MULTIPERIODICITY IN RRs AND δ SCUTI STARS: AN OBSERVATIONAL VIEW

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I. Introduction

On reviewing the observational literature of the recent past on RRs (or dwarf Cepheid or AI Velorum) and δ Scuti stars, it appeared that most present controversy is centered on three specific questions:

(1) Are δ Scuti stars really periodic, or only quasi-periodic?

(2) Are tidal modulations responsible for the slow cyclic variations observed in many of these stars, or not?

(3) Are there any real physical differences between the members of these two groups, or are they actually the same kind of object?

I would like to restrict this review primarily to these questions; and since some of our recent observational results bear directly on questions (1) and (2), and indirectly on question (3), I hope I will be forgiven for describing briefly here some of our current relevant findings.

For the most recent and comprehensive survey on δ Scuti stars, one should consult the excellent review paper by Baglin et al. (1973), which also contains a very extensive bibliography. The Annotated Catalog and Bibliography on δ Scuti Stars, by Seeds and Yanchak (1972) is quite useful for a variety of reference problems.

II. δ Scuti Variability, Periodic or Quasi-Periodic?

It has long been known that many δ Scuti stars exhibit light curves

variable in phase, shape, and amplitude, so that a stable light curve is
perhaps the exception to the general rule. Most presently known group
members were discovered relatively recently by survey work designed for that
purpose, so that only a few of these stars have been intensively observed
over any extended period of time. Le Contel et al. (1974) and Valtier
et al. (1974), in reporting on their photometry of HR 432, 515, 812, 8006,
and 9039, first suggested and later insisted that periods in most δ Scuti
stars have meaning only in a statistical sense, and that intrinsic irregu-
larities, due probably to nonlinear coupling between convection and pulsa-
tion in the upper layers, usually dominate the variability. Smyth et al.
(1975) concluded that while the primary frequency remained constant and
recognizable in HR 1653 and HR 3265, in ρ Phe the frequency spectrum is an
apparently continuously and rapidly varying function of the time of observa-
tion. In a private communication, Stobie has informed me that of the four
stars 1 Mon, 21 Mon, ρ Phe, and σ Tuc, only 1 Mon has a stable frequency
spectrum.

TABLE 1. STEWARD OBSERVATORY[1] PHOTOMETRY OF δ SCUTI STARS

Variable	Comparison	Filter	P_o(day)	Nights	Measures	Hours	Cycles	Years	Observer[3]
CC And	BD+41°120	V	0.12491	33	2230	138.2	46.1	1955-57	F
CC And	BD+41°120	V	0.12491	16	3344	92.0	30.7	1974	F[4]
14 Aur	18 Aur	b	0.08809	41	6743	160.0	75.7	1972-75	F,W
4 CVn	BD+43°2221	b	?	28	6173	149.1	?	1974	W
DQ Cep	HD199938	V	0.07886[2]	6	399	23.2	12.2[2]	1958	F
δ Del	ζ Del	b	0.15679	29	5431	114.8	30.5	1972-74	F,W
1 Mon	2 Mon	b	0.13613	5	837	18.2	5.6	1974	W
δ Sct	HR7055	b	0.19377	24	4855	111.0	23.9	1972-73	F
				Totals	30012	806.5	224.7		

1: Observations in 1972 made while Fitch was guest observer at Observatorio Nacional
de Mexico, San Pedro Martir, Baja California, Mexico.

2: Primary period, but perhaps not fundamental period.

3: F = W. S. Fitch; W = W. Z. Wisniewski.

4: Observations on J.D.2442339 and 2442340 made by F. E. Brengman.

These conclusions are contradictory to my own experience. Table 1 gives a summary of our photometry on δ Scuti stars. Six of the seven stars we have observed show a well-defined and unambiguous primary period, which for simplicity I assume is the fundamental radial mode. This mode identification seems secure for CC And, δ Sct, and 1 Mon, because they also have an excited second radial overtone; but I have no evidence that the primary frequency in 14 Aur, DQ Cep, and δ Del is correctly identified. Only in the case of 4 CVn have I not yet found a stable primary frequency, and I attribute at least part of the difficulty in this case to the fact that our observations of 4 CVn were all made near full moon, so the aliasing problem with both diurnal and monthly sidelobes is very severe. It may still develop that 4 CVn is not strictly periodic, but at present I don't expect this result. I think an adequate explanation for the difference in our experience and that of the Nice group (Le Contel et al. 1974, Valtier et al. 1974) lies in the fact that δ Scuti stars with nonstable light curves usually have an extremely complex system of variability, which requires a very great amount of observing time to decode. I hope to demonstrate this conclusion here in the cases of 14 Aur, δ Sct, and CC And. For the five stars which the Nice group discussed, they had 5, 5, 4, 13, and 8 nights of observations, respectively, and the actual observing runs (and coverage of the light variation) were usually rather brief, so their failure to find regularity in the light variations is perhaps understandable.

I don't think it is presently possible to formulate any general rules concerning data adequacy for analysis of multiperiodicity, since the requirements can vary so markedly from star to star, but it is usually fairly easy to estimate when a data set is inadequate. My own prejudice is that fewer than ten long and closely spaced nights are probably not worth

analysing, unless the changes in the light curve are fairly simple and easily guessed. The observations of ρ Phe in 1968 by Cousins et al. (1969), and in 1972 by Smyth et al. (1975), were made during 21.4 hours on 12 nights spanning 23 days in 1968 and during 41.4 hours on 8 nights spanning 63 days in 1972. The fractional coverage of the variability was 4.2% in 1968 and 2.7% in 1972, and the aliasing problem in both data sets is fairly severe. Having examined the data distribution and their published frequency spectra, I think that while their conclusion, that the frequency spectrum of ρ Phe is basically unstable, may be correct, it has not yet been proven. I can easily simulate the frequency spectrum behavior they found in ρ Phe by appropriate subdivisions of our data on 14 Aur, and in fact did so several years ago. Nevertheless, the primary frequency in 14 Aur is statistically a very stable entity. In Figure 1 I illustrate, with 7 typical light curves, the characteristic light variation of 14 Aur A. Normally there are about 11 1/3 cycles per day (c/d), but on some pairs of successive nights there are
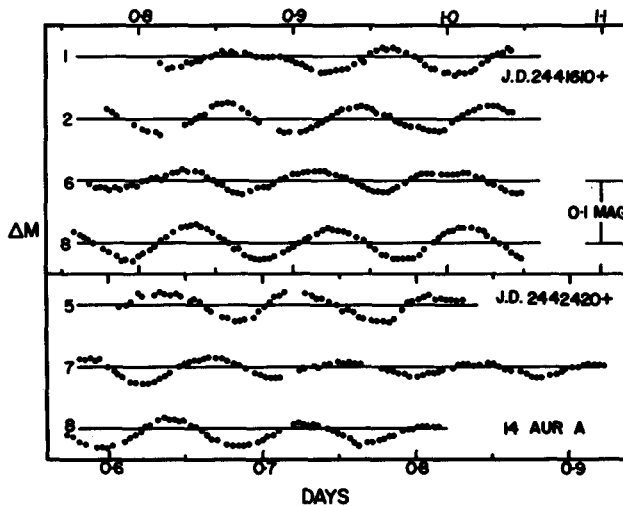

Fig. 1 – Representative observed b-magnitude light curves of 14 Aur A minus 18 Aur.

instead an integral number of c/d linking two nights. Normally there is very little cycle-to-cycle change, but sometimes the amplitude changes rapidly during one night. My present conclusions concerning 14 Aur are that the light variation is basically periodic but very complex, and I cannot pretend I yet understand it.

In Table 2 are given four different solutions approximating the blue magnitude light variation of 14 Aur A. I've not illustrated any of these

TABLE 2.   b-MAGNITUDE VARIATION OF 14 AUR A: 4 REJECTED SOLUTIONS[*]

Name	f_i(c/d)	A_i	σ_A	ϕ_i	σ_ϕ	σ(1 obs)
$2f_L$	0.527904	0.0030 ±	0.0003	0.774 ±	0.016 ±	8.4 mmag
f_0	11.35227	0.0168	0.0003	0.795	0.003	
$2f_L$	0.527904	0.0030	0.0003	0.776	0.015	8.3 mmag
$43f_L$	11.349936	0.0033	0.0004	0.079	0.021	
f_0	11.35227	0.0146	0.0004	0.779	0.005	
$2f_L$	0.527904	0.0032	0.0002	0.776	0.013	7.2 mmag
$43f_L$	11.349936	0.0042	0.0004	0.056	0.014	
f_0	11.35227	0.0133	0.0004	0.772	0.005	
f_{nr}	11.62581	0.0054	0.0002	0.076	0.007	
$(f_0-43f_L)/2$	0.00117	0.0048	0.0005	0.451	0.012	6.3 mmag
$f_{nr}-f_0-f_L$	0.02242	0.0028	0.0003	0.906	0.017	
$2f_L$	0.527904	0.0035	0.0002	0.792	0.010	
$43f_L$	11.349936	0.0034	0.0004	0.059	0.017	
f_0	11.35227	0.0137	0.0004	0.780	0.004	
f_0+f_L	11.616222	0.0013	0.0002	0.325	0.026	
$f_{nr}-F$	11.56180	0.0009	0.0003	0.619	0.055	
f_{nr}	11.59380	0.0025	0.0004	0.743	0.026	
$f_{nr}+F$	11.62580	0.0042	0.0004	0.028	0.015	
$f_{nr}+2F$	11.65780	0.0014	0.0004	0.681	0.047	
$f_{nr}+3F$	11.68980	0.0013	0.0003	0.396	0.040	

[*] $m_{14} = m_{18} - 1.507 - 2.5 \log [1 + \Sigma A_i \sin 2\pi (f_i t + \phi_i)]$
t = Hel.J.D. - 2441502.2600
1748 observations, averaged from 6743 with Δt = 0.003 day

in the observed light curves, because I don't consider any of them as acceptable representations of the star's behavior. However, while I'm not satisfied with these approximations, there are several fairly firm conclusions one can draw. First, the primary is slightly prolate, being 0.006 mag brighter when seen at velocity extrema than when seen in conjunction.

Second, there appears to be some kind of a resonance problem between the primary pulsation and the orbital motion. Third, tidal modulation such as seen in CC And and σ Sco is <u>not</u> present in this 3.8 day period spectroscopic binary, unless the orbital period changed suddenly and strongly between the 1968 radial velocity measures of Chevalier et al. (1968) and our own photometric measures which began in 1972. This last possibility seems much too improbable to consider further.

III. Tidal Modulation, Yes or No?

I was unhappy with my first published analysis of the light variation of CC And (Fitch 1960), because the adopted analytic representation did not adequately reproduce the observations. Later (Fitch 1967), when I found that the short and long term variability of both CC And and the β Cep star σ Sco could be understood in terms of intrinsic radial pulsation perturbed tidally by a companion in a binary orbit, I rashly suggested that whenever a short period pulsator also shows long period variations in pulsation characteristics, these long period complications are probably caused by tidal modulation. That this suggestion is not universally true has now been clearly demonstrated by Broglia and Marin (1974) for the case of Y Cam, and by myself in the cases of 14 Aur and the β Cep star 16 Lac (Fitch 1969).

Table 3 presents the orbital elements we derived from all the published velocities of 14 Aur which we could find. The period was determined by a

TABLE 3. ORBITAL ELEMENTS FOR 14 AURIGAE A

$P = 3.78857 \pm 0.00003$ days (estimated error)
$e = 0.0$
$\gamma = -9.8$ km/sec
$K = 22.5 \pm 0.7$ km/sec (m.e.)
$T_o = $ J.D.2420003.10 ± 0.02 (m.e.) at R.V. Max.
$f(M) = 0.0045 \pm 0.0004\ M_\theta$ (m.e.)

Fourier transform amplitude spectrum, giving an estimated uncertainty of 0.000002 c/d, and the rest of the elements were gotten by least-squares fits of sine curves in harmonics of the orbital period. There is no evidence for a noncircular orbit, and we do not confirm the modulation characteristic suggested by the observations of Hudson et al. (1971). Rather, given the orbital period as known, we find that the orbital phase of minimum light amplitude varies with time, just as did Broglia and Marin (1974) in the eclipsing binary Y Cam.

Table 4 presents a summary of all the material I could find on short period pulsators in close binaries (excluding pulsars, white dwarfs, etc.), arranged to emphasize the apparent correlation between long term variability and orbital eccentricity. I consider the top eleven stars as definitely established binaries, though skeptics may wish to exclude CC And from this category. The inclusion of the last three stars in this list is specula-tive. KP Aql (Ibanoglu and Gülmen 1974) is an eclipsing binary with an A type primary, and the published light curves suggest to me that it may also be a δ Scuti star, but this suggestion needs definite testing. I include δ Scuti and 1 Mon as binaries, because their observed characteristics are consistent with other pulsators known to be binaries, but I cannot prove my assumptions here. The assignment of the modulation characteristics to the five stars in the middle group is also speculative, by an obvious extrapola-tion of the apparent pattern shown by the first six stars in Table 4. GX Peg is apparently misnamed TW Lac in Table 2 of Baglin et al. (1973).

I would like to emphasize two points concerning Table 4. First, Lomb (1975) has definitely established tidal modulation of the pulsation ampli-tude in α Vir, so I am no longer the only one to have found this kind of behavior in a short period pulsating star. Second, I could not find any

RRs stars to include in this list, though the RRs star SZ Lyn is the primary
in a wide binary with period 3.14 years (Barnes and Moffett 1975), and the
RRc star RW Ari is the primary of an eclipsing binary (Wisniewski 1971).

TABLE 4. SHORT PERIOD PULSATORS IN CLOSE BINARIES

Name*	Orbit			Pulsation			
	Type	P_L(days)	e	Type	P_0(day)	Tidal Modulation	Nonradial Modes
CC And	Photom.	10.469	$\approx$ 0.15?	δ Sct	0.12491	Yes	No
14 Aur	Sp. 1	3.7886	0.0	δ Sct	0.08809	No	Yes
Y Cam	Ecl.	3.3055	0.0	δ Sct	0.063	No	Yes?
16 Lac	Sp. 1	12.096	0.0	β Cep	0.16917	No	Yes
σ Sco	Sp. 1	33.13	0.40	β Cep	0.24684	Yes	No
α Vir	Sp. 2	4.0142	0.13	β Cep	0.1733	Yes	No?
AB Cas	Ecl.	1.3669	0.0	δ Sct	0.058	No?	Yes?
ZZ Cyg	Ecl.	0.6286	0.0	δ Sct	0.1	No?	Yes?
δ Del	Sp. 2	40.58	0.7	δ Sct	0.15679	Yes?	No?
UX Mon	Ecl.	5.9045	0.0	δ Sct	0.2	No?	Yes?
GX Peg	Sp. 1	2.3409	0.0	δ Sct	0.06	No?	Yes?
KP Aql	Ecl.	3.3675	0.0	δ Sct?			
1 Mon	Photom.	15.492?	> 0.0?	δ Sct	0.13612	Yes?	No?
δ Sct	Photom.	$\approx$ 10?	0.0?	δ Sct	0.19377	No	Yes

    * CC And, Fitch 1967, present paper; 14 Aur, present paper; Y Cam, Broglia
and Marin 1974; 16 Lac, Fitch 1969; σ Sco, Fitch 1967; α Vir, Lomb 1975; AB Cas,
Tempesti 1971; ZZ Cyg, Hall and Cannon 1974; δ Del, Preston, in Leung 1974,
present paper; UX mon, Scaltritti 1973, Lynds 1957; GX Peg, Breger 1969, Harper
1933; KP Aql, Ibanoglu and Gülmen 1974; 1 Mon, Millis 1973, Shobbrook and Stobie
1974; δ Sct, Fath 1935, 1937, 1940, present paper.

If the suggested correlation of nonradial mode excitation and zero
orbital eccentricity or of tidal modulation and nonzero eccentricity is con-
firmed by future work, then one possible explanation of these correlations
might involve the pulsation amplitude growth rates for the δ Scuti stars.
According to Chevalier (1971), the e-folding time for the fundamental
radial mode amplitude is about 2×10^4 yr, or, as quoted in Baglin et al.
(1973), about 10^3 yr. I don't know what the nonradial mode growth rates
are, but these modes are presumably driven by energy leakage from the radial
modes in the case of nonspherical symmetry, so I would expect all nonradial

modes to be damped out by the continuously changing surface deformations of a primary with a close companion in an elliptical orbit. In this case only the radial modes should survive, though with phase and amplitude continuously variable in a zonal pattern over the surface of the primary. Further, of course, the integrated pulsation amplitude should be much smaller than in a single star of the same structure. If, however, the postulated close companion of a pulsating primary moves in a circular orbit, then the nonspherical deformations will appear static in the rotating frame, particularly if the primary rotates synchronously, and over a sufficient length of time the nonradial modes should grow to significant strength. This is apparently true in 16 Lac, and it may also be the explanation of the 3 nonradial modes excited in δ Sct.

Fath (1935, 1937, 1940) observed δ Sct on 63 nights in 1935, 1936, and 1938, while we have obtained a coverage of 23.9 cycles on δ Sct during 24 nights in 1972 and 1973. Fath used α Sct as a comparison star, and later found it to be variable in brightness. This introduces an unfortunate amount of noise into his measures, which are further complicated by his short (41 days maximum) observing seasons. Therefore, there is a significant uncertainty in the annual cycle counts for all but the fundamental period.

The annual sidelobes are even stronger in our own measures, where the 1972 and 1973 seasons spanned only 20 days and 14 days, respectively. It was, therefore, somewhat surprising to find that these two independent data sets, separated by 34 years, agreed on the annual cycle counts for the fundamental radial mode frequency f_o and its second harmonic $2f_o$, the second radial overtone f_2, and the strongest nonradial mode f_{n1}. Both data sets agreed on the presence of two more nonradial modes f_{n2} and f_{n3}, though f_{n3}

13 Astronomical I.

TABLE 5. ADOPTED SOLUTIONS FOR THE LIGHT VARIATION OF δ SCUTI[*]

Name	Fath's White Light Measures					1972-73 b-Magnitude Measures				
	f_i(c/d)	A_i	σ_A	$\emptyset_i$	$\sigma_\emptyset$	f_i(c/d)	A_i	σ_A	$\emptyset_i$	$\sigma_\emptyset$
f_{n3}						4.73582	0.0046 ±	0.0003	0.624 ±	0.010
f_0	5.16078	0.0783 ±	0.0007	0.534 ±	0.001	5.16070	0.0717	0.0003	0.428	0.001
f_{n2}	5.27946	0.0039	0.0007	0.215	0.028	5.27885	0.0036	0.0003	0.791	0.014
f_{n1}	5.35401	0.0176	0.0007	0.760	0.006	5.35446	0.0150	0.0003	0.732	0.003
f_2	8.59388	0.0054	0.0007	0.544	0.020	8.59355	0.0058	0.0003	0.679	0.008
$2f_0$	10.32156	0.0084	0.0007	0.915	0.013	10.32140	0.0067	0.0003	0.754	0.007
f_0+f_{n2}	10.44024	0.0025	0.0007	0.637	0.044					
$2f_{n2}$	10.55892	0.0013	0.0007	0.753	0.087					
f_0+f_{n1}	10.51479	0.0037	0.0007	0.201	0.030	10.51516	0.0025	0.0003	0.931	0.019
$2f_{n1}$	10.70802	0.0016	0.0007	0.518	0.070	10.70892	0.0016	0.0003	0.740	0.030

Comparison = α Scuti	Comparison = HR 7055
σ (1 obs) = ± 15.6 mmag	σ (1 obs) = ± 7.6 mmag
919 measures	1158 measures, averaged from 4855 with
Δm_o (1935) = -0.048 mag	Δt = 0.003 day
Δm_o (1936) = -0.037 mag	
Δm_o (1938) = +0.014 mag	Δm_o = -1.189 mag

[*] $m_\delta = m_{comp} + \Delta m_o - 2.5 \log \left[1 + \Sigma A_i \sin 2\pi (f_i t + \emptyset_i) \right]$
t = Hel.J.D. - 2427900.0

is very weak in Fath's data but stronger than f_{n2} in our own data. Our adopted values for f_{n2} and f_{n3} are rather sensitive to small changes in the whitening parameters for the stronger terms and may each be in error by 1 cycle/year, but this doesn't matter for the representation of these observations.

We originally assumed that the frequencies were strictly constant, and consequently experienced much difficulty in choosing the correct annual cycle counts. This occurred because while the two data sets agreed on the highest peaks for the strongest terms, they disagreed on the precise frequency values, and when we tried to force phase-locking from 1935 to 1973 we had to settle for peaks one annual sidelobe off the symmetry center of

the frequency spectra. We also experienced problems in fitting to the data, and could not achieve a satisfactory analytic representation of the accurate modern photometry. We finally abandoned the assumption of constant periods and only compared the spectra of the two data sets to look for the strongest terms. Our adopted representations are compared graphically to a selection of typical observations in Figure 2, and are shown analytically in Table 5.
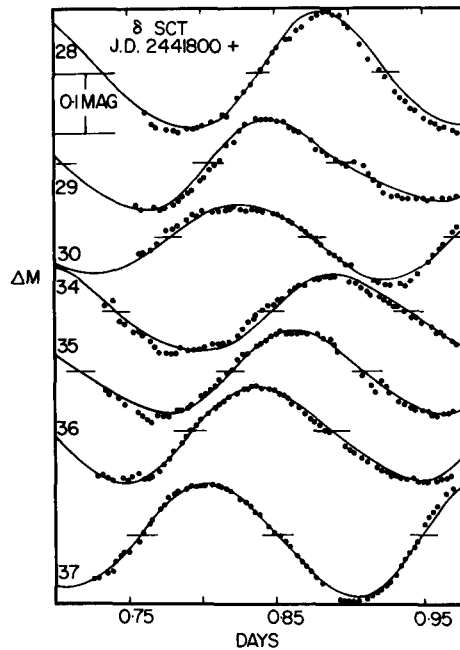

Fig. 2 - Representative observed and computed b-magnitude light curves of δ Sct minus HR7055.

We think that while these solutions could probably still be improved, they represent a reasonable approximation to the true behavior of δ Sct.

Please note, in Table 5, that the nonradial modes cluster about f_o and, in Fath's data, are apparently coupled nonlinearly to f_o, but they do

13$^{\textbf{x}}$ I.

not bear any simple frequency difference relation to f_o or to each other.
Since this behavior is very similar to our earlier findings on 16 Lac, it
prompted the suggestion that δ Sct may also be a binary with a circular
orbit. Note also that the frequency pattern is not easily explained by the
usual theory of rotational coupling (Ledoux 1951) or by the R- and S- mode
theory of Chandrasekhar and Lebovitz (1962).

With all of the uncertainties involved, our adopted frequency changes
must be regarded as highly provisional and subject to independent confirma-
tion. If, however, they are correct, and if they represent a secular change
due to evolution, then they indicate that $\delta \ln f_o = -0.000016$ in 36 yr. If
we postulate evolution at constant mass M, and assume the pulsation constant
$Q_o = 0.033$ day, we obtain $\delta \ln R \approx + 10^{-5}$ in 36 yr, and deduce an e-folding
time for the radius R of about 4×10^6 yr, with evolution toward the red
side of the instability strip. There are so many possible explanations for
our adopted frequency changes, including simply observational error, that
little reliance should be placed on this estimate.

Because Shobbrook and Stobie (1974), in their excellent paper on 1 Mon,
questioned the reality of the double cycle I found in CC And (Fitch 1967),
we reobserved CC And on 16 nights in 1974. Figure 3 presents the fundamen-
tal radial mode pulsation amplitudes A_o and phases ϕ_o (obtained by fitting a
sine and cosine curve to individual cycles of pulsation and then deriving
the equivalent sine term $A_o \sin 2\pi (f_o t + \phi_o)$) plotted against the phases
of the best 5-day period (such as is found by periodogram analysis). The
phase variation in 1955-57 and the amplitude variation in 1974 both clearly
show differences between even and odd cycles of the 5-day period, and they
also show that the detailed modulation characteristics have changed in the
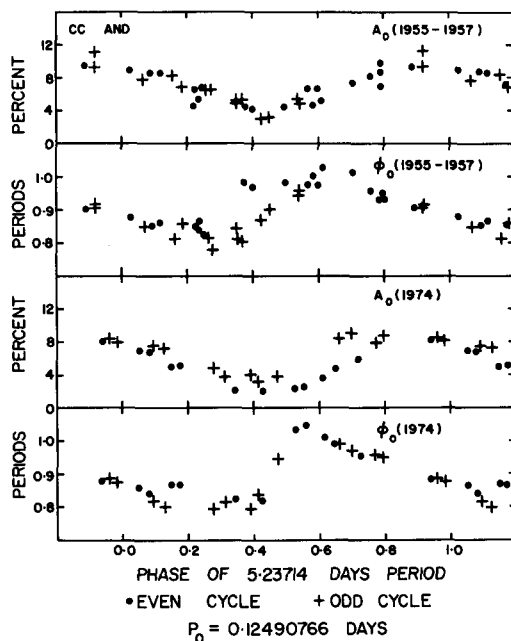interval between the two data sets. Therefore, since it is not possible to

Fig. 3 - Observed phase (ϕ_o) and yellow amplitude (A_o) variation of the fundamental pulsation in CC And as functions of phase of the long period (= 5.23714 day). Even and odd cycles are differentiated.

match precisely these two sets, I derived the best value for the 10.5-day double cycle by the condition of minimum scatter in the 1955-57 data, and didn't worry about phase-locking the long period over the complete 19 yr span, though the fundamental mode is unambiguously phase-locked over this interval. Figure 4 presents the modulation characteristics of A_o and ϕ_o with phase of our adopted 10.469 day period. The smoothed A_o curves and the 1955-57 ϕ_o curve are my estimates of the real variation. The smoothed 1974 ϕ_o curve is the best 8-term analytic approximation (using a Fourier expansion including all terms through $8 f_L$, where f_L = 0.09552 c/d is our adopted modulating frequency) to the smoother variation I estimated. It

was necessary to obtain the analytic expansion for the 1974 phase modulation

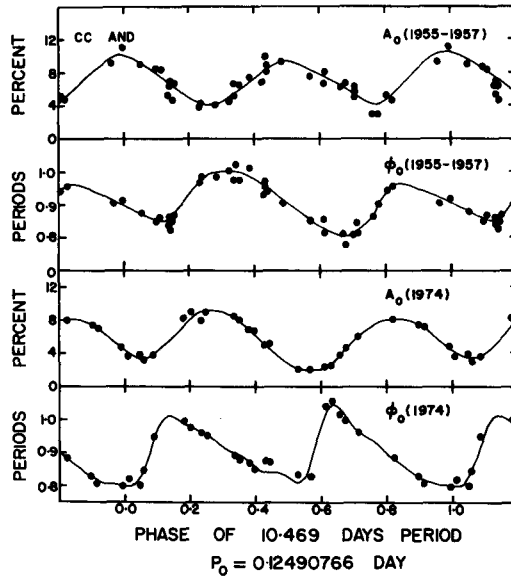by fitting to a smoothed curve, since there is significant scatter in the

Fig. 4 - Observed (solid circles) and adopted (full line) ϕ_0 and A_0 variations of CC And as functions of phase of the 10.469 day period.

limited number of data points and a very high order fit is required for

accurate prediction of the observed modulation. The adopted phase modula-

tions are given in Table 6. Please note the markedly different modulation

TABLE 6. PHASE MODULATION OF CC AND*

	1955-57		1974	
i	B_i	β_i	B_i	β_i
1	0.04105	0.0073	0.01524	0.8193
2	0.06684	0.5275	0.09420	0.8621
3	0.01014	0.4913	0.00510	0.1798
4	0.01433	0.0486	0.03386	0.6772
5	0.01515	0.1390	0.00495	0.0177
6			0.01847	0.4896
7			0.00439	0.7840
8			0.01038	0.3057

* $\Delta\phi_0(t) = \Sigma B_i \sin 2\pi(i f_L t + \beta_i)$

$t = \text{Hel.J.D.} - 2435000.0$

$f_L = 0.09552$ c/d

characteristics in the two data sets, which I attribute to revolution of the apse of the elliptical orbit. Both minimum and maximum amplitude are smaller in 1974 than previously, and the 1974 phase variation approaches a sawtooth pattern. If an observer only sampled the real variation on one of the branches of the 1974 phase modulation, he would by periodogram analysis derive frequency estimates for f_o ranging between 7.96 and 8.31 c/d, depending on which branch he observed. If he only sampled the real variation on two successive branches, he would conclude that the star had an unstable and continuously variable frequency spectrum. For this reason, and from my experience with δ Sct itself, I do not consider conclusions drawn from short data strings convincing, and I am extremely skeptical about the suggestion that δ Scuti stars are nonperiodic.

The second radial overtone f_2 is not directly modulated by f_L (i.e., there are no terms with frequencies $f_2 + k\,f_L$), so in our adopted representation, shown in Table 7, this mode is given by a single sine term. However,

TABLE 7. ADOPTED V-MAGNITUDE SOLUTIONS FOR CC ANDROMEDAE[*]

	1955-57					1974			
j, k	A_{jk}	σ_A	$\emptyset_{jk}$	$\sigma_\emptyset$	j, k	A_{jk}	σ_A	$\emptyset_{jk}$	$\sigma_\emptyset$
1,-7	0.0017 ±	0.0005	0.603 ±	0.045	1,-5	0.0008 ±	0.0004	0.574 ±	0.069
1,-6	0.0010	0.0005	0.755	0.084	1,-4	0.0008	0.0004	0.731	0.071
1,-3	0.0022	0.0005	0.912	0.040	1,-3	0.0026	0.0004	0.554	0.022
1,-2	0.0151	0.0005	0.928	0.005	1,-2	0.0139	0.0004	0.492	0.004
1,-1	0.0013	0.0005	0.159	0.060	1,-1	0.0067	0.0004	0.973	0.009
1, 0	0.0676	0.0006	0.904	0.001	1, 0	0.0576	0.0004	0.901	0.001
1,+1	0.0010	0.0005	0.505	0.078	1,+1	0.0068	0.0004	0.763	0.009
1,+2	0.0134	0.0005	0.884	0.006	1,+2	0.0152	0.0004	0.288	0.004
1,+3	0.0037	0.0005	0.814	0.024	1,+3	0.0015	0.0004	0.501	0.040
1,+6	0.0026	0.0005	0.906	0.031	1,+4	0.0026	0.0004	0.190	0.022
1,+7	0.0029	0.0005	0.226	0.027	1,+5	0.0029	0.0004	0.073	0.020
2,-2	0.0044	0.0005	0.782	0.016	2,-2	0.0048	0.0003	0.329	0.011
2, 0	0.0087	0.0005	0.676	0.008	2, 0	0.0066	0.0003	0.620	0.008
2,+2	0.0044	0.0004	0.617	0.016	2,+2	0.0037	0.0003	0.070	0.015
f_2	0.0078	0.0005	0.638	0.009	f_2	0.0047	0.0003	0.636	0.011

Δm_o = +0.263 mag $\qquad\qquad$ Δm_o = +0.264 mag

σ (1 obs) = ± 10.6 mmag $\qquad$ σ (1 obs) = ± 7.4 mmag

1041 measures, averaged from 2230 with $\quad$ 865 measures, averaged from 3344 with

Δt = 0.0035 day $\qquad\qquad\qquad$ Δt = 0.0035 day

[*] $m_{CC} = m_{comp} + \Delta m_o - 2.5 \log \{ 1 + \Sigma A_{jk} \sin 2\pi [(jf_0 + kf_L)t + j\Delta\emptyset_o(t) + \emptyset_{jk}]$
$\quad + A_2 \sin 2\pi (f_2 t + \emptyset_2) \}$

f_0 = 8.005914, f_2 = 13.346146, f_L = 0.09552 c/d. t = Hel.J.D. - 2435000.0

182

the systematic nature of the residuals from our adopted solution, shown for

the 1974 data in Figure 5, strongly suggests that most of the remaining fit-

ting errors above the noise level are due to the neglect of modulation on

f_2. Time limitations in the preparation of this paper prevented following

up this point, but I plan to see whether one can represent the modulation of

f_2 through coupling to f_o (i.e. with terms having phases of the form $jf_o t +$
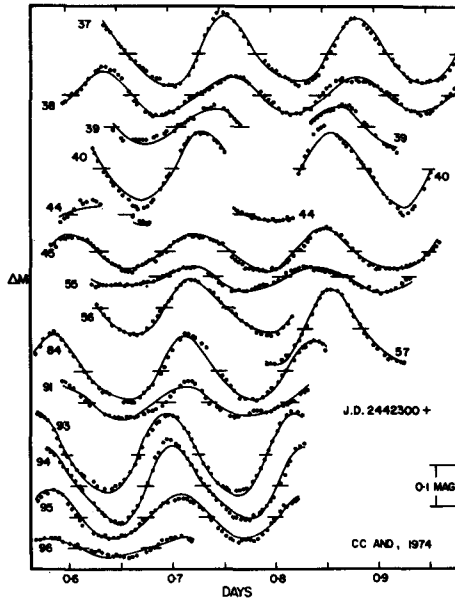
$j\Delta\phi_o(t) + kf_L t + mf_2 t$).

Fig. 5 — Observed and computed yellow magnitude variation of CC And minus
BD + 41°120 in 1974.

In concluding this section, I should like to emphasize the following

points:

(1) At present, it appears that a short period pulsator in a close

binary will exhibit tidal modulation of radial pulsation modes if the orbit

is elliptical, and will show both radial and nonradial mode excitation if

the orbit is circular. However, in this last case the nonradial mode

frequency spacing is not at a simple constant frequency difference.

(2) In at least the case of CC And, the accurate representation of the observed light variation requires a complex and highly nonlinear modulation of amplitude and phase, separately. It is not possible to achieve a good approximation to the observed variation using only a simple amplitude modulation function involving just the sum of a set of sine terms each having a constant frequency and phase. This implies that the observed variation is actually an integral over the apparent disk of areas which are moving instantaneously with different phases and amplitudes, but with the same (fundamental) frequency. This seems to me physically plausible, since the radial mode variations in the deep interior will be little affected by tides, whereas both the phase and amplitude of the emergent wave are sensitive and highly nonlinear functions of the detailed structure of the HeII (and possibly also H and HeI) ionization zone, which must itself be zonally perturbed by any close companion.

(3) Aside from possible slow secular or very long period changes in frequencies and relative mode strength, the variations of δ Sct are strictly periodic. The variations of CC And are also strictly periodic (or very slowly changing in secular manner), and they require at most the fundamental and second radial overtone period, the orbital period, and the apse precessional period to explain them.

(4) Ordinary Fourier transform calculations, whether with data whitening in the time domain (Wehlau and Leung 1967) or with spectrum whitening in the frequency domain (Gray and Desikachary 1973), are not adequate tools for dealing with the complexities of variation displayed in such stars as CC And. Much better analytical methods, allowing for nonlinear phase and amplitude modulations, are urgently needed. Realistic models for such stars as CC And

will also require provision for the lag between the instantaneous amplitude and phase, and the instantaneous growth rates of the pulsation, all of which I think vary in a continuous manner over the stellar surface.

IV. Are RRs and δ Scuti Stars Physically Distinct?

Eggen (1956) first suggested that there was a real difference between the low amplitude δ Scuti stars and the higher amplitude RRs stars, though both groups show spectral types A to F and have periods less than 0.25 day (< 0.21 day, according to the General Catalog of Variable Stars, though VX Hya does not conform to this definition). More recently Eggen (1970), Breger and Bregman (1975), Chevalier (1972), and Baglin et al. (1973) have all argued, from various points of view, that there is no meaningful difference between the two types. Petersen and Jørgensen (1972) agreed with the prevailing view that δ Scuti stars are main-sequence and early post-main-sequence Pop. I stars with masses $1.5 \leq M/M_\odot \leq 2.5$, while concluding that the RRs stars are post-red-giant Pop. II objects with $M < M_\odot$. In a recent communication, Petersen has informed me that he now thinks all RRs stars, including SX Phe, may have $M > M_\odot$, in agreement with the δ Scuti stars. Breger (1975), from a Wesselink analysis based on published velocities and new uvbyβ photometry, has derived a radius $R = 3.0\ R_\odot$ and mass $M = 2.9\ M_\odot$ for the RRs star AD CMi. The mass determination in this case depends on log g measures, which I think are generally unreliable. If instead one adopts Breger's value for the radius, and a fundamental radial mode pulsation constant $Q_0 = 0.033$ day, a more reasonable mass $M = 1.9\ M_\odot$ follows. In either case, Breger's evidence for a Pop. I mass in AD CMi seems to me very convincing.

Walraven's (1955) classic paper on SX Phe and AI Vel pioneered all more recent investigations of multimode excitation of Cepheid strip stars. While

trying to develop suitable analytic procedures for computer simulation of the observed variations in VX Hya (Fitch 1966), I found it is only necessary to use a doubly-harmonic Fourier expansion to reproduce the light variation in that doubly-periodic star. Because I have since verified that the method also works very well on the published photometry (and velocities) of the double radial mode variables SX Phe and AI Vel (Walraven 1955) and VZ Cnc (Fitch 1955), I assume it will also work on all other such stars which lack additional complications such as, for example, tidal modulation. Very recently, Fitch and Szeidl (1976) found that the peculiar RR Lyrae star AC And has the fundamental and first and second radial overtone modes all excited and nonlinearly coupled together, so that the analytic representation of the light variation requires a triply-periodic harmonic expansion. To obtain even a rough approximation to the observed variation requires a fifth order expansion containing 115 different frequencies generated from the three incommensurable radial mode periods.

Table 8 is an extract from Table 3 of our paper on AC And, listing those observed radial mode periods and period ratios for δ Scuti, RRs, and

TABLE 8.   SHORT PERIOD MULTIMODE VARIABLES IN THE CEPHEID STRIP

Name[*]	Type	P_0(day)	P_1(day)	P_2(day)	P_1/P_0	P_2/P_1	P_2/P_0	Log P/P_c	$M/M_\odot$	$R/R_\odot$
SX Phe	RRs	0.05496	0.04277		0.7782			-0.50		
CY Aqr	RRs	0.06104	0.04543:		0.7443:			-0.52:		
AE UMa	RRS	0.08602	0.06653		0.7734			-0.87		
RV Ari	RRs	0.09313	0.07195		0.7726			-0.94		
21 Mon	δ Sct	0.09991	0.07500		0.7507			-0.95		
BP Peg	RRs	0.10954	0.08451		0.7715			-1.07		
AI Vel	RRs	0.11157	0.08621		0.7727			-1.09		
CC And	δ Sct	0.12491		0.07493			0.5999	-1.15		
1 Mon	δ Sct	0.13612		0.08261			0.6069	-1.23		
V703 Sco	RRs	0.14996	0.11522		0.7683			-1.33		
δ Sct	δ Sct	0.19377		0.11636			0.6005	-1.52	≈ 1.9	≈ 4.0
VX Hya	RRs	0.22339	0.17272		0.7732			-1.68		
VZ Cnc	RRs		0.17836	0.14280		0.8006		-1.70		
AC And	RRab	0.71124	0.52513	0.42107	0.7383	0.8018	0.5920	-2.61	3.1	10.7

: P_1 for CY Aqr is uncertain

* SX Phe, Walraven 1955; CY Aqr, Fitch 1973; AE UMa, Szeidl 1974; RV Ari, Detre 1956; 21 Mon, Gupta 1973; BP Peg, Broglia 1959; AI Vel, Walraven 1955; CC And, present paper; 1 Mon, Shobbrook and Stobie 1974; V703 Sco, Oosterhoff 1966; δ Sct, present paper; VX Hya, Fitch 1966; VZ Cnc, Guman 1955, Fitch 1955; AC And, Fitch and Szeidl 1976.

RR Lyr stars which I consider to be reliably determined (the overtone
period in CY Aqr is still open to question). The tabulated densities are
calculated from the observed periods by fitting formulae developed from
Cogan's (1970) published pulsation models. These formulae also permit
direct calculation of mass M and radius R in favorable cases, and the
observed P_0 and P_2 periods of AC And lead to the mass and radius we adopted,
but they do not permit such a calculation for the shorter period variables
here. The mass and radius of δ Sct were adopted from the density (inferred
from the observed periods P_0 and P_2), the trigonometric parallax (Jenkins
1952), mean apparent magnitude (Hoffleit 1964), surface temperature (Bessell
1969), and metal rich model star evolutionary tracks (Robertson 1971, 1975).
Evolution theory, pulsation theory, and observation all agree if δ Sct and
AC And both have Z ≈ 0.044 and are on their first left-to-right crossing of
the instability strip. Pulsation theory, as represented by the models of
Cogan (1970) and of Petersen and Jørgensen (1972) agrees with observation,
as represented by the observed ΔS values (Preston 1959) and periods for
these stars. But since Chevalier (1972) has challenged the model calcula-
tions of Cogan and of Petersen and Jørgensen, and since Petersen has appar-
ently now changed his stand regarding the nature of the RRs stars, I cannot
predict what the final outcome of the argument will be.

Please note that of the 14 stars listed in Table 8, only VZ Cnc does
not have an excited fundamental radial mode, that 5 of these stars have
firmly established excitation of the second radial overtone, and that the
three δ Scuti stars CC And, 1 Mon, and δ Sct have a weakly excited second
overtone and quiescent first overtone. I conclude that there are no obvious
differences between the radial mode periods of the RRs and δ Scuti stars,
except those due to the (perhaps debatable) dependence on heavy element
composition Z.

Breger (1970) argued that A-F pulsators and Am stars are mutually exclusive groups, and that perhaps metallicism inhibits pulsation. This view has been pursued further by Baglin (1972), by Vauclair et al. (1974), and by others, and will be described by Baglin at this meeting. The crux of the argument seems to be that in slowly rotating single stars, element separation by diffusion can lead to a He deficiency in the He II ionization region, so that such stars lack the capability for self-excited oscillations, whereas stars maintained with a homogeneous envelope by rotationally driven mixing currents will start to pulsate as they pass through the instability strip. I've nothing to add to this discussion, but I should like to offer one other simple suggestion for consideration.

The correlation of increasing pulsation amplitude with increasing period and luminosity and decreasing surface temperature, when moving up the center of the instability strip from the main sequence, is well known, and reasonably well explained by model calculations. If we assume that the RRs and δ Scuti stars are all Pop. I or disk population objects with $M > M_{\odot}$, and if we compare, from Tables 3 and 8 (and SZ Lyn), stars with the same fundamental period, perhaps the amplitude differences between RRs and δ Scuti stars merely reflect the presence or absence of complications caused by close companions. That is, comparing the RRs stars VX Hya or VZ Cnc, SZ Lyn, RV Ari, and AE UMa with the δ Scuti stars δ Sct, CC And, 21 Mon, and 14 Aur A, respectively, it may well be that the larger amplitudes in the first group are those normal to essentially single stars, while the smaller amplitudes of the second group result from partial damping of the radial modes by the nonspherical deformations caused by close companions. If so, then here is one more mechanism for inhibiting pulsation in the instability strip. I am not now foolish enough to suggest that _all_ δ Scuti stars with long period

complications are members of binary systems, but I do think it possible that there are many more of them in close binaries than we presently recognize. In any case, it seems unlikely that any one mechanism will explain the great diversity of characteristics displayed by these very interesting short period variables.

In conclusion, I would venture to suggest that in any future discussion of these stars, we could greatly profit by closer collaboration with our colleagues in IAU Commissions 26, 30, and, especially, 42.

The new observations of δ Sct and CC And herein described will be available from the archives in London and Odessa as files IAU(27).RAS-42 and 43, respectively.

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190

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14 Astronomical I.

Discussion to the paper of FITCH

DESIKACHARY: Do you identify the nonradial oscillations you propose
               for cases like δ Sct with any specific modes such as
               gravity and f modes?

FITCH: No. That problem I leave to the theoreticians.

DESIKACHARY: Would you like to explain the clustering of the nonradial
               modes around the fundamental?

FITCH: In δ Sct, there is evidence that the nonradial modes
               are coupled to the fundamental, and I assume they derive
               their excitation by energy leakage from f_o. If this is
               correct, it seems to me natural that the nonradial modes
               observed will be those relatively close to the strongest
               radial mode excited.

BAGLIN: If nonradial modes are the dominant ones, this could
               have some consequences on the observational parameters.
               The fact that the different parts of the surface do not
               pulsate in phase would perturb the amplitudes and the
               relation between the measured rotational velocity and
               light variations. The relation between ΔV_R and Δm
               for high amplitude δ Scuti stars seems to agree with
               the Cepheids, which are radial pulsators. For example,
               the β CMa stars follow a very different relation – Lucy
               has shown in this meeting that a mixture of nonradial
               modes could look more like macroturbulence than like
               pulsation. The light curves analysed in this paper
               correspond to evolved stars (class IV and III) and
               slowly rotating stars (V ≃ 30–60 km/sec) – they differ
               very much from the classical ordinary variables close
               to the main sequence, as, for example HR 8006.

LE CONTEL: I want to make a suggestion for the general discussion
               on these δ Scuti stars. I think we have to distinguish
               at least 3 subgroups: "dwarf" δ Scuti stars (rapid
               rotators, small amplitudes, and complicated variations),
               for example HR 8006; giants (rapid rotators, low amplitude

variations) for example HR 515, HR 432; and a third
group you present, which are almost all binaries or slow
rotators (14 Aur).

In my opinion, if all these stars have the same
mechanism at work from the point of view of internal
structure, their observed differences are probably real
due to the fact that we only observe superficial layers.
So you are probably right in the third case where tidal
interaction can produce the main effect, but in the other
two cases we have to look for other physical phenomena
which could be at work in the atmosphere. This is why
we suggest examination of the coupling between pulsation
and convection.

GEYER: I do not agree with you that RRs stars and δ Scuti
stars are closely connected. There are two RRs stars
observed in globular clusters, the membership of which
has up to now not completely been excluded nor establish-
ed. V65 in ω Centauri is, according to the Greenwich
proper motion investigation perhaps a non-member. On
the other hand, it falls so well within the RR Lyrae
gap of this cluster and shows the same UV excess as
these RR Lyrae stars, that it might as well be a cluster
member.

FITCH: Personally, I think there must be a continuum of proper-
ties for the RRs stars, with masses and compositions
ranging from those of the δ Scuti stars to those of the
extreme halo population.

14[x] I.