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A technique is presented for characterizing the ionization structure and consequent thermal x-ray emission of a SNR when non-equilibrium ionization effects are important. The technique allows different theoretical SNR models to be compared and contrasted rapidly in advance of detailed numerical computations. In particular it is shown that the spectrum of a Sedov remnant can probably be applied satisfactorily in a variety of SNR structures, including the reverse shock model advocated by Chevalier (1982) for Type I SN, the isothermal similarity solution of Solinger, Rappaport and Buff (1975), and various inhomogenous or 'messy' structures.

### 1. NON-EQUILIBRIUM SPECTRAL FORMATION

The ionization structure of a SNR is solved (cf. Hamilton et al. 1983) by integrating the time-dependent collisional ionization equations in Lagrangian gas elements behind the shock front:

$$\frac{Dn(X^i)}{n_e Dt} = C(X^{i-1}, T_e) n(X^{i-1}) - \{C(X^i, T_e) + \alpha(X^{i-1})\} n(X^i) + \alpha(X^i, T_e) n(X^{i+1}) \quad (1)$$

Here  $n(X^i)$  is the density of ion  $X^i$ ,  $n$  and  $T_e$  are the electron density and temperature, and  $C(X^i, T_e)$  and  $\alpha(X^i, T_e)$  are respectively ionization and recombination rate coefficients out of and into ion  $X^i$ .

Equations (1) imply that the ionization structure of a Lagrangian gas parcel depends on its ionization time  $\tau$ ,

$$\tau = \int n_e Dt \quad (2)$$

(the integration being started from the moment the gas parcel is shocked), and on the thermal history of the gas parcel.

The thermal emission from a gas parcel  $dM$  is proportional to  $dE = n_e dM$ , and depends otherwise on the composition, ionization structure and electron temperature of the gas parcel. For a spherically symmetric SNR it is useful to define

$$E = E(<r) = \int_0^r n_e dM_r \quad (3)$$

Now suppose that the temperature dependence of ionization and emission rates could be ignored. Then the ionization structure at any point would depend only on the ionization time at that point, and the spectrum of the entire SNR would depend entirely on the variation of the ionization time  $\tau$  with  $E$  through the remnant (the contribution of any element scaling linearly with its abundance). One might expect rates to be insensitive to temperature in a strongly ionizing plasma, in which  $kT_e$  exceeds the ionization potential of dominant ions so that Boltzmann factors are unimportant (a notable exception is lines excited through inner shell processes, such as the 7keV line of iron in an ionizing plasma; such lines remain temperature sensitive and are useful as temperature diagnostics).

The main source of temperature variation in the rate coefficients is the Boltzmann factor. The effect of thermal history on ionization structure can therefore crudely be accounted for by incorporating a Boltzmann factor into ionization times, defining therefore modified ionization times  $\tau_\chi$ :

$$\tau_\chi = \int \exp(-\chi/kT_e) n_e dt \quad (4)$$

where  $\chi$  is an ionization potential. Obviously  $\tau = \tau_0$ .

## 2. COMPARISON OF SNR MODELS

Fig.1 shows graphs of modified ionization times  $\tau_\chi$  and temperature  $T$  (assuming  $T_e = T_{ion}$ ) versus  $E$  for:

- a) the Sedov solution
- b) the isothermal similarity solution of Solinger, Rappaport and Buff (1975);
- c) the outer and
- d) inner shocks in the self-similar reverse shock solution advocated by Chevalier (1982) for Type I SN, i.e. an ejected density profile  $\rho \propto r^{-7}$  moving into a uniform density medium.

Consider first the Sedov solution. Fig.1 indicates that the degree of ionization, as characterized by the ionization times, advances inward from the shock front, reaches a maximum, then declines toward the center of the SNR. At early times when Boltzmann factors are unimportant ( $\chi \ll kT$ ) the maximum ionization occurs at a point where  $E = .11 E_{total}$ , corresponding to an interior mass fraction of .30, or a fractional radius of .89. Precisely this behaviour can be seen explicitly in the detailed ionization curves for early models traced by Itoh (1979) and Gronenschild and Mewe (1981). Later on, when Boltzmann factors become

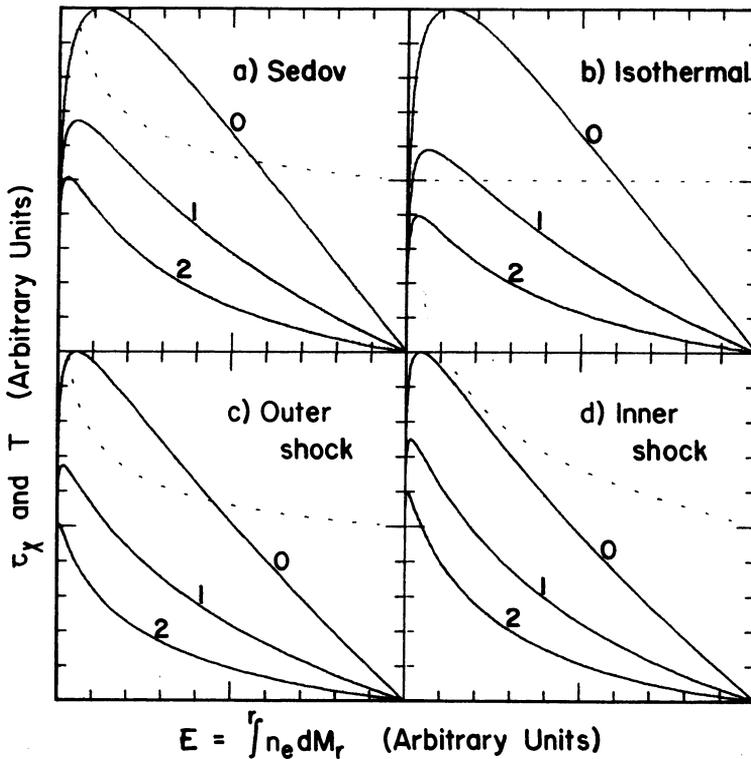


Figure 1. Modified ionization times  $\tau_X$  (—) and temperature  $T$  (---) versus  $E$  for various SNR models.  $\tau_X$  curves are labelled with values of  $\chi/kT_{\text{shock}}$ . The shock front in each case is to the right.

important, the maximum degree of ionization moves relatively inward.

The ionization curves for the other SNR models shown in Fig.1, when suitably normalized, look remarkably similar to the Sedov curves. Moreover the runs of temperature are quite similar, except that the isothermal model lacks a high temperature inner region. The inference is that the resulting x-ray spectra should be quite similar. The major differences will occur in temperature sensitive emission, i.e. in general at the harder end of the spectrum. For example the isothermal spectrum will be relatively weaker in hard x-rays than the Sedov spectrum, but the soft x-ray spectrum will be almost indistinguishable from a Sedov spectrum.

The normalization of the curves in Fig.1 is fixed by three parameters: a characteristic ionization time, a characteristic temperature, and a total emission integral. Table 1 shows the normalizations used for each model in Fig.1.

Table 1. Normalization of quantities in Fig.1.

Model	$\frac{E_{total}^{1,2}}{n_e M}$	$\frac{\tau_{max}^{1,3}}{n_e t}$	$\frac{T_{inner}^4}{T_{outer}}$
Sedov	2.07	1.37	-
Reverse shock	1.92	1.79	-
{ Outer			
{ Inner	1.58	3.25	.353
Isothermal	1.38	.92	-

<sup>1</sup>Here  $n_e$  is the 'electron' density in the ambient medium,  $n_e \approx 1.23n_H$  if  $n_H$  is the ambient hydrogen number density.

<sup>2</sup> $M$  is the total mass of shocked material. In the reverse shock model the outer shock accounts for .67 of the mass  $M$ .

<sup>3</sup> $\tau_{max}$  is the maximum value of  $\tau$ .  $t$  is the age.

<sup>4</sup>Ratio of post-shock temperatures in the reverse shock model assuming equal molecular weights.

### 3. INHOMOGENEITIES

The technique described can be used to consider emission from inhomogeneities distributed for example in some statistically homogenous way through a SNR (see Hamilton and Sarazin 1983 for further details). If the density contrast between clumps and ambient medium is not too large, then clumps can be shocked to x-ray emitting temperatures, and non-equilibrium ionization may be important.

The characteristic ionization times and temperatures of the clumped and unclumped components, having density contrast  $\kappa$ , are obviously related by

$$\begin{aligned} \tau_{clumped} &\approx \kappa \tau_{unclumped} \\ T_{clumped} &\approx \kappa^{-1} T_{unclumped} \end{aligned} \tag{5}$$

which is a constraint on possible combinations of spectra. A further constraint follows from requiring that the filling factor of the clumped component be less than unity.

### 4. STEADY STATE APPROXIMATION

In a steady state approximation in which mass enters a blast wave at a constant rate, and the density and temperature profiles behind the shock remain constant, then

$$\frac{dE}{d\tau} = \frac{n_e dM}{n_e dt} = \frac{dM}{dt} = \text{constant} \tag{6}$$

i.e.  $\tau$  varies linearly with  $E$  behind the shock front. Reference back to Fig.1 shows that a linear increase of  $\tau$  occurs through most of the emitting region in each model, suggesting that a steady-state is a good approximation there.

In the steady state approximation the emission from a line in any ionizing ion stage is just

$$\text{Emission} = \frac{\text{Excitation rate coefficient}}{\text{Ionization rate coefficient}} \times \frac{dE}{d\tau} \quad (7)$$

## 5. CONCLUSIONS

A simple technique has been presented for characterizing the ionization structure and consequent thermal x-ray emission of a SNR when non-equilibrium ionization effects are important.

It has been shown that the (soft, at least) x-ray spectrum of a Sedov remnant is probably a good approximation for  
 -each of the outer and inner shock waves in the self-similar reverse shock model advocated by Chevalier (1982) for Type I SN;  
 -the isothermal model of Solinger, Rappaport and Buff (1975);  
 -statistically uniform clumpy SNRs (= 2-component spectrum).  
 Much of the reason for the agreement is the approximate validity of a steady-state approximation in most of the emitting region. Sedov spectra will presumably be satisfactory approximations wherever this is the case.

A particular spectrum (for a given composition) is essentially characterized by three parameters, a characteristic ionization timescale, a characteristic temperature, and an emission measure. The first two parameters fix the shape of the spectrum, while the emission measure fixes the absolute level of the observed flux.

Finally it should be noted that there are many cases where a Sedov spectrum does not apply, notably where there are large systematic density gradients (e.g. a circumstellar wind) in the ambient medium.

## REFERENCES

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## DISCUSSION

WINKLER: If the Sedov model approximately reproduces the results of so many other models including the isothermal model, then could one instead use the even simpler isothermal model as the basis for approximating a more complex reality?

HAMILTON: Recall the isothermal model is isothermal in Eulerian coordinates, not Lagrangian coordinates. However, I think you could get a good approximation on the ionization structure this way if you wanted to.