

Part 7. Summary



**The meteoritic inclusions in the AGB community, FANTASTIC !,
(Sachiko Amari)**

AGB Theory — A Retrospective

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1. Introduction

My assignment was to listen to all of the talks at the symposium and to discover and summarize results that have appeared since the last conference on AGB stars. Since the entire meeting was devoted to new results, discovery was a simple task. Because of the large volume of new results, an adequate summary is impossible. I therefore do not attempt to provide a comprehensive survey, but confine my remarks to a small subset of the discoveries and advances which have enriched and modified our understanding of AGB stars over the past decade.

2. Interrelationships

One of the most delightful aspects of a conference such as this, at which experts from many different specialities interact, is to see unexpected interrelationships exposed. One which particularly intrigued me was the formation of ^{26}Al at the base of the convective envelope of relatively massive AGB models, as described by Lattanzio & Forestini, and the use which Glassgold made of the positrons from the decay of ^{26}Al in the circumstellar matter at 10^{17} cm from the star to convert molecular hydrogen and the molecule CO into the radical HCO^+ . The abundance of ^{26}Al expected in matter ejected in the wind of the real analog of the AGB model appears to be sufficient to account for the abundance of HCO^+ inferred from spectroscopy and a simple spherically symmetric circumstellar shell formed by the wind.

And this is just the beginning! According to Zinner & Amari, it appears that, in meteoritic inclusions in which the $^{12}\text{C}/^{13}\text{C}$ ratio is similar to that found in AGB model envelopes, the $^{26}\text{Mg}/^{27}\text{Al}$ ratio is about the same as the $^{26}\text{Al}/^{27}\text{Al}$ ratio in the model envelopes.

These coincidences, if true, show remarkable correspondences between model predictions and observations. They imply further that isotopic abundances in some chunks of matter formed in stellar interiors and injected into a stellar wind manage to survive the arduous trip through space to the protosolar nebula and to survive further processing in this nebula. FANTASTIC!

Of course, the connection between nucleosynthesis in AGB stars and abundances in meteoritic inclusions extends far beyond this example. The beautiful work by Gallino and his coworkers on s-process nucleosynthesis has been matched by equally beautiful detective work by Amari, Anders, Lewis, Zinner, and others on inclusion compositions.

3. Luminosity core-mass relationships

In several talks, mention has been made of the “failure” of the luminosity core-mass relationship. This is misleading. First of all, since, at any given core mass, thermally pulsing AGB (TPAGB) stars vary quasi periodically in luminosity over the pulse cycle (by as much as a magnitude in low mass models), there is, strictly speaking, no such thing as a time-independent luminosity core-mass relationship.

On the basis of models by himself and by Uus (1970) in which thermal pulses were suppressed, Paczyński (1970) constructed a luminosity core-mass relationship, $L \sim 60\,000 (M_{\text{core}}/M_{\odot} - 0.5) L_{\odot}$, which is actually a very good approximation to the maximum luminosity of low mass thermally pulsing models near the end of the quiescent hydrogen shell burning phase, just prior to the helium shell flash which marks the onset of a thermal pulse. The Paczyński-Uus (PU) relationship has played an important role in the development of our understanding of real TPAGB stars by providing a useful approximation to one portion of the real time-varying relationship for low mass models. It also describes well the luminosities of the central star of a planetary nebula during the plateau phase. The relationship has on occasion been misused, when it has been forgotten that it describes only a particular point in the thermal pulse cycle.

As can be seen from several of Uus’ models and those of subsequent researchers, the PU relationship underestimates the luminosity maximum when the base of the convective envelope extends into the hydrogen-burning region. This is particularly true of relatively massive TPAGB models, the more so the larger the mixing-length to scale-height parameter l/H . This is because, for l/H larger than some critical value, a considerable fraction of the luminosity of the model during the hydrogen-burning phase is produced within the convective envelope; the larger l/H , the larger is the luminosity of the model relative to that given by the PU relationship. However, this does *not* mean that there is not a luminosity core-mass relationship analogous to the PU relationship. When sufficiently many pulses are followed (typically ~ 30 pulses are enough), the pulse amplitude reaches a locally asymptotic value, and, for every given choice of l/H , one can construct a perfectly valid luminosity core-mass relationship in the spirit of the PU relationship (see, e.g., Iben 1977). Further, the more massive the core, the larger is the fraction of the thermal pulse cycle that this relationship faithfully describes. Thus, use of the phrase “failure of *the* luminosity core-mass relationship” should be avoided.

4. Nucleosynthesis, dredge-up, and carbon stars

Nice discussions of the third dredge-up process and its consequences for carbon-star formation were given by Lattanzio & Forestini and Mowlavi. It was, however, somewhat amusing to learn that “dredge-up cannot occur in models when the Schwarzschild criterion for convection is employed” while remembering that this criterion was in fact employed in the models used to discover the third dredge-up process in the first place (Iben 1974, 1975, 1976)!

Blöcker and Lattanzio & Forestini described element and isotope nucleosynthesis in the convective envelopes of AGB models (e.g., ${}^7\text{Li}$, ${}^{13}\text{C}$, ${}^{17}\text{O}$, ${}^{18}\text{O}$, ${}^{26}\text{Al}$) and debated the relative effectiveness of hot bottom burning and dilution in the convective envelope in accounting for the apparent paucity of very luminous carbon stars. It was pointed out that luminous (“dust-enshrouded”) carbon stars, although rare, do exist (Kastner et al. 1993), and that this can be explained by the fact that the temperature at the base of the convective envelope of a mass-losing model at first increases, but then decreases as the mass of the hydrogen-rich envelope decreases below some critical value.

Not highlighted at the symposium was the very important discovery by Straniero et al. (1995) that, during the quiescent hydrogen-burning phase, ${}^{13}\text{C}$ existing in the radiative helium zone between the CO or ONe core and the hydrogen-burning shell actually experiences the ${}^{13}\text{C}(\alpha, n){}^{16}\text{O}$ reaction and serves as a neutron source for s-process nucleosynthesis prior to the formation of the convective shell during a helium shell flash. Blöcker described the equally important discovery (Herwig et al. 1997) that overshoot at the base of the convective envelope during the third dredge-up mixes hydrogen and carbon together in such a way that a pocket of ${}^{13}\text{C}$ is formed by hydrogen burning after dredge-up, and this provides the first step in the now standard way of producing s-process isotopes in low mass, TPAGB stars of population I (Gallino et al. 1998).

Herwig et al. pointed out that, if overshoot occurs at the base of the convective envelope, it should also occur at the base of the convective shell during the helium shell flash. This allows carbon and oxygen to diffuse into the convective shell, decreasing the abundance by mass of ${}^4\text{He}$ and raising the abundances of both ${}^{12}\text{C}$ and ${}^{16}\text{O}$ above the abundances by mass ($X_4 \sim 0.75^+$, $X_{12} \sim 0.20^+$, $X_{16} \sim \text{a few} \times 0.01$) found when overshoot is neglected. This might help in understanding the abundances ($X_{12} \sim 0.5$, $X_4 \sim 0.35$, $X_{16} \sim 0.17$) typical of PG1159 stars (Werner et al., this volume), if they are made according to the born-again scenario (Iben et al. 1983; Iben & MacDonald 1995). It is reasonable to call this additional mixing process at the base of the convective shell the “fourth” dredge-up process.

5. Pulsation mode

The debate about the dominant radial pulsation mode of Mira variables has gone on for at least three decades and, judging from the presentations by Wood et al., Tuchman, and Feast, it promises to continue. During the seventies, Lee Ann Willson consistently and forcefully defended the fundamental mode and Peter Wood consistently favored the first overtone. At some point Peter experienced a conversion to the fundamental, and he presented observational evidence here to support this conversion. Tuchman described results of theoretical hydrodynamical calculations which suggest that, for model parameters chosen to fit those of Mira variables, the full-amplitude mode surviving after many pulsation periods tends to be the fundamental. However, the first overtone still has its champions, and Michael Feast made the case for the first overtone, using stellar angular diameters measured by interferometry and/or lunar occultations coupled with distances determined by HIPPARCOS to estimate stellar radii.

A possible resolution of the discrepancy in its present incarnation may lie in a fuller understanding of what “radius” the different types of observation are measuring, as compared to the theoretical radius R used in the determination of Q in the equation $P = Q R^{3/2}/M^{1/2}$. A large Q implies pulsation in the fundamental mode; a small Q implies pulsation in the first overtone. The theoretical pulsation equation assumes that R is the radius at optical depth $2/3$, and this corresponds to the value of R obtained from $\sigma T_e^4 = L/4\pi R^2$, where T_e is estimated photometrically. On the other hand, the angular diameter measurements may estimate something larger than R at optical depth $2/3$, for the same reason that, when viewing a thin spherical shell which is optically thin in the radial direction, one does not see a disk, but rather a ring which has a larger diameter than the inner diameter of the ring (optical images of the Ring Nebula are classical examples of this effect). A smaller R implies a larger Q , which moves in the direction of favoring pulsation in the fundamental mode.

6. Winds — Pulsations, shocks, dust, radiation pressure, etc.

The basic ingredients of the currently most popular theory of mass outflow from AGB stars were established by the early eighties. Acoustical pulsation calculations by Wood (1976, 1979), Hill & Willson (1979), and Willson & Hill (1979) demonstrated that, although shocks formed during pulsations can by themselves drive matter from the surface, they can not do so at a sufficiently high rate to account for the observations. Some other mechanism such as radiation pressure on grains formed in the atmosphere must assist in the mass-loss process. That dust grains form in atmospheres of cool giants was known from the observations (Woolf & Ney 1969; Gehrz & Woolf 1971), and the theory of grain nucleation and growth was evolving (e.g., Draine 1981). Bowen (1988) put all of the ingredients together to produce the first self-consistent theoretical paradigm for AGB winds: shocks due to large amplitude acoustical pulsations create a dynamic, inflated atmosphere in which grains can form and grow, radiation pressure drives the grains outward, and the grains drag the gas along.

At this meeting, Fleischer et al. and Höfner described results of extensive hydro-dynamical calculations which have developed the paradigm to a high degree of sophistication. For example, in the case of carbon-rich atmospheres, the interplay of shocks, dust formation, dust evaporation, radiative transfer and absorption, and grain acceleration can lead to multimode, asymmetric light curves, even when the “piston” which replaces the acoustical pulsations in the real AGB analog is driven with a perfectly symmetric sinusoidal velocity (Winters et al. 1994).

Crucial to the further development of the paradigm is a theory of grain nucleation and growth consistent with results of laboratory experiments. Thanks to the importance of carbon-rich molecules in the fuel industry, an extensive experimental foundation for understanding nucleation and grain growth is available for situations in which $C/O > 1$ (Henning, this volume). Unfortunately, the classical nucleation theory which has been used extensively in the context of AGB atmospheres apparently fails completely to explain the results of the laboratory experiments. An adequate laboratory foundation for a theory of grain nucleation and growth when $O/C > 1$ is not yet available, but the experience

with carbon-rich environments suggests that classical theory may fail also when $O/C > 1$. The assessment by Henning led to spirited interchanges between members of the audience and speakers Cherchneff, Jeong, and Kozasa who presented theoretical models for nucleation and grain growth.

Lodders & Fegley gave a very nice review of the results of thermochemical equilibrium calculations under average conditions found in the detailed model atmospheres given by the hydrodynamical calculations, emphasizing that, for compounds which are not expected to be affected by processing in the ISM, the solar nebula, and in meteorites, the relative abundances found in some meteoritic inclusions are consistent with those given by the equilibrium calculations. This is encouraging support for the utility of the equilibrium calculations, but it does not mean that time-dependent calculations can be avoided in developing atmospheric models and understanding the process of mass loss and the observed spectral variations of real TPAGB stars. Winters (Winters, Le Bertre & Keady, this volume) showed quite convincing evidence for the temporal evolution of the grain-size distribution function in stars such as IRC +10216, and this variation must play an important role in the details of mass loss.

There may be mechanisms other than the pulsation-shock-grain-radiation pressure paradigm at work in driving mass loss from TPAGB stars. Bujarrabal pointed out that, for hundreds of variable AGB stars for which reliable estimates have been made, inferred mass-loss rates lie typically in the range of a few $\times 10^{-8} M_{\odot} \text{ yr}^{-1}$ to a few $\times 10^{-5} M_{\odot} \text{ yr}^{-1}$, whereas indirect evidence provided by the distribution and kinematics of circumstellar matter in post-AGB, proto-planetary stars suggest a final mass-outflow rate of the order of $\sim 5 \times 10^{-4} M_{\odot} \text{ yr}^{-1}$, perhaps 10-20 times larger than the mass-outflow rate of IRC +10216, everyone's favorite Mira wind. One solution to the puzzle may lie in the structure which develops near the hydrogen-helium discontinuity in the interior following a helium shell flash when the mass of the electron-degenerate core becomes large enough and the mass of the hydrogen-rich envelope declines below a critical value. In a poster paper which describes results of his theoretical quasi-static model calculations, Alan Sweigart shows that, under appropriate conditions of core mass and envelope mass, the gas pressure relative to the radiation pressure ($P_{\text{gas}}/P_{\text{rad}}$) at a point near the hydrogen-helium discontinuity reaches a minimum which decreases in value with each successive pulse. He infers that the minimum will go to zero (which numerical difficulties prevent him from demonstrating explicitly) and that matter beyond the minimum is effectively detached from the interior and flies off as the superwind proposed by Renzini (1979).

7. Mass transfer and chemical peculiarities in binary systems with TPAGB donors

The less evolved component in a close binary can acquire chemical peculiarities by accreting matter from a TPAGB star companion which has developed its own chemical peculiarities in consequence of nucleosynthesis during successive helium shell flashes followed by dredge-up. In turn, the nebular material ejected by the TPAGB star can be shaped by interaction with the companion star. If the TPAGB star is single or in a wide binary, the ejected material can also be

shaped by a planetary system, a cometary system, or a combination of these systems. Here, I will review only the mass-transfer phenomenon.

Ba stars and their population II counterparts, the CH stars, occur in binary systems in which the secondary is typically a low mass white dwarf (McClure, Fletcher & Nemec 1980; McClure 1983; McClure & Woodsworth 1990). The fact that the typical orbital separation of binaries with Ba star components is ~ 2 AU (McClure 1983), coupled with the fact that a TPAGB star with a hot white dwarf core of mass $\sim 0.6 \pm 0.05 M_{\odot}$ has a mean radius of the order of $\sim 1.06 \pm 0.37$ AU, shows that mass transfer from the TPAGB progenitor of the white dwarf to its companion does not, in general, result in a dramatic decrease in orbital separation, as might be expected if the donor fills its Roche lobe with a deep convective envelope and/or if the mass ratio q of donor to accretor exceeds ~ 0.67 during the mass-transfer episode. A possible inference is that, in some instances, the TPAGB star does not fill its Roche lobe during most of the mass-transfer episode, with mass transfer being primarily by accretion from the wind of the TPAGB star (Boffin & Jorissen 1988; Jorissen & Boffin 1992; Jorissen et al. 1998). In other instances, even when the TPAGB star fills its Roche lobe, the self-sustained wind from the donor may result in much more matter being lost from the system than is transferred to the accretor, with the tendency toward orbital shrinkage by common envelope (CE) action being countered by a tendency toward expansion occasioned by system mass loss; in the absence of any accretion, the product of the orbital separation and the total mass of the system would remain constant. For still other systems, because of extensive mass loss from the TPAGB star prior to Roche-lobe filling, $q < 0.67$ when the donor eventually fills its Roche lobe, permitting mass transfer to continue stably. In this case, the mass-transfer rate may be accelerated during the brief periods of radial expansion that follow helium shell flashes.

S stars (giants with C/O ratios intermediate between M stars and C stars) which do not show lines of Tc form another class of objects, all of which appear to be in binaries (Brown et al. 1990; Jorissen et al. 1993; Johnson, Ake & Ameen 1993). At this meeting, Jorissen reviewed the most recent findings regarding the binary characteristics of S stars, including the fact that, in the Galaxy, S stars showing lines of unstable Tc form a class whose members are, in general, brighter than members of the class of S stars which do not show Tc lines (Van Eck et al. 1998). Thus, S stars with no Tc are first giant branch stars with a white dwarf companion and S stars with Tc are TPAGB stars which have dredged up Tc from their own interiors. The S stars with Tc may be single stars, members of binaries with less evolved companions, or members of binaries with white dwarf companions.

The distributions of binary Ba, CH, and S stars in the eccentricity-period (e - P_{orb}) diagram, when compared with the distribution in this same plane of binaries with spectroscopically normal G and K giant components, contain important information about the nature of mass transfer when the donor is a TPAGB star (Jorissen et al. 1998). For periods $P_{\text{orb}} > 300$ days, binaries with normal G and K giants are distributed roughly uniformly in e over the range $0.05 < e < 0.9$, and, for periods $P_{\text{orb}} < 300$ days, roughly half have $e \sim 0.0$. This shows that binary stars are born with a wide distribution of eccentricities and that orbits of binaries containing giants which do not support a strong wind

are circularised by tidal forces when the radius of the giant is comparable to its Roche-lobe radius. Binaries containing Ba, CH, or S giants and having periods as large as ~ 5000 days have $e < 0.5$, suggesting that a frictional interaction between component stars and matter in the wind from a TPAGB precursor may have succeeded in reducing the eccentricity of the primordial binary (remember that the lifetime of a TPAGB star is ~ 100 times shorter than the lifetime of a first red giant branch star and that a typical TPAGB star in a 5000 day binary is far from filling its Roche lobe).

Binaries containing Ba stars and with $P_{\text{orb}} < 600$ days have $e < 0.2$. Roughly half of the binaries containing Ba stars and most binaries containing strong Ba stars and with $P_{\text{orb}} = 200\text{--}2000$ days have orbits consistent with $e \sim 0.0$. Most binaries containing mild Ba stars and with $P_{\text{orb}} > 600$ days and most binaries containing strong Ba stars and with $P_{\text{orb}} > 2000$ days have $0.1 < e < 0.4$. For S stars and CH stars, the statistics are not as good, but there is a conspicuous break in the e distribution at $P_{\text{orb}} \sim 1000$ days; binaries with $P_{\text{orb}} < 1000$ days have distinctly smaller e than those with $P_{\text{orb}} > 1000$ days. Even for $P_{\text{orb}} > 1000$ days, however, binaries containing CH stars have $e < 0.2$ and binaries containing S stars have $e < 0.4$. These trends reinforce the suspicion that a frictional interaction between stellar components and the intrinsic TPAGB wind contributes to a reduction in the eccentricity of the primordial orbit.

Jorissen et al. (1998) argue that, in the Galaxy, strong Ba stars belong to a more metal-weak population than do mild Ba stars. This inverse correlation may be related, in part, to the fact that the frequency of carbon stars decreases in passing from the SMC to the LMC and then to the Galaxy (Blanco, McCarthy & Blanco 1980); the abundances of oxygen and of iron increase in passing through the stellar aggregates in the stated order, but the abundance of carbon (and possibly also of s-process isotopes) in matter dredged up following a thermal pulse is (may be) more or less independent of the metallicity. The relative ease with which metal-weak TPAGB stars produce carbon stars has an important consequence for comparisons between theoretical models of the third dredge-up and the distribution of carbon stars in low metallicity systems. As Jorissen emphasized, the carbon star distribution in the SMC (Westerlund et al. 1995) extends approximately two magnitudes fainter (to ~ -2.3 mag) than the distribution found in earlier, less deep surveys (Blanco et al. 1980; Westerlund et al. 1991). The earlier surveys gave a low luminosity cutoff which is more or less consistent with the luminosity of low mass TPAGB stars at minimum during a thermal pulse cycle at the onset of the TPAGB phase. The significance of the newer survey is that (1) most of the SMC carbon stars with luminosity in the range $M_{\text{bol}} \sim -3.5$ mag to -2.3 mag are first red giant branch stars with white dwarf companions and that (2) the carbon star distribution does not clearly define the low luminosity cutoff for TPAGB stars of low metallicity.

Among binaries consisting of a white dwarf and a chemically peculiar giant, there are many which are close enough that the TPAGB precursor of the white dwarf must have filled its Roche lobe, but which are wider than would be expected if unstable mass transfer led to the formation of a CE. HD 77247 with $P_{\text{orb}} \sim 80.5$ days and $e = 0.09$ is a conspicuous example. A similar phenomenon may account for the orbital characteristics of the double white dwarf

L870-2, which has a relative long period of ~ 1.5 days (Saffer, Liebert & Olszewski 1987, 1988) even though the precursor binary has passed through two phases of mass transfer during which the donor overfilled its Roche lobe. The mass donor in the first mass-transfer event was probably a TPAGB star, and the energy required to expel the envelope matter from the binary system is greater than can be explained by the increase in the orbital binding energy (Iben & Webbink 1989; Iben & Livio 1993). The inference is that, when the Roche-lobe filling star is a TPAGB star with a strong intrinsic wind, $\dot{M}_{\text{wind}} \gg \dot{M}_{\text{transferred}}$, and the tendency for expansion associated with mass loss competes successfully with the frictional interaction between stars and wind matter to reduce the degree of orbital shrinkage which would occur in the absence of the intrinsic wind. Further support for this interpretation comes from the fact that close binary central stars in planetary nebulae originate typically in initial binaries in which the primary fills its Roche lobe in an early case C event, before the TPAGB, dusty-wind phase (Iben & Tutukov 1993).

Using data obtained from infrared and MACHO observations, Wood et al. presented the distribution of LMC long period variables in the magnitude-period plane. Stars in his sample fall into five rather clearly separated sequences, three of which correspond to radial pulsation in the fundamental mode and in the first two overtone modes. A fourth (poorly populated) sequence consists of contact binaries on the first giant branch for low mass stars. In addition, some objects with contact binary-like light curves exist in the equivalent evolutionary phase for intermediate mass stars, i.e., at the end of the core helium-burning loops. The fifth, longest period sequence, is interpreted by Wood et al. as an orbital period sequence, with median period ranging from $P_{\text{orb}} \sim 100$ days at the smallest luminosities to $P_{\text{orb}} \sim 1000$ days at the largest luminosities. If one doubles the periods as plotted by Wood for members of the fourth sequence, these members define a low-luminosity extension of the fifth sequence. Fully 25% of all variables in the sample are in the fifth sequence. At any luminosity, the scatter in the logarithm of the orbital period about the median is only $\sim \pm 0.15$; stars in the fifth sequence appear to be binaries with a period such that the TPAGB star component only slightly overfills or underfills its Roche lobe (by, say, $\sim \pm 25\%$). Once again, the inference is that, when the donor is a TPAGB star with a strong intrinsic wind, and whether or not the donor fills its Roche lobe, the conventional picture of mass transfer through a Roche lobe is superseded by wind mass loss from the system and accretion by the secondary at a rate small compared with the wind mass-loss rate.

The most puzzling aspect of Wood's fifth (binary) sequence is the small dispersion in orbital period at a given luminosity. One might expect initial main-sequence binaries to be distributed rather evenly in $\log A$ (and therefore in $\log P_{\text{orb}}$). Yet orbital separations of the evolved binaries in the fifth sequence are very sharply concentrated at a value apparently determined by the radius of the TPAGB component. What could funnel binaries into this narrow range? And how could there be so many of them?

8. AGB stars with Oxygen-Neon cores

Not discussed at this meeting except in this summary is work that has been done to estimate (1) the initial mass marking the transition between stars which become AGB stars with CO cores and those which become AGB stars with ONe cores and (2) the initial mass marking the transition between stars which ultimately become white dwarfs and those which become neutron stars after first being TPAGB stars.

There are only two sets of studies which address the first of these transitions by actually calculating the conversion by carbon burning of a partially electron-degenerate CO core into a fully electron-degenerate ONe core. The first set (Nomoto 1984, 1987) examines the evolution of the core and the helium layer above it, treating the hydrogen-rich envelope as a boundary condition. The second set explores the evolution of complete models for a population I composition and masses $9 M_{\odot}$ (García-Berro, Ritossa & Iben 1997, paper III), $10 M_{\odot}$ (Ritossa, García-Berro & Iben 1996, paper II), $10.5 M_{\odot}$ (Iben, Ritossa & García-Berro 1997, paper IV), and $11 M_{\odot}$ (Ritossa, García-Berro & Iben 1999, paper V). The paucity of relevant models is probably due to the fact that, when carbon ignites off center, as it does in most of the models, the burning front narrows significantly as it evolves toward the center, and resolving the front to the extent necessary for model convergence requires that the time step be reduced for each successive model; Zeno's paradox is in practice averted because the thickness of the front remains finite as the distance of the front from the center goes to zero. In any case, when $Z = 0.02$ and $Y = 0.28$, the transition mass for models which become TPAGB stars with ONe cores rather than CO cores is slightly smaller than $9 M_{\odot}$.

The condition for the second transition (between white dwarf and neutron star products) is that, when the mass of the ONe core reaches $\sim 1.37 M_{\odot}$, electron captures on ^{24}Mg and ^{16}O lead to core collapse to neutron star dimensions (Miyaji et al. 1980; Miyaji & Nomoto 1987). Paper V finds that, when $Z = 0.02$ and $Y = 0.28$, the second transition mass is slightly smaller than $11 M_{\odot}$.

Table 1 summarizes several characteristics of the models described in papers II-V at the end of the carbon-burning phase. In this table, M_{*} is the model mass, M_{ONe} is the mass of the ONe core, defined as the point at which the abundances of ^{12}C and ^{20}Ne are equal, ΔM_{CO} is the mass of the CO layer above the ONe core, and $\Delta M_{\text{Ne free}}$ is the mass of a "neon-free" zone, defined as the region over which all neon isotopes are less abundant by number than 10^{-4} . All masses are in solar units. The neon-free zone resides between the CO layer and the hydrogen-rich envelope. Its significance is that, if a white dwarf of this composition is in a cataclysmic variable, there will be a sequence of nova outbursts such that Ne will be underabundant in the nova ejecta.

Also shown in Table 1 are the mass M_{WD} of the white dwarf expected if the real analog loses mass at the average rate of $10^{-4} M_{\odot} \text{ yr}^{-1}$ during the TPAGB phase. The real analog of the $11 M_{\odot}$ model will become a neutron star before it loses its hydrogen-rich envelope.

The final entry in Table 1, $(X_{\text{C}})_{\text{max}}$, is the maximum abundance by mass of ^{12}C left in the ONe core at the end of carbon burning. Its significance is that, if a white dwarf of this composition is one of a merging pair of white dwarfs or

if it can accrete matter from a companion with a hydrogen-rich or helium-rich envelope until its mass reaches the Chandrasekhar limit, heat released when carbon is ignited in the ONe core could trigger the ignition of oxygen, with a resulting disruption of the star rather than a collapse into a neutron star.

Table 1. Properties of models at the end of the carbon-burning phase

M_*	M_{ONe}	ΔM_{CO}	$\Delta M_{\text{Ne free}}$	M_{WD}	$(X_{\text{C}})_{\text{max}}$
9	1.066	0.050	0.014	1.16	0.048
10	1.190	0.015	0.009	1.25	0.012
10.5	1.263	0.0065	0.0036	1.31	0.006
11	1.36764	0.00082	0.00065	NS	0.000018

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**Thibaut Le Bertre, only now realizing the forthcoming work
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