 3.2. Some other examples are also mentioned.

In Section 4 we deal with Kähler manifolds of constant scalar curvature and the relation between the group of projective transformations, the group of affine transformations and the isometry group of the Levi-Civita connection.

1. Preliminaries

Let us recall some facts from the projective geometry of manifolds. A map $f : \left(\widetilde{M}, \widetilde{\nabla}\right) \longrightarrow (M, \nabla)$ of manifolds with symmetric connections is called projective if for each geodesic γ of $\widetilde{\nabla}$, $f \circ \gamma$ is a reparameterisation of a geodesic of ∇, that is, there exists a strictly increasing C^∞ function h on some open interval such that $f \circ \gamma \circ h$ is a ∇-geodesic. Linear connections $\widetilde{\nabla}$ and ∇ on M are projectively equivalent if the identity map of M is projective. A projective transformation of (M, ∇) is a diffeomorphism which is projective (see [**12**]). The transformation s is projective on M, if the pull back $s^*\nabla$ of the connection is projectively related to ∇, that is, if there exists a global 1-form $\pi = \pi(s)$ on M such that

$$s^*\nabla_X Y = \nabla_X Y + \pi(X)Y + \pi(Y)X, \tag{1}$$

for arbitrary smooth vector fields $X, Y \in \mathcal{X}(M)$. Having (1) in mind, if s and t are two projective transformations, we find

$$\pi(st) = \pi(s) + \widehat{s} \cdot \pi(t),$$

where $\widehat{s}$ is the cotangent map, that is, $[\widehat{s} \cdot \pi]_{s(p)} = \widehat{s} \cdot [\pi]_p$.

If a transformation s of M preserves geodesics and the affine character of the parameter on each geodesic, then s is called an affine transformation of the connection ∇ or simply of the manifold M, and we say that s leaves the connection ∇ invariant.

Similarly, we can introduce the holomorphically projective transformation. In an almost complex manifold Tashiro [**15**] has studied a change of a symmetric affine connection ∇ with respect to which the almost complex structure J is covariant constant.

The transformation s is holomorphic projective on M if it preserves the system of holomorphically planar curves, that is, if the pull back $s^*\nabla$ of the connection is holomorphically projective related to ∇, that is, if there exists a global 1-form $\pi = \pi(s)$ on M such that

$$s^*\nabla_X Y = \nabla_X Y + \pi(X)Y + \pi(Y)X - \pi(JX)JY - \pi(JY)JX,$$

for arbitrary smooth vector fields X and Y.

Ishihara studied in [5] the group of projective transformations, the group of affine transformations and the group of isometries of M. He also studied in [6] the group of holomorphically projective transformations. The topology of compact complex surfaces, holomorphic projective connections and holomorphic affine connections were studied by Kobayashi and Ochiai in [7], Inoue, Kobayashi and Ochiai in [4] respectively. Ishihara has proved the following theorems.

THEOREM A. *If (M,∇) is a compact manifold and the Ricci tensor of ∇ vanishes identically in M, then the group of projective transformations of M coincides with its subgroup of affine transformations.*

If M is also irreducible then Ishihara has proved that the group of projective transformations of M coincides with its group of isometries.

THEOREM B. *If a complex manifold M of complex dimension $n > 1$ is complete with respect to a symmetric connection ∇ $(\nabla J = 0)$ and the Ricci tensor of M vanishes identically in M, then the group of holomorphically projective transformations of M coincides with its subgroup of affine transformations.*

Moreover, if M is a compact Kählerian manifold then Ishihara has proved that the identity component of its group of holomorphically projective transformations for the Levi-Civita connection coincides with the identity component of its group of isometries.

2. The Chern characteristic class c_1^n of a Hermitian manifold

Let M be a complex manifold, of complex dimension n and let an endomorphism J $\left(J^2 = -I\right)$ be the corresponding almost complex structure on TM. We denote by $\mathcal{X}_{\mathbf{C}}(M)$ and M_m the Lie algebra of C^∞ complex vector fields on M and the real tangent space to M at m. Let ∇ be an arbitrary complex symmetric connection, that is, a connection such that

$$\nabla J = 0, \quad \text{and}$$
$$\nabla_X Y - \nabla_Y X = [X,Y],$$

for $X, Y \in \mathcal{X}_{\mathbf{C}}(M)$. The curvature R of ∇ is defined by

$$R(X,Y) = \nabla_{[X,Y]} - [\nabla_X, \nabla_Y] \quad \text{for} X, Y \in \mathcal{X}_{\mathbf{C}}(M),$$

and satisfies the following relations:

$$R(X,Y) = -R(Y,X), \tag{2}$$

the first Bianchi identity

$$R(X,Y)Z + R(Y,Z)X + R(Z,X)Y = 0,$$

and the Kähler identity,

$$R(X,Y) \circ J = J \circ R(X,Y), \tag{3}$$

for $X, Y \in \mathcal{X}_{\mathbf{C}}(M)$.

Especially, if $R(X,Y) = -R(JX,JY)$, we say ∇ is a holomorphic affine connection or if $R(X,Y) = R(JX,JY)$, ∇ is an affine Kähler connection (see [**7**, **11**]).

Let E_1, $JE_1, \cdots, E_n$, JE_n, be a real base for tangent space M_m and $\omega^1, \overline{\omega}^1, \cdots$, $\omega^n, \overline{\omega}^n$ the corresponding dual base for M_m^*. Then we shall write $E_{n+p} = JE_p = E_{\overline{p}}$ and similarly $\omega^{n+q} = \overline{\omega}^q$, $1 \leqslant p,q \leqslant n$. In this paper we suppose summation for every pair of repeated indexes. Also, we use the following ranges for indexes i, j, p, $q = 1, 2, \cdots, n$, and I, J, P, $Q = 1, 2, \cdots, 2n$. We denote $JE_P = E_{\overline{P}}$ and $R(X,Y)E_P = R_{XYP}{}^Q E_Q$. For $X = E_I, Y = E_J$ we simplify notation and write $R_{E_I E_J P}{}^Q = R_{IJP}{}^Q$.

It will be useful for our consideration of Chern classes to introduce the following traces:

$$\begin{aligned} \mu(X,Y) &= \frac{1}{2} \operatorname{tr} \{V \to R(X,Y)V\} = R_{XYi}{}^i, \\ \overline{\mu}(X,Y) &= \frac{1}{2} \operatorname{tr} \{V \to J \circ R(X,JY)V\} = R_{XJY\overline{i}}{}^i \end{aligned} \tag{4}$$

for $X, Y \in M_m \otimes \mathbf{C}$ and $V \in M_m$.

The Ricci tensor of type $(0,2)$ is

$$\rho(X,Y) = \operatorname{tr} \{V \to R(V,X)Y\} = R_{IXY}{}^I.$$

Generally, ρ is neither symmetric nor antisymmetric. After some computations, we can prove

$$2\overline{\mu}(X,Y) = \rho(X,Y) + \rho(JY,JX), \tag{5}$$

$$2\mu(X,Y) = \rho(Y,X) - \rho(X,Y). \tag{6}$$

We put

$$\Omega_I^J(X,Y) = R_{XYI}{}^J, \qquad \text{that is} \qquad \Omega_I^J = R_{PQI}{}^J \omega^P \wedge \omega^Q,$$

and $$\Theta_i^j(X,Y) = -\left(\Omega_i^j(X,Y) - \sqrt{-1}\,\Omega_{\overline{i}}^j(X,Y)\right),$$

for $X, Y \in M_m \otimes \mathbf{C}$. Then (Θ_p^q) is a matrix of complex 2-forms and

$$\det\left(\delta_p^q - \frac{1}{2\pi\sqrt{-1}}\Theta_p^q\right) = 1 + \gamma_1 + \cdots + \gamma_n$$

is a globally defined closed form which represents the total The Chern class of M via de Rham's theorem (see [1]). The Chern classes determined by γ_1, γ_2 are denoted by c_1, c_2 respectively. In particular, the Chern forms γ_1 and γ_1^2 are given by

$$\gamma_1 = \frac{\sqrt{-1}}{2\pi}\sum \Theta_i^i = \frac{\sqrt{-1}}{2\pi}(\Omega_i^i - \sqrt{-1}\Omega_{\bar{i}}^i), \tag{7}$$

$$\begin{aligned}\gamma_1^2 &= -\frac{1}{4\pi^2}\sum_{1\leqslant i<j\leqslant n} \Theta_i^i \wedge \Theta_j^j \\ &= -\frac{1}{4\pi^2}\sum_{1\leqslant i<j\leqslant n}\left\{\left(\Omega_i^i \wedge \Omega_j^j - \Omega_{\bar{i}}^i \wedge \Omega_{\bar{j}}^j\right)\right. \\ &\qquad \left. -\sqrt{-1}\left(\Omega_i^i \wedge \Omega_{\bar{j}}^j + \Omega_{\bar{i}}^i \wedge \Omega_j^j\right)\right\}.\end{aligned} \tag{8}$$

We consider Chern numbers $c_2(M) = \int_M \gamma_2$ and $c_1^2(M) = \int_M \gamma_1^2$ for a compact complex surface M and similarly $c_1^n(M) = \int_M \gamma_1^n$ for an arbitrary complex compact n-dimensional manifold.

Here we study properties of some Chern numbers when complex manifolds admit some symmetric, complex (not necessarily metric) connection.

Now, from the relation (7) we have directly

PROPOSITION 2.3. *Let M be a complex manifold with a complex symmetric connection ∇, and $\mu = 0$. Then γ_1^n is given by*

$$\gamma_1^n = \frac{1}{(2\pi)^n}\underbrace{\left(\Omega_{\bar{i}}^i \wedge \cdots \wedge \Omega_{\bar{j}}^j\right)}_{n \text{ times}}.$$

3. PROJECTIVE TRANSFORMATIONS OF A COMPLEX MANIFOLD

The group of projective and the group of affine transformations of an affine torsion free connection have been studied in some papers (see [5, 10, 14]). Mainly, it was assumed that the corresponding Ricci curvature tensor is parallel or definite. Here we consider a compact complex manifold and make some weaker assumptions for the Ricci-type tensors of a complex connection but we assume the manifold satisfies some topological conditions. We need the following lemma.

Lemma 3.1. *Let M, $\dim_C M = n$, be a compact complex manifold with a complex symmetric connection ∇. If*

(i) $\mu = 0$,

(ii) $\overline{\mu}$ *is a semi-definite bilinear form of rank* 0 *or* n, *nonnegative if* n *is odd,*

(iii) $c_1^n(M) \leqslant 0$,

then ∇ is the holomorphic affine connection.

Proof: First, because of (i) we use (7) to write

$$\gamma_1(X, JY) = \frac{1}{2\pi}\overline{\mu}(X, Y),$$

and by (6) we have the symmetry of ρ, that is, $\rho(X,Y) = \rho(Y,X)$. Then $\overline{\mu}(X,Y)$ is a symmetric bilinear form compatible with the complex structure J, which can be seen in the following way:

$$2\overline{\mu}(X,Y) = \rho(X,Y) + \rho(JY,JX) = \rho(Y,X) + \rho(JX,JY) = 2\overline{\mu}(Y,X)$$

and

$$\begin{aligned} 2\overline{\mu}(JX,JY) &= \rho(JX,JY) + \rho\big(J^2Y, J^2X\big) = \rho(Y,X) + \rho(JX,JY) \\ &= \overline{\mu}(Y,X) = \overline{\mu}(X,Y). \end{aligned}$$

So, we can choose a basis such that $\overline{\mu}(E_P, E_Q) = 0$, for $P \neq Q$. Then

$$\begin{aligned} \gamma_1^n(E_1, JE_1, \cdots, E_n, JE_n) &= n!\gamma_1(E_1, JE_1)\cdots\gamma_1(E_n, JE_n) \\ &= n!(2\pi)^{-n}\overline{\mu}(E_1,E_1)\cdots\overline{\mu}(E_n,E_n). \end{aligned}$$

Now, we use (ii) to see $\gamma_1^n(E_1, JE_1, \cdots, E_n, JE_n) \geqslant 0$, which implies

$$\int \gamma_1^n \geqslant 0$$

and by (iii), $\gamma_1^n = 0$. Therefore $\overline{\mu} = 0$ and by (5) $\rho(X,Y) = -\rho(JY,JX)$. We use the symmetry of ρ to write $\rho(X,Y) = -\rho(JX,JY)$. Finally, Theorem 3.3 in [9] implies $R(X,Y) = -R(JX,JY)$ and therefore ∇ is a holomorphic affine connection. ∎

Remark. The idea of diagonalisation of the bilinear form $\overline{\mu}$ is already used in [3] for a formally holomorphic connection.

Theorem 3.2. *Let M, $\dim_C M = n$, be a compact complex manifold with a complex symmetric connection ∇. If*

(i) $\rho(X,Y) = \rho(Y,X)$, and $\rho(JX,JY) = \rho(X,Y)$,

(ii) $\overline{\mu}$ *is a semi-definite bilinear form of rank* 0 *or* n, *nonnegative if* n *is odd,*

(iii) $c_1^n(M) \leqslant 0$,

then the group of all projective diffeomorphisms of the connection ∇ coincides with the group of all affine diffeomorphisms of the same connection.

PROOF: We use Lemma 3.1 to see $\rho(X,Y) = -\rho(JX,JY)$. Then, because of (i) $\rho = 0$, and Theorem A implies our Theorem. ∎

COROLLARY 3.3. *Let M be a surface of general type with a complex symmetric connection ∇. If*

(i) $\rho(X,Y) = \rho(Y,X)$, and $\rho(JX,JY) = \rho(X,Y)$,

(ii) *$\overline{\mu}$ is a semi-definite bilinear form of rank* 0 *or* n, *nonnegative if n is odd,*

(iii) $c_2(M) \leqslant 0$,

then the group of all projective diffeomorphisms of the connection ∇ coincides with the group of all affine diffeomorphisms of the same connection.

PROOF: We use the Miyaoka inequality $c_1^2(M) \leqslant 3c_2(M)$ and Theorem 3.2 to prove the corollary. ∎

REMARK. Under the assumptions of Theorem 3.2 or Corollary 3.3, because of Theorem B, we have the group of holomorphically projective transformations coincides with the group of affine transformations of ∇.

EXAMPLE 1. We construct here a family $\mathcal{E}$ of a symmetric complex connections on $\mathbf{C}P^1$ such that $\mu = 0$, $\rho(JX,JY) = \rho(X,Y)$ and $\rho(X,X) < 0$ for $X \neq 0$, $X,Y \in M_m$. Then, $(M,\nabla) = \left(\mathbf{C}P^1 \times \mathbf{C}P^1, \nabla_1 \times \nabla_2\right)$, satisfies (i) and (ii) in Theorem 3.2 for $\nabla_1, \nabla_2 \in \mathcal{E}$.

We represent the complex projective line $\mathbf{C}P^1$ as the complexification of the standard sphere (S^2, g) embedded in $\mathbf{R}^3$. Then, $S^2 \setminus \{N,S\}$, (N and S are the north and south pole respectively), is parameterised as

$$x = \cos\alpha\sin\beta, \quad y = \sin\alpha\sin\beta, \quad z = \cos\beta,$$

$0 < \alpha < 2\pi$, $0 < \beta < \pi$.

In terms of the base $f_1 = \partial/\partial\alpha$ and $f_2 = \partial/\partial\beta$ of S^2_m the corresponding almost-complex structure J is determined by $Jf_1 = \sin\beta f_2$, $Jf_2 = -(1/\sin\beta)f_1$.

Let $\widetilde{\Gamma}^k_{ij}$ $(i,j,k = 1,2)$ be the Christoffel symbols for a symmetric complex connection $\widetilde{\nabla}$. It has to satisfy the following conditions:

$$\widetilde{\Gamma}^1_{11} = \widetilde{\Gamma}^2_{12} = \widetilde{\Gamma}^2_{21}, \qquad \sin^2\beta\widetilde{\Gamma}^1_{12} = -\widetilde{\Gamma}^2_{11},$$
$$\widetilde{\Gamma}^1_{21} - \widetilde{\Gamma}^2_{22} = \cot\beta, \qquad \sin^2\beta\widetilde{\Gamma}^1_{22} = -\widetilde{\Gamma}^2_{21}.$$

The symbols $\widetilde{\Gamma}^1_{22}$ and $\widetilde{\Gamma}^2_{22}$ can be arbitrary chosen and all the others depend on these two. We denote $\widetilde{\Gamma}^2_{22} = h$, $\widetilde{\Gamma}^1_{22} = f$, where h and f are some smooth functions periodic with respect to α.

Let $E_1 = (1/\sin\beta)\partial/\partial\alpha$, $E_2 = \partial/\partial\beta$ be the modified base of the S^2_m. For the almost-complex structure J we have $JE_1 = E_2$, $JE_2 = -E_1$.

By straightforward computations we find for the corresponding Ricci tensor $\widetilde{\rho}$

$$\widetilde{\rho}(E_1, E_1) = -\frac{\partial f}{\partial\alpha} + \frac{\partial h}{\partial\beta} + h\cot\beta - 1, \tag{10}$$

and

$$\widetilde{\rho}(E_1, E_2) = -\frac{\partial f}{\partial\beta}\sin^2\beta - f\sin 2\beta - \frac{\partial h}{\partial\alpha}. \tag{11}$$

Notice, that for $f \equiv 0$ and $h = \varepsilon\sin\beta$

$$\widetilde{\rho}(E_1, E_1) = -1 + 2\varepsilon\cos\beta, \text{ and } \widetilde{\rho}(E_1, E_2) = 0$$

on $S^2 \setminus \{N, S\}$. Therefore, for $|\varepsilon| < 1/2$, $\widetilde{\rho}(E_1, E_1) < 0$ and $\widetilde{\rho}(E_1, E_2) = 0$.

Because of the properties of functions $h = h(\alpha, \beta)$, $f = f(\alpha, \beta)$ we see that our complex connection can be also extended to a global connection on S^2 and $\widetilde{\nabla} \in \mathcal{E}$.

Let us remark that the product of two manifolds satisfying (i) and (ii) in Theorem 3.2 also satisfies the same conditions.

For $f = 0$ and an arbitrary function $h = h(\beta)$ we obtain the connection $\widetilde{\nabla}$ satisfying (i) in Theorem 3.2.

REMARK. It is simple to prove that the complex connection given in the previous example is not projectively equivalent to the Levi-Civita connection, as by straightforward computations we see the geodesics for these connections are different. Moreover, using the Lie derivative $\mathcal{L}_V\widetilde{\Gamma}^k_{ij}$ we can find a vector field V such that $\mathcal{L}_V\widetilde{\Gamma}^k_{ij} \neq 0$. This implies that the projective transformations for this complex connection do not coincide with the affine transformations.

REMARK. Let us remark that $\widetilde{\Gamma}$ can be written in the following way:

$$\widetilde{\Gamma}^i_{jk} = \Gamma^i_{jk} + \delta^i_j p_k + \delta^i_k p_j - g_{kj}P^i, \quad p_i = g_{ik}P^k, \tag{**}$$

where

$$P^1 = -f, \qquad P^2 = h.$$

The connection $\widetilde{\nabla}$ given by (**) was studied by Simon in [**13**]. He has proved this connection is affine conformally invariant on a hypersurface of an affine space with relative normalisation.

EXAMPLE 2. Let us now find a connection satisfying all the conditions in Theorem 3.2. Nomizu and Podesta have constructed complex non-flat Levi-Civita connections on the Hopf surfaces $H = \mathbf{C}^2 - \{0\}/\{z \to kz\}$ $(k \in \mathbf{R} - \{0\})$ satisfying the condition $R(JX, JY, Z, W) = R(X, Y, Z, W)$. As $c_1(H) = 0$ this example satisfies (i) and (iii) of Theorem 3.2. The holomorphic normal projective connection (in the sense of [7]) for these complex connections satisfies all the conditions of our theorem. The curvature tensor of the holomorphic normal projective connection is the corresponding Weyl holomorphic projective curvature tensor of the Levi-Civita connection. Since the Hopf surfaces have no constant holomorphic sectional curvature the curvature tensor of the holomorphic normal projective connection is not zero.

4. PROJECTIVE TRANSFORMATIONS OF KÄHLER MANIFOLDS

In the general case the group of projective diffeomorphisms, the group of affine diffeomorphisms and the isometry group do not coincide for an Riemannian manifold (M, g). Nagano [10] has proved that if M is a complete Riemannian manifold with parallel Ricci tensor then the largest connected group of projective transformations of M coincides with the largest connected group of affine transformations of M unless M is a space of positive constant sectional curvature.

In this section we use basic facts and notations for Kähler manifolds from [1]. We prove the following result.

THEOREM 4.1. *Let M be a compact Kähler manifold of complex dimension $n > 1$. If*

(i) *τ is constant,*
(ii) $c_1 = 0$

then

(a) *the group of all projective diffeomorphisms of the Levi-Civita connection coincides with the group of all affine diffeomorphisms of the same connection;*
(b) *the identity component of the group of all holomorphically projective diffeomorphisms of the Levi-Civita connection coincides with the identity component of its group of isometries.*

PROOF: Since τ is constant and $\omega^{n-1}c_1(M) = 0$, using the formula

$$\omega^{n-1}c_1(M) = \frac{1}{n\pi}\int_M \tau\Phi^n,$$

we find $\tau = 0$. From the condition $c_1 = 0$ it follows $c_1^2(M) = 0$ and therefore using the formula

$$\omega^{n-2}c_1^2(M) = \frac{1}{4n(n-1)\pi^2}\int_M \left(\tau^2 - 2\|\rho\|^2\right)\Phi^n,$$

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we find

$$\frac{1}{4\pi^2}\int_M \|\rho\|^2\Phi^n = 0,$$

that is, $\|\rho\|^2 = 0$, or $\rho = 0$. Hence, having Theorem A in mind it follows that the group of all projective diffeomorphisms of the Levi-Civita connection coincides with the group of all affine diffeomorphisms of the same connection. $\square$

Furthermore, having Ishihara's results in mind, (b) follows.

REMARK. The class of Kähler manifolds with constant scalar curvature is larger than the class of Kähler manifolds with parallel Ricci tensor (see Theorem 6.1 in [**2**] and [**8**]). Therefore our assumptions for the geometry of the manifold in Theorem 4.1 are weaker than in the Theorem of Nagano [**10**], but the topological assumptions are stronger.

Now, from Theorems 4.1 and B follows directly

COROLLARY 4.2. *If a Kähler manifold M is a compact irreducible manifold and*

(i) τ *is constant,*
(ii) $c_1 = 0$,

then the group of all projective diffeomorphisms of the Levi-Civita connection coincides with the group of all isometries of M.

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