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Orbit propagation around small bodies using spherical harmonic coefficients obtained from polyhedron shape models

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Abstract. Missions to asteroids have been the trend in space exploration for the last years. They provide information about the formation and evolution of the Solar System, contribute to direct planetary defense tasks, and could be potentially exploited for resource mining. Be their purpose as it may, the factor that all these mission types have in common is the challenging dynamical environment they have to deal with. The gravitational environment of a certain asteroid is most of the times not accurately known until very late mission phases when the spacecraft has already orbited the body for some time.

Shape models help to estimate the gravitational potential with a density distribution assumption (usually constant value) and some optical measurements of the body. These measurements, unlike the ones needed for harmonic coefficient estimation, can be taken from well before arriving at the asteroid's sphere of influence, which allows to obtain a better approximation of the gravitational dynamics much sooner. The disadvantage they pose is that obtaining acceleration values from these models implies a heavy computational burden on the on-board processing unit, which is very often too time-consuming for the mission profile.

In this paper, the technique developed on [1] is used to create a validated Python-based tool that obtains spherical harmonic coefficients from the shape model of an asteroid or comet, given a certain density for the body. This validated software suite, called *AstroHarm*, is used to analyse the accuracy of the models obtained and the improvements in computational efficiency in a simulated spacecraft orbiting a small body.

The results obtained are shown offering a qualitative comparison between different order spherical harmonic models and the original shape model. Finally, the creation of a catalogue for harmonics is proposed together with some thoughts on complex modelling using this tool.

Keywords. Spherical harmonics; Shape model; Propagation; Small bodies; Polyhedron.

1. Introduction

The number of missions targeting small bodies, such as asteroids or comets, has risen in the past decades. The Near-Earth Asteroid Rendezvous (NEAR) mission [2] flew to Eros and rendezvoused the asteroid to an approach distance of around 500 km, to then lower that distance to 35 km. This mission included a large set of instruments to perform different measurements of the environment around Eros, including a Multispectral Imager (MSI), a Near-Infrared Spectrograph (NIS), an X ray/Gamma Ray Spectrometer (XRS/GRS), a Magnetometer (MAG), and a NEAR Laser Rangefinder (NLR). After that, the objective of the missions shifted towards sampling the small body they were focused on, by means of small lander probes or even by performing short sampling manoeuvres (often called Touch-and-Go (TaG) manoeuvres).

The Rosetta mission [3; 4], for instance, targeted comet 67P/Churyumov-Gerasimenko close to its aphelion and studied the physical and chemical properties of the nucleus and its coma, and the development of the interaction region of the solar wind and the comet. Additionally, Rosetta deployed Philae, a small lander, onto the surface of the comet. Although the landing was not successful, Philae helped retrieving information about the gravitational field of the comet and its surface properties.

With a different approach, Hayabusa [5], Origins, Spectral Interpretation, Resource Identification, and Security-Regolith Explorer (OSIRIS-REx) [6], and Hayabusa2 [7] missions collected samples by making touch-down -or TaG- sequences. Both Hayabusa missions visited asteroid Itokawa, while OSIRIS-REx visited Bennu. The retrieval of surface samples from the small body they were orbiting enriches the scientific outcome of the mission by allowing the mission to bring back to Earth, for further analysis, materials directly extracted from the asteroid. To do that, however, an extremely high level of knowledge about the asteroid's mass distribution is needed. Without a proper gravitational potential model, attempting such manoeuvres would risk the end of the mission. Thus, and due to the non-sphericity that these bodies often present, very detailed shape models are constructed from observations to feed the trajectory design process.

Future missions to asteroids include Double Asteroid Redirection Test (DART) [8] and Hera [9] missions, which are part of the Asteroid Impact and Deflection Assessment (AIDA) program [10]. This program will attempt to deflect Dimorphos, the smaller body of the Near-Earth Asteroid (NEA) binary system Didymos. Another example would be the Psyche mission [11], which will rendezvous the namesake asteroid (16 Psyche) and will investigate its metallic composition.

In this work, a method to aid future missions accomplish their objectives is presented that involves the use of a precise gravitational model using spherical harmonics, obtained from early observations of a certain celestial body. This method trades-off accuracy of the solution and computational effort, to meet the mission requirements (on-board modelling, navigation uncertainty...).

Throughout this document, Section 2 will introduce the fundamentals of the technique and its implementation. Section 3 will present the validation results for the developed tool, and Section 4 will display the obtained results for two study-cases, namely, Bennu and Lutetia. Finally, Section 5 will sum up the main points of this research and will draw directions for future work.

2. From polyhedra to spherical harmonics

As the aforementioned shape models become more accurate, thanks to the images and observations taken on-board as the Spacecraft (S/C) gets closer to the small body, the computational burden of calculating the gravitational potential becomes heavier. When using shape models, polyhedron dynamics methods [12] are employed to compute the gravity potential at a certain coordinate, and given a set of vertices, edges, and faces for a constant density shape model. These methods, albeit very accurate even for irregularly-shaped bodies, require to run through all of the faces included in the shape model, which, for high-definition models, are in the order of hundreds of thousands, rendering them not fitting for on-board applications.

This is not compatible with the high degrees of autonomy a mission to an asteroid needs. Telecommanding every manoeuvre from Earth is not feasible due to the long time of travel of the signal (order of minutes) in comparison to the short reaction time needed in such delicate sequences (order of seconds). Alternatives exist, that reduce the computational effort of the orbital dynamics calculations, while maintaining high levels of accuracy, such as mascons or spherical harmonics. In [13], the authors perform an evaluation of how different mascons models perform in terms of both accuracy and computational effort. However, in this work, the analyses will be directed towards the spherical harmonics approach.

The main reference for this work is [1], where the authors developed an algorithm to compute the spherical harmonics coefficients of a given constant density polyhedron (which is not an unrealistic assumption for a wide set of asteroids), following a similar process to what was done in [14]. The key idea is to use recurrence relations for the integrands that appear when one computes the spherical harmonics coefficients for a given body. Such integrands usually involve Legendre functions and polynomials. The recurrence relations are presented in both their normalized and non-normalized form.

Finally the polyhedron is partitioned into a collection of simplices and the integration of the integrands is performed. These simplices are tetrahedra whose bases are the triangular faces of the polyhedron and the vertices are at the centre of the reference system used to define the coordinates of the points that form the shape model. A change of variable is used to ease the integration of the aforementioned simplices.

In terms of implementation, the main issues identified by [1] are to represent homogeneous polynomials in three variables and to operate with them without using symbolic manipulators. This is achieved by representing trinomials of degree n as arrays of length $(n+1)(n+2)/2$ whose elements are the coefficients of the trinomials ordered in such a way that each coefficients correspond to the right trinomial.

AstroHarm (AstroSim Harmonics) is a Python suite and module that takes the theory developed on [1] to a software materialisation. The capabilities that this module offers include: wavefront (.obj) files management, geometric assessment of shape models (reference radius, volume, centre of mass...), triplet management and operations, C and S matrices recursive computation, normalisation routines for coefficients, and file I/O.

3. Validation

In order to validate the coefficients obtained by *AstroHarm*, semi-analytical methods were used to obtain spherical harmonic coefficients from accurately known geometric bodies as a cube, a tetrahedron, and a double pyramid (octahedron). Using Wolfram Mathematica® [15] software and symbolic formulation, the semi-analytically-computed coefficients for various n and m † values were compared to the ones obtained from *AstroHarm*. The results for the cube can be observed in Figure 1.

Errors in the validation of the coefficients for the C and S matrices for the cube and the octahedron range from 1e-19 to 1e-17, which represents a totally negligible value, only due to floating-point precision errors. For the tetrahedron, the errors are also very small but significantly larger than the other cases, ranging from 1e-11 to 1e-10. This is due to the lower *sphericity* of the geometry, which, albeit being perfectly convex, leaves more empty volume when circumscribed by a sphere.

4. Results

Once these coefficients are obtained, a spherical harmonic model has to be implemented. Following the method shown in [16], the gravitational acceleration given by these coefficients can be computed and used as internal dynamics for a given propagator. To evaluate the accuracy of this spherical harmonics model, a ground truth trajectory is required. This ground truth is obtained from a polyhedron dynamics suite, developed by [17] and based on [18]. This choice is supported by the fact that this method is exact up to the surface of the given shape model, assuming constant density distribution.

† n and m refer to the order and degree of the spherical harmonics model, respectively.

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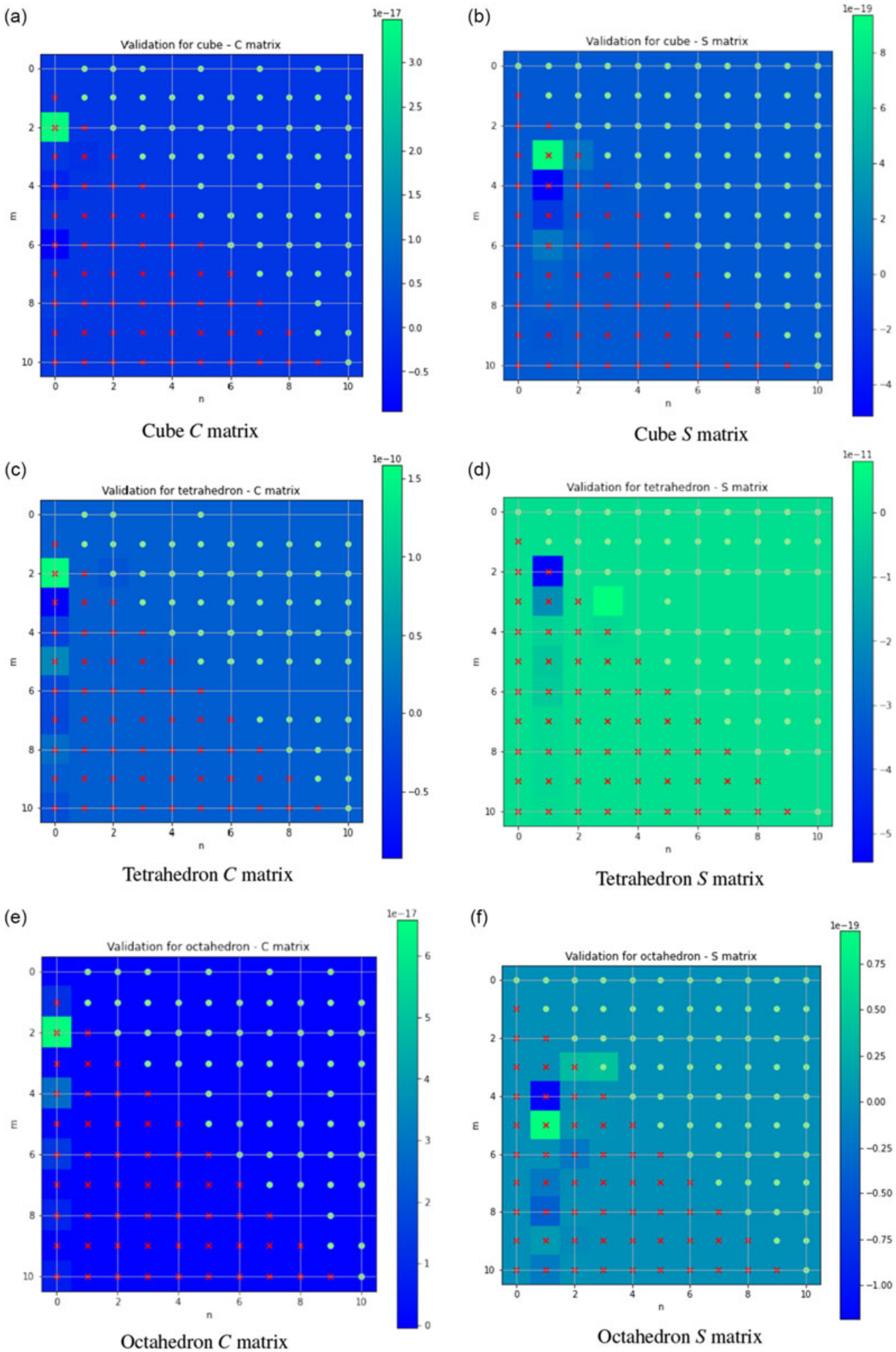

Figure 1. Validation heatmaps for C and S matrices for the different shape models. Red crosses point out cases were $m > n$, which should be disregarded due to its lack of conceptual meaning. Green dots represent values that analytically equal to zero.

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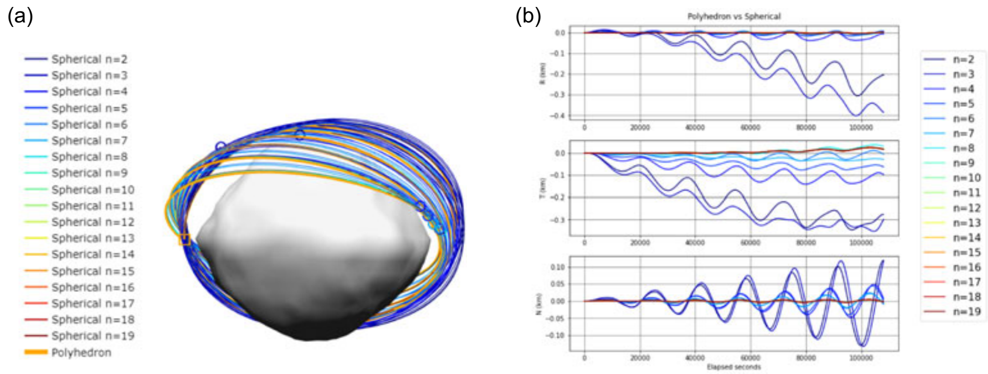
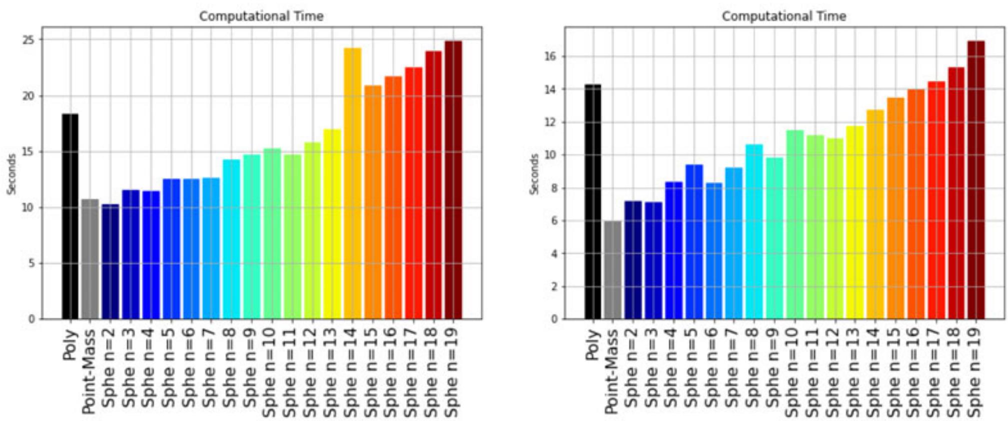


Shape model of asteroid Bennu used for simulation. Comparison of propagated trajectories using spherical harmonics with respect to the Polyhedron model (orange) for reference.

Radial, tangential, and normal component errors for different spherical harmonics models w.r.t. polyhedral model for Bennu.

Figure 2. Trajectories around Bennu for the different values of n after the 30 h propagation. $sma = 350$ m (1.20 Bennu radii), $ecc = 0.1$, $inc = 45°$, where sma, ecc, inc are the semi-major axis, eccentricity and inclination.



Bennu

Lutetia

Figure 3. Computational effort for the different values of n after the 30 h propagation.

The two main drivers of this research are computational performance and results accuracy. Once the computation of the coefficients is validated, the next step is to take the experimentation to a real asteroid for which a shape model is given.

Starting with Bennu, the algorithm is run for different maximum orders for the coefficients going from 2 to 19, obtaining the results shown in Figure 2 in terms of trajectory difference w.r.t. the polyhedral model, which is taken as ground truth.

It can be clearly seen that, as the order of the coefficients increases, the error is reduced, arriving at values close to no error for $n > 10$ in this case. In terms of computational effort, Figure 3a shows that the computational effort for these levels of accuracy at $n > 10$ is reduced by around 17%, w.r.t. the baseline polyhedral model.

4.1. *Remarks*

The limitations of this algorithm were also explored within this project. In particular, two specific parameters were investigated: body shape and orbital distance. The former

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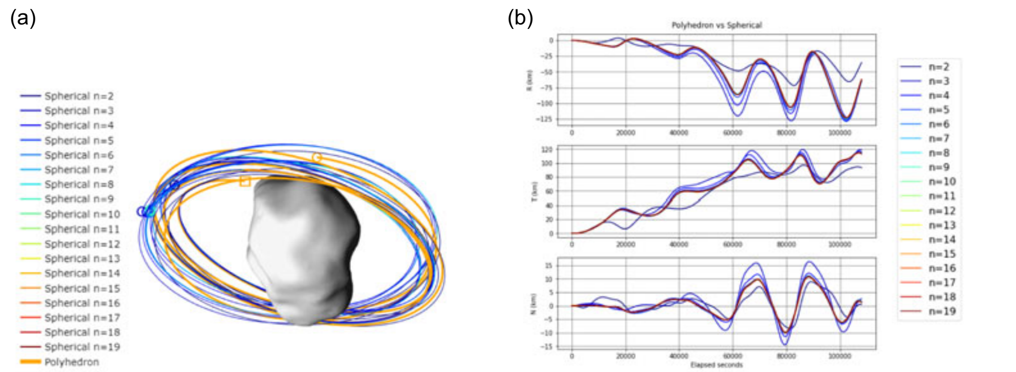

Shape model of asteroid Lutetia used for simulation. Comparison of propagated trajectories using spherical harmonics with respect to the Polyhedron model (orange) for reference.

Radial, tangential, and normal component errors for different spherical harmonics models w.r.t. polyhedral model for Lutetia.

Figure 4. Trajectories around Lutetia for the different values of n after the 30 h propagation. $sma = 150$ km (1.55 Lutetia radii), $ecc = 0.1$, $inc = 45°$, where sma, ecc, inc are the semi-major axis, eccentricity and inclination.

tried to gain some insight into how the oblateness of a body could make it more difficult to obtain a set of spherical harmonics coefficients that could get as accurate as the polyhedron model. Using Lutetia, a less spherical body than Bennu, as subject, the results in Figure 4 were obtained.

Even though there is a clear convergence on the trajectories as the order of the spherical harmonics model rises, this convergence is far from the polyhedral model. Further investigations are required to determine if this problem is solely caused by the high non-spherical shape of the attracting body, or if it can be solved using different approaches. A possibility is to change the origin of the system of reference, which is used to compute the spherical harmonic coefficients. The usual choice is to use the barycenter as the origin, in order to make the first degree coefficient vanish. Choosing a different origin would result in non-zero first degree coefficients, but also in a different reference radius, which could benefit the convergence and accuracy of the method. Computational effort follows the same behaviour seen before, (see 3b).

When analysing the effect of the orbital distance (semi-major axis), the behaviour shown in Figure 5 was observed. When distance from the body grows larger, the significance of its irregular shape decays until it becomes no longer noticeable. This analysis serves to indicate up to which point these models can be used depending on the gravitational environment.

5. Conclusions and future work

A tool has been developed to obtain spherical harmonics coefficients from polyhedron shape models. The coefficients computed have been validated using semi-analytical methods. The further usage of these coefficients in an orbital propagator is assessed by comparing the integrated trajectories with the ones obtained using polyhedron dynamics.

The results show that, in terms of computational effort, the spherical harmonics implementation is far superior to the polyhedron dynamics implementation. Accuracy-wise, for more spherical bodies, the trajectories converge to the ground-truth.

However, this method finds it difficult to replicate the actual gravitational accelerations when orbiting too close to a highly non-spherical body. Future work will investigate on

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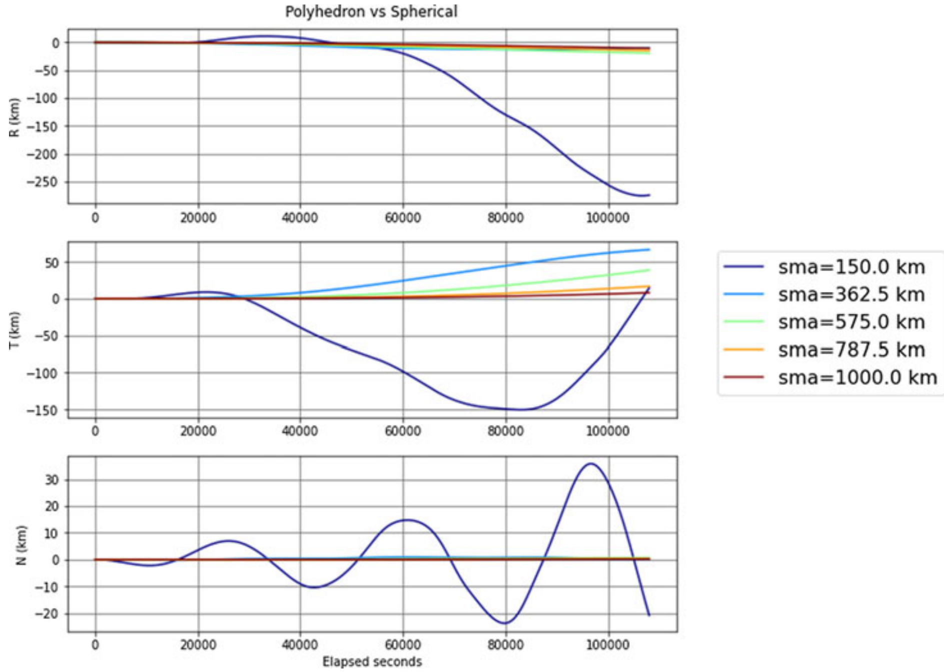

Figure 5. Radial, tangential, and normal component errors for different semi-major axis w.r.t. polyhedral model for Lutetia.

the influence of the choice of the origin of the reference frame for a certain body affects the accuracy of the propagated trajectories.

Pursuing the enhancement of this method could lead to the creation of a catalogue of spherical harmonic coefficients for bodies for which only shape models are available, given a certain constant density assumption. Such a catalogue could help future mission during their early stages by providing them with preliminary gravity potential models, which can be updated as more observations are gathered.

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