Macroeconomic Dynamics (2025), **29**, e101, pp. 1–10
doi:10.1017/S1365100525000276

NOTES

A note on the size distribution of government debt

Behzod B. Ahundjanov[1], Sherzod B. Akhundjanov[2], and Botir B. Okhunjanov[3]

[1]Department of Economics, University of Illinois Chicago, Chicago, IL, USA
[2]Department of Applied Economics, Utah State University, Logan, UT, USA
[3]Department of Economics, Denison University, Granville, OH, USA
Corresponding author: Sherzod B. Akhundjanov; Email: sherzod.akhundjanov@usu.edu

Abstract

The cross-sectional distribution of government debt is often approximated by a lognormal distribution. This note empirically demonstrates that government debt is more accurately characterized by the double Pareto-lognormal (dPLN) distribution, which features a lognormal body with two Pareto tails. The dPLN assuredly surpasses alternative parametric distributions and passes goodness-of-fit tests. With its analytical tractability, flexibility, and parsimony, coupled with a theoretical foundation, the dPLN may be appealing for different computational and empirical applications.

Keywords: debt; lognormal; Pareto tails; power law; random growth

JEL classifications: C46; H63

1. Introduction

This note concerns the cross-country size distribution of government debt, commonly measured as the debt-to-GDP ratio (in %). Traditionally, the size distribution of government debt has been parameterized as lognormal. From a theoretical perspective, Barro (1979), constructing a model of public debt for a large national government, states:

> "The theory predicts that the level of debt or the debt-to-income ratio would be irrelevant for current debt issue. [. . .] This result supports the surprising proposition of the theory that the debt-to-income ratio does not have a 'target' value but rather moves 'randomly'."

From an empirical standpoint, Barro (1979) documents size-independent proportional growth of the debt-to-GDP ratio by regressing the growth rate of public debt against the level of debt and finding the estimated coefficient to be insignificantly different from zero. Much of the ensuing literature has also treated the size of the debt-to-GDP ratio as a unit root process and hence nonstationary (Bohn, 1998).

The above characterizations and empirical evidences for the size and growth rate of the debt-to-GDP ratio are largely consistent with a process adhering to random multiplicative growth, more commonly referred to as Gibrat's law of proportionate random growth (Sutton, 1997; Gabaix, 2009). It is well known that random multiplicative growth, in its purest form, gives rise to a lognormal distribution, which conceivably justifies the prevailing practice of adopting the lognormal distribution for modeling the size distribution of government debt. However, Gibrat's law can similarly generate a power law (Pareto) distribution (Gabaix, 1999; Reed, 2001), and the *double Pareto-lognormal* (dPLN) in particular, which is the product of independent double Pareto and lognormal distributions (Reed, 2003; Reed and Jorgensen, 2004).

For power law, and dPLN, to arise as the stationary distribution, a random multiplicative growth process needs some friction (Gabaix, 2009). Among several generative mechanisms of power law and dPLN (Reed, 2001, 2003; Toda, 2014, 2017; Beare et al. 2022; Beare and Toda, 2022), a random multiplicative growth process with *random resets* (Reed, 2001; Beare and Toda, 2022) stands out. The theory essentially posits that economic units experience random multiplicative shocks to their size through time until stopped (perishing) at random and being replaced by a new unit. For our empirical context, this would imply government debt evolves over time according to random multiplicative growth and is occasionally reset, perhaps due to sovereign default. This appears entirely reasonable since, as Barro (1979) pointed out:

> "[T]here may be a wide range within which the debt-income ratio can vary essentially freely [. . .] but there may be some eventual limits that come into play. A limit on the high side would arise when the debt-income ratio rises sufficiently to affect the probability of the government's default."

The plausibility of the random resetting mechanism in capturing the key features of the empirical context positions it as a valid alternative to the conventionally adopted pure random multiplicative growth process. This, in turn, raises the need for a thorough empirical examination of the size distribution of government debt across the entire cross-section and its evolution over time. If the debt-to-GDP ratio follows a dPLN distribution, it would provide indirect evidence suggesting that debt-to-GDP obeys the generative mechanism of dPLN—a random resetting mechanism. Conversely, if the debt-to-GDP ratio is lognormal, it would offer indirect evidence in favor of a pure random multiplicative growth process. The present article seeks to address this issue.

Our analysis demonstrates that the lognormal distribution tends to provide a reasonable fit to the cross-sectional distribution of debt-to-GDP ratios for the period between 1980 and 2000. However, significant departures from lognormality are observed in the post-2000 debt data. In contrast, the dPLN fits the data remarkably well across *all* periods, whether in terms of model fit criteria or goodness-of-fit tests. While the dPLN fits the data at least as well as the lognormal during 1980–2000, it straightly outperforms the lognormal and other candidate distributions in the post-2000 period.

Why does this finding matter? The finding has several noteworthy economic, econometric, and social implications, as elaborated next.

First, it indicates that the debt-to-GDP ratio follows a ubiquitous empirical regularity, the *power law*,[1] in both its upper and lower tails. For the upper tail, this means that the fraction of units above size x is roughly proportional to $x^{-\alpha}$, where $\alpha > 0$ is the upper-tail power law exponent. Similarly, for the lower tail, it implies that the fraction of units below size x is approximately proportional to x^{β}, where $\beta > 0$ is the lower-tail power law exponent. Several recent studies, including those by Toda (2012) on income and Toda (2017) on consumption, have shown the dPLN's superiority in fitting size distributions previously described by lognormal or Pareto distributions. Methodologically, our work contributes to this literature by documenting the dPLN's robust fit to, and the power law behavior of, the size distribution of government debt.

Second, this empirical regularity is intriguing in its own right and warrants explanation. The strong support we find for the dPLN provides indirect evidence for the hypothesis that the debt-to-GDP is governed by a random multiplicative growth process with friction (random resets). Moreover, it is important to note that under a pure random multiplicative growth mechanism, the size distribution remains lognormal, with an ever-increasing log variance. However, empirical evidence contradicts this, as we find the log variance of the debt-to-GDP ratio to be stable over time. This further highlights the lack of support for the lognormal distribution and its theoretical basis. Consequently, our work helps reconcile the empirical evidence with the underlying theory of the size distribution of government debt.

Third, it suggests heavy-tailedness and tail risk in the debt-to-GDP ratio. Economically, this indicates that the system's behavior is strongly influenced by its largest units, with the power law exponents α, β describing the extent of concentration (inequality) at the top and bottom

https://doi.org/10.1017/S1365100525000276 Published online by Cambridge University Press

of the distribution (Toda, 2012, 2014). Econometrically, this stresses the implausibility of thin-tailed distributions (e.g., the normal) for the debt-to-GDP ratio, which dismiss extremequely large cases as improbable. In fact, even some common heavy-tailed distributions (e.g., the lognormal) are not adequate in this context. For sound and credible analysis, it is essential to employ statistically rigorous methods that properly account for heavy-tailedness and tail risk properties. Toward this end, the dPLN emerges as a reasonable choice both empirically and theoretically.

Fourth, it provides insights into the existence of moments. Given that power-law distributed variables possess only a finite number of moments, econometric techniques that assume the existence of all moments may be invalid (Kocherlakota, 1997; Toda and Walsh, 2015). For the debt-to-GDP ratio, we find that only the first and second moments (i.e., mean and variance, respectively) generally exist. While higher-order sample moments (e.g., skewness and kurtosis) can always be computed for the observed data, these moments are non-convergent. Therefore, the distribution of the debt-to-GDP ratio cannot be characterized by higher-order moments beyond the mean and variance. This is consequential for descriptive and econometric analysis (e.g., Generalized Method of Moments (GMM)) of government debt.

Finally, the dPLN is as analytically tractable as the lognormal, with many of the lognormal's desirable properties generalizable to the dPLN (Reed, 2003; Reed and Jorgensen, 2004). Recent techniques for solving heterogeneous-agent models frequently parameterize cross-sectional distributions in order to reduce computational complexity. Given the dPLN's robust empirical fit, the distribution may be especially suitable for calibrations and econometric applications because it is analytically tractable, flexible, parsimonious, and grounded in theory. Accordingly, we recommend adopting the dPLN to more accurately specify the size distribution of government debt.

2. Methods

2.1 Data

Similar to Reinhart and Rogoff (2011), government debt is defined as total gross central government debt measured as a percentage of GDP.[2] Our sample covers the period from 1980 to 2020 and includes the entire IMF membership, which ranges from 117 to 175 countries, depending on the year. Table 1 provides summary statistics for each year. Using yearly data on the debt-to-GDP ratio, we fit each candidate distribution described below to each year separately by maximum likelihood estimation (MLE).

2.2 Double Pareto-Lognormal (dPLN)

Let X represent a random variable for the debt-to-GDP ratio, with its outcome denoted by x, where $x > 0$. The probability density function of the dPLN is given by

$$f_{\text{dPLN}}(x;\mu,\sigma,\alpha,\beta) = \frac{\alpha\beta}{\alpha+\beta}\left[x^{-\alpha-1}\exp\left(\alpha\mu+\frac{\alpha^2\sigma^2}{2}\right)\Phi\left(\frac{\log(x)-\mu-\alpha\sigma^2}{\sigma}\right)\right.$$
$$\left. + x^{\beta-1}\exp\left(-\beta\mu+\frac{\beta^2\sigma^2}{2}\right)\left(1-\Phi\left(\frac{\log(x)-\mu+\beta\sigma^2}{\sigma}\right)\right)\right], \quad (1)$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. The dPLN has four parameters: μ, σ are the mean and standard deviation of the lognormal component, and $\alpha, \beta > 0$ are the power law (Pareto) exponents for the upper and lower tails, respectively.[3]

Similar to the Pareto distribution, the dPLN has finitely many moments, a feature of empirical relevance (Kocherlakota, 1997; Toda and Walsh, 2015). Specifically, the rth moment about origin

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Table 1. Summary statistics for debt-to-GDP ratio (in %)

Year	Obs.	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
1980	117	40.092	34.353	0.600	20.000	50.570	249.940
1981	119	43.419	39.001	2.420	21.900	50.715	280.190
1982	122	50.666	49.139	1.700	27.000	63.700	399.170
1983	122	59.196	57.326	3.500	29.065	72.835	468.920
1984	124	61.035	59.025	1.010	28.922	72.100	478.250
1985	126	65.592	59.465	0.000	31.775	76.438	520.810
1986	128	67.937	59.939	0.000	36.172	79.050	562.500
1987	125	72.364	56.989	0.000	40.080	89.020	482.170
1988	126	72.432	65.288	0.000	38.392	86.050	472.680
1989	126	72.870	62.426	0.000	37.550	86.248	472.210
1990	130	82.701	137.142	0.000	37.770	88.462	1,472.050
1991	132	73.945	68.673	0.000	38.942	83.978	614.780
1992	139	78.066	72.229	0.000	40.640	93.240	557.960
1993	147	75.991	64.203	0.000	38.850	90.315	489.530
1994	152	75.282	64.256	0.000	36.665	97.413	412.880
1995	159	70.125	55.779	0.000	32.870	88.265	350.890
1996	162	66.348	49.039	0.000	32.787	84.350	324.750
1997	165	62.694	45.389	0.000	32.090	78.200	318.300
1998	170	70.063	58.200	0.000	35.645	82.975	438.070
1999	170	71.716	56.130	0.000	38.547	88.537	384.520
2000	173	71.490	59.606	0.000	37.990	85.410	435.320
2001	174	70.751	59.402	2.040	37.157	88.748	432.620
2002	175	70.626	56.736	1.980	40.045	86.440	431.170
2003	175	69.004	61.409	0.990	38.675	83.555	558.120
2004	173	64.792	57.760	0.400	34.180	83.290	505.370
2005	175	58.572	52.101	0.300	29.040	73.560	454.270
2006	174	51.504	49.313	0.990	22.475	65.547	419.400
2007	174	45.700	42.140	0.690	19.407	58.370	341.140
2008	174	44.515	38.825	0.730	20.040	56.233	250.250
2009	175	46.918	35.129	0.660	23.470	59.055	206.900
2010	175	45.965	33.317	0.620	25.215	57.725	205.560
2011	174	46.589	34.057	0.570	24.523	58.185	219.190
2012	174	47.795	34.450	0.540	25.255	61.087	226.150
2013	174	49.718	35.759	0.520	27.527	61.562	229.710
2014	174	50.928	34.861	0.070	27.105	65.398	233.540
2015	174	53.971	34.288	0.060	32.585	68.343	228.450
2016	174	56.305	33.977	0.060	35.938	71.493	232.560
2017	174	57.260	34.658	0.060	36.383	69.400	231.420
2018	174	58.547	35.556	0.050	37.575	71.273	232.600
2019	174	59.865	37.766	0.270	38.222	72.338	237.950
2020	173	69.591	40.623	0.290	44.730	84.770	266.180

Note: The number of observations corresponds to the number of countries in the IMF's historical public debt database (Ali Abbas et al. 2011).

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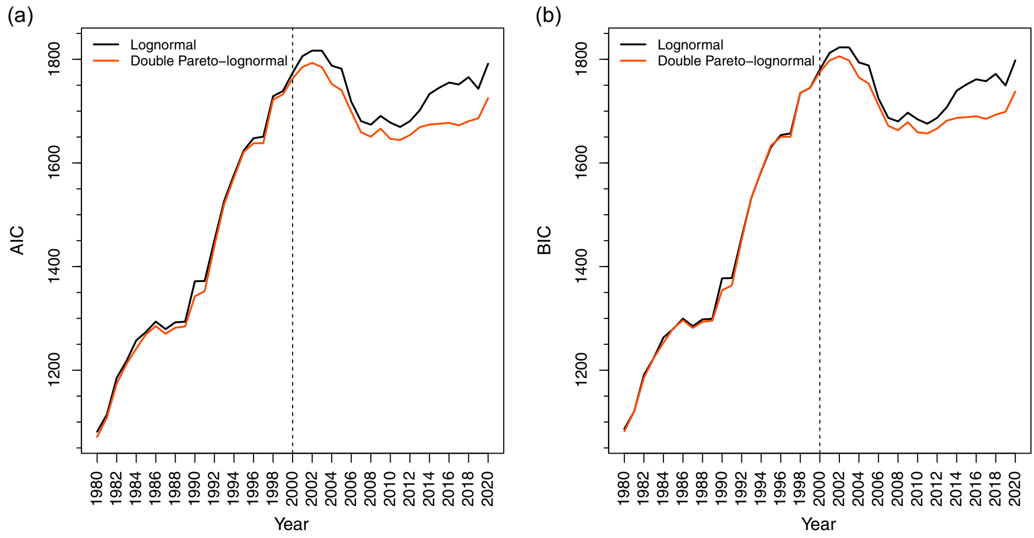

Figure 1. Relative quality of lognormal and dPLN in fitting the debt-to-GDP ratio, 1980–2020.

for the dPLN takes the following form:

$$\mu_r' = \mathrm{E}(X^r) = \begin{cases} \frac{\alpha\beta}{(\alpha-r)(\beta+r)} \exp\left(\mu r + \frac{1}{2}\sigma^2 r^2\right) & \text{if } -\beta < r < \alpha \\ \infty & \text{otherwise} \end{cases}. \quad (2)$$

This implies that μ_r' does not exist for $r \geq \alpha$. For instance, the variance exists only if the upper-tail power law exponent satisfies $\alpha > 2$.

There are several features of the dPLN that make the distribution attractive for computational works. First, similar to the lognormal and Pareto distributions, the dPLN is analytically tractable (Reed, 2003; Reed and Jorgensen, 2004), as its moments generally have closed-form expressions. Second, the dPLN is flexible, capturing the lognormal and double Pareto distributions as limiting cases (for $\alpha, \beta \to \infty$ and $\sigma \to 0$, respectively). Third, the distribution is parsimonious, with only three ($\alpha = \beta$) or four ($\alpha \neq \beta$) parameters. Fourth, the dPLN allows for fat tails, making it convenient for modeling heavy-tailed data compared to other, more complex mixture distributions. Fifth, since α, β describe the concentration at the top and bottom of the distribution, respectively, they can be used in applied work to decompose inequality at the top and bottom of the size distribution (Toda, 2012, 2014).

2.3 Diagnostics

To assess the relative quality of alternative models, we report the log-likelihood, Akaike information criterion (AIC), and Bayesian information criterion (BIC). While both AIC and BIC account for overfitting, BIC penalizes the number of model parameters more heavily than AIC. Generally, the model with the lowest AIC or BIC is preferred.

To evaluate the goodness of fit of individual models, we also perform the Kolmogorov–Smirnov (KS) and Anderson–Darling (AD) tests, and produce quantile–quantile (Q–Q) and distributional plots. The KS and AD tests, commonly used in the size distribution literature (e.g., Toda, 2017), compare the empirical distribution with that of a fitted model. The AD test is more robust than the KS test to deviations in the tails, which makes it more relevant for the analysis of tail heaviness. The KS and AD p-values formally test the null hypothesis that the sample is drawn from the fitted distribution.

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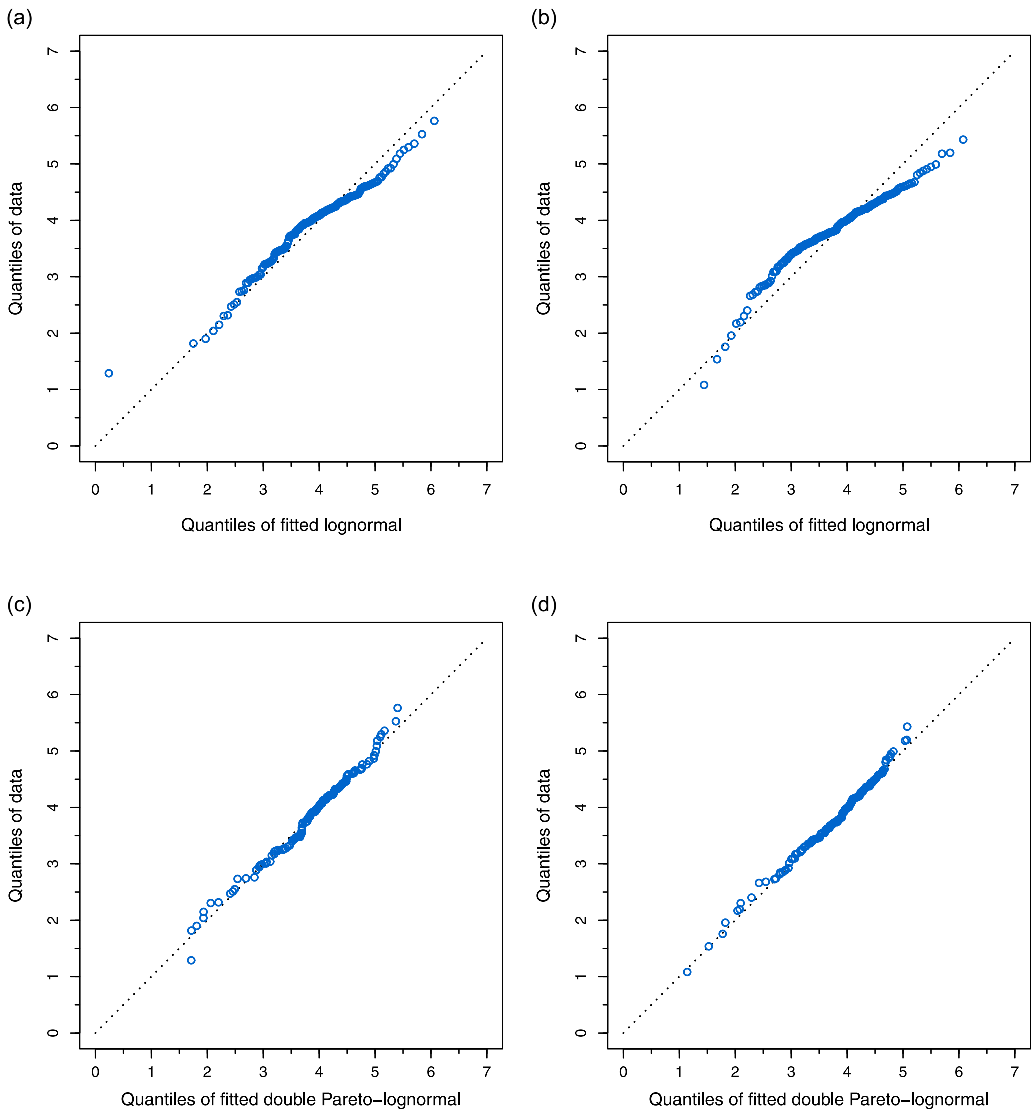

Figure 2. Q–Q plots of fitted lognormal and dPLN: (*a*) lognormal fit for 1997; (*b*) lognormal fit for 2015; (*c*) dPLN fit for 1997; (*d*) dPLN fit for 2015.

3. Results

3.1 Lognormal vs. dPLN

Figure 1 summarizes the relative quality of the lognormal and dPLN distributions in fitting the debt-to-GDP ratio for the period between 1980 and 2020.[4] According to both AIC and, particularly, BIC, the performances of the two distributions are largely comparable during 1980–2000, though dPLN exhibits a slight edge over the lognormal. After 2000, however, the performances diverge significantly, with dPLN starkly outperforming the lognormal.

The goodness-of-fit test results support the plausibility of dPLN. The KS and AD tests do not reject dPLN at the 0.05 significance level in 40 and 41, respectively, out of 41 years, whereas the lognormal is not rejected in 28 years, 20 of which are before 2000. This indicates that dPLN performs at least as well as the lognormal before 2000, but post-2000, dPLN's performance remains strong while the lognormal's performance deteriorates.

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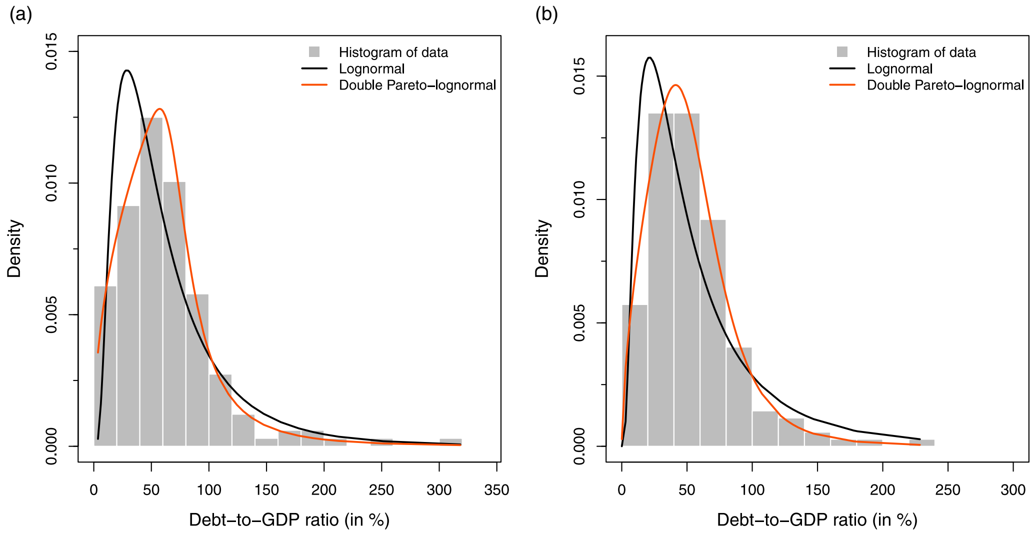

Figure 3. Histogram of data and density plots of fitted lognormal and dPLN: (*a*) densities for 1997; (*b*) densities for 2015.

Visual evidence for this can be found in Figures 2 and 3, which compare the fits of the two distributions to data from before and after 2000, using 1997 and 2015 as examples. From Q–Q plots in Figure 2, it is apparent that the lognormal fit to 1997 data (panel (*a*)) is fairly good, with only a small deviation in the upper tail. However, for the 2015 data (panel (*b*)), there are significant departures in both the upper and lower quantiles of the fitted lognormal distribution. On the other hand, dPLN's quantiles align closely with the data in both years (panels (*c*) and (*d*)). Figure 3 further illuminates this with density plots, where again one observes the striking fit of dPLN to the debt-to-GDP ratio.

Overall, both model fit criteria and goodness-of-fit tests strongly favor dPLN over the lognormal.

Figure 4 shows the MLE estimates of the dPLN parameters for each year. Several salient observations come to light. First, the upper-tail power law exponent α hovers around 2.5, with an average of 2.76 across all years. The lower-tail power law exponent β has an average of 1.63 across all years.[5] Econometrically, this suggests the tail heaviness of government debt, which explains the lognormal's poor fit in figures 2 and 3. Economically, this suggests concentration at the top and bottom of the distribution, which has implications for inequality and policy design. Second, for $\alpha \approx 2.76$ and $r < \alpha$, only the first and second moments (i.e., mean and variance, respectively) exist for the debt-to-GDP ratio. This is instructive for descriptive and empirical analysis (e.g., GMM) of government debt. Third, the log variance parameter σ remains generally stable, with an average of 0.28 across all years. This further illustrates the implausibility of the lognormal distribution and its theoretical basis, as under a pure random multiplicative growth mechanism, the size distribution would be lognormal, with an increasing variance over time. For the debt-to-GDP ratio, we find a lack of support for this to be the case.

3.2 Other Candidate Distributions

To compare the performance of dPLN within a broader class of parametric distributions, we consider two additional flexible distributions with varying tail heaviness: first, generalized beta II (Toda, 2017), and second, Pareto-tails lognormal (PTLN) (Luckstead and Devadoss, 2017).

The generalized beta II is a four-parameter distribution that nests many common distributions, including exponential, (generalized) gamma, lognormal, Weibull, chi-square, Lomax, Rayleigh,

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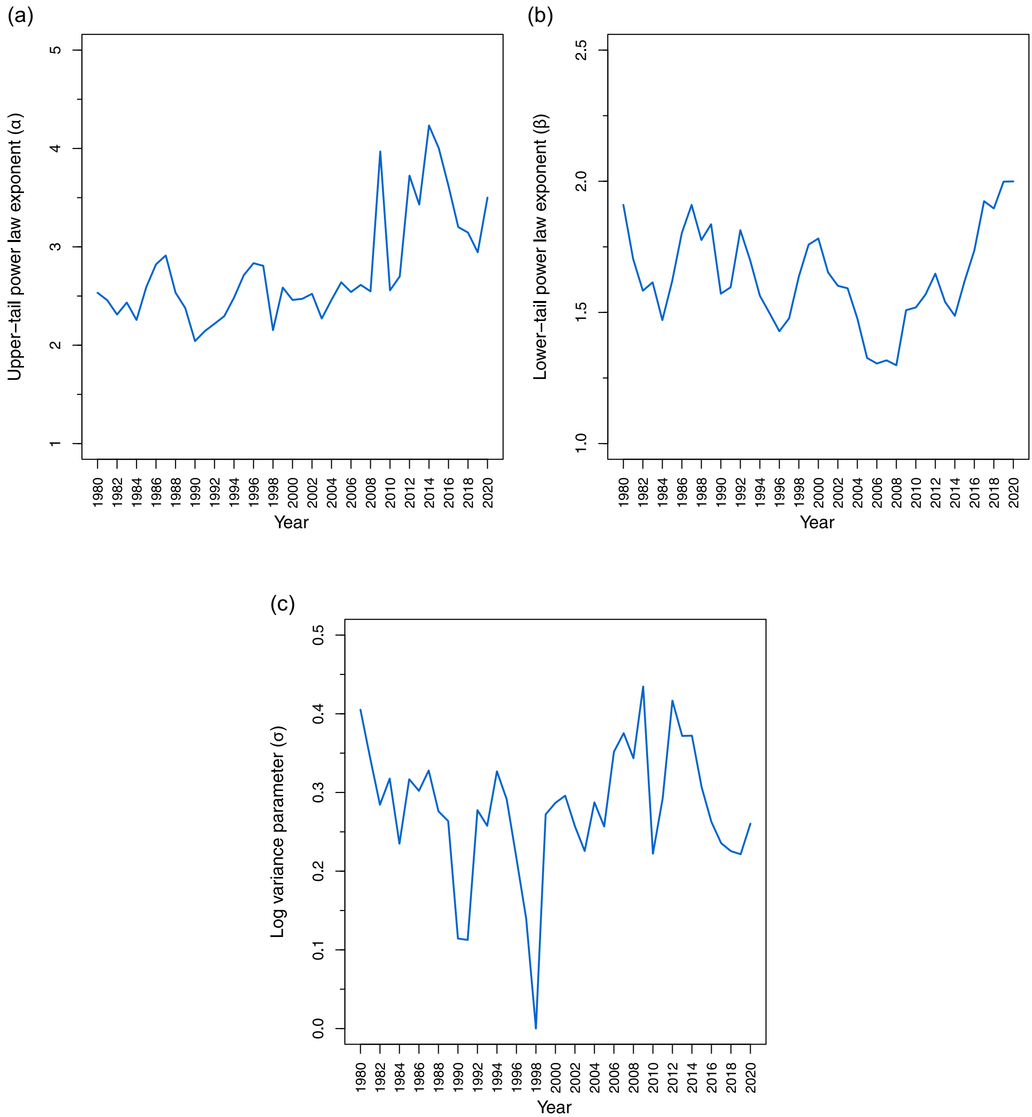

Figure 4. DPLN parameter estimates: (*a*) upper-tail power law exponent α; (*b*) lower-tail power law exponent β; (*c*) log variance parameter σ.

Laplace, and log-logistic, among others. The PTLN is a close alternative to dPLN and shares many of its attractive features but has six parameters: in addition to the four parameters of dPLN, PTLN includes τ_l and τ_u, which are the transition (threshold) points from the lower tail Pareto to the lognormal body and from the lognormal body to the upper tail Pareto, respectively. PTLN also nests the lognormal-upper tail Pareto distribution of Ioannides and Skouras (2013) (for $\tau_l = x_{\min}$).

The relative quality of these distributions is presented in Figure 5.[6] Evidently, both AIC and BIC favor dPLN. The generalized beta II does not perform as well as the dPLN, with the KS and AD tests failing to reject the distribution in 11 and 35, respectively, out of 41 years. Therefore, dPLN remains dominant among a large class of parametric distributions. However, PTLN is a close contender. The additional two parameters in PTLN improve its AIC performance compared to dPLN, but not its BIC performance, as expected. The KS and AD tests do not reject PTLN for

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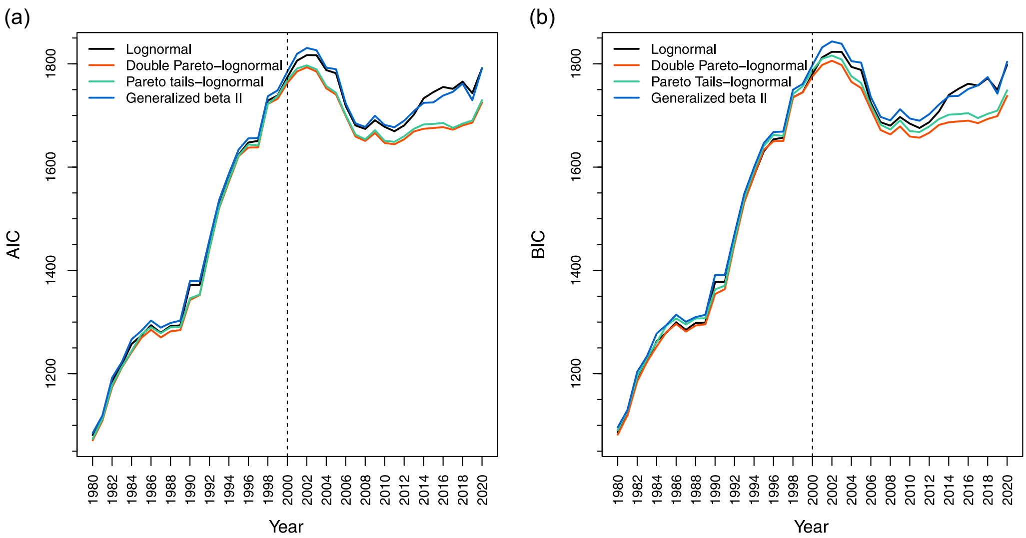

Figure 5. Relative quality of alternative distributions in fitting the debt-to-GDP ratio, 1980–2020.

any of the years analyzed, indicating that PTLN fits the government debt data quite well, though it is less parsimonious and less tractable than dPLN.

Taken together, the analytical tractability, flexibility, and parsimony of dPLN, along with its remarkable performance in terms of model fit criteria and goodness-of-fit tests, rightfully validate the distribution as one of the benchmarks for fitting the debt-to-GDP ratio.

Supplementary material. The supplementary material for this article can be found at https://doi.org/10.1017/S1365100525000276.

Notes

1 Power laws are remarkably common in economics, finance, and social and natural sciences. See, for instance, Gabaix (1999), Reed (2001), Gabaix (2009), Devadoss et al. (2016), Akhundjanov et al. (2017), Ahundjanov and Akhundjanov (2019), Akhundjanov and Chamberlain (2019), Akhundjanov and Toda (2020), Ahundjanov et al. (2022), and Akhundjanov and Drugova (2022).
2 See Ali Abbas et al. (2011) for further data information.
3 For the theory and properties of the dPLN, see Reed (2003) and Reed and Jorgensen (2004).
4 Detailed MLE and goodness-of-fit test results for the lognormal and dPLN are presented in Supplementary Materials.
5 In principle, one can obtain the implied power law exponent using an approach similar to that of Beare and Toda (2020), exploiting the panel structure of the data to estimate the distribution of the debt-to-GDP growth rate and the resetting (sovereign default) probability. However, the limited cross-sectional sample size and the rarity of sovereign defaults preclude credible estimation and inference.
6 Detailed MLE and goodness-of-fit test results for the generalized beta II and PTLN are presented in Supplementary Materials.

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Cite this article: Ahundjanov BB, Akhundjanov SB and Okhunjanov BB (2025). "A note on the size distribution of government debt." *Macroeconomic Dynamics* **29**(e101), 1–10. https://doi.org/10.1017/S1365100525000276